Analytical approach to the photon statistics in the thermal Jaynes-Cummings model with an initially unexcited atom

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Simple analytical expressions for the time evolution of the mean photon number and its normally ordered variance for the case of an initially thermal cavity field coupled to an initially unexcited two-level atom are proposed. The possibility of appearance of sub-Poissonian field statistics is discussed.

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I. INTRODUCTION

Despite its simplicity, the standard Jaynes-Cummings model (JCM) [1] of a two-level atom coupled to a single-mode coherent quantized cavity field via one-photon transitions is able to produce a variety of unique phenomena such as collapses and revivals [2], sub-Poissonian photon statistics [3], squeezing [4], and higher-order squeezing [5,6], in the sense of both Hong and Mandel's [7] and Hillery's [8] definitions. The multiphoton JCM [9] is capable of producing not only higher-order squeezing but also higher-order intrinsic squeezing [7] for the photon multiplicity of the atomic transition ≥ 4 [5].

Knight and Radmore [10] have shown a collapse of the oscillations in the standard JCM at chaotic classical and chaotic quantum pumping. Only in the latter case does a collapsed atom revive which means that solely revivals are immediately related with the graininess of light.

Hillery [11] has determined the bounds on sub-Poissonian field statistics in the standard JCM for the case of an initially excited atom interacting with the thermal photons of a not totally cooled cavity. In a totally cooled cavity this system produces sub-Poissonian photon number statistics for all times because of the one-photon transition at every atomic jump. If the spontaneous emission takes place in a cavity in the presence of thermal photons, depending on their mean number they suppress sub-Poissonian photon statistics of the emitted field or cancel it altogether. For the one- and two-photon JCM's with an unexcited atom at $t=0$ interacting with an initially chaotic (thermal) field the bounds on sub-Poissonian statistics of the evolving field have been recently found numerically [12] and, as it could be expected, they are more restrictive than for an initially excited atom. In this case, at the onset of the interaction the atom absorbs a photon of the super-Poissonian field.

II. PHOTON-NUMBER STATISTICAL MOMENTS

The rotating-wave-approximation Hamiltonian for the model in question in the case of exact resonance reads as

$$\begin{aligned} &H=H_{\text{free}}+H_{\text{int}} , \\ &H_{\text{free}}=\hbar\omega S^3+\hbar\omega a^\dagger a , \\ &H_{\text{int}}=g\hbar[a^\dagger S^-+aS^+] , \end{aligned} \tag{1}$$

where $a^\dagger$ (a) is the photon creation (annihilation) operator and S^-, S^+, and S^3 are pseudospin lowering, raising, and inversion operators of the atom, respectively; ω denotes the frequency of the field mode while g is the atom-field coupling.

If the atom starts initially in its lower state and the field $|-\rangle$ is in a photon number state $|n\rangle$, the resulting interaction-picture wave function of the system is as follows:

$$|\Psi(t)\rangle=|-,n\rangle C_-^{(n)}(t)+|+,n-1\rangle C_+^{(n)}(t) , \tag{2}$$

where $|+\rangle$ denotes the upper state of the atom and the probability amplitudes $C_-^{(n)}(t)$ and $C_+^{(n)}(t)$ read as

$$\begin{aligned} &C_-^{(n)}(t)=\cos\Omega_n t , \\ &C_+^{(n)}(t)=-i\sin\Omega_n t , \end{aligned} \tag{3}$$

where Ω_n is the Rabi frequency of the oscillations of the system:

$$\Omega_n=g\sqrt{n} . \tag{4}$$

Due to conservation of the excitation number, the mean photon number $\langle n(t)\rangle$ carries the same information as atomic inversion. The expectation value of the photon number takes the form

$$\langle n(t)\rangle_n=\langle\Psi(t)|a^\dagger a|\Psi(t)\rangle_n=n-|C_+^{(n)}(t)|^2, \tag{5}$$

where the subscript n denotes that the field is initially in a Fock state.

In turn, the second-order statistical moment reads as

$$\langle n(t)^2\rangle_n=n^2+(1-2n)|C_+^{(n)}(t)|^2 . \tag{6}$$

In order to obtain the photon-number statistical moments for an initially chaotic field one has to sum Eqs. (5) and (6) over the geometrical photon-number distributions P_n,

$$P_n=\frac{\bar{n}^n}{(\bar{n}+1)^{n+1}} , \tag{7}$$

where $\bar{n}$ is the initial mean photon number.

On summation of the constant terms of Eqs. (5) and (6) we get

$$\langle n(t)\rangle=\bar{n}-\tfrac{1}{2}[q-S_0(t)] , \tag{8}$$

$$\langle n(t)^2\rangle=2\bar{n}^2+\bar{n}-\tfrac{1}{2}[\bar{n}(1+q)+S_0(t)-2S_1(t)], \tag{9}$$

where

$$q=\frac{\bar{n}}{\bar{n}+1} , \tag{10}$$

$$S_0(t)=(1-q)\sum_{n=1}^{\infty} q^n\cos 2\Omega_n t , \tag{11}$$

$$S_1(t)=(1-q)\sum_{n=1}^{\infty} q^n n\cos 2\Omega_n t , \tag{12}$$

and we used for the thermal field the relation $\overline{n^2}=2\bar{n}^2+\bar{n}$.

The above sums are simply calculated by means of a computer. On the other hand, it is always tempting to obtain an analytical solution to the problem. A number of successful efforts has been made in this direction for the JCM [2,10,13–18] for various types of the initial photon statistics. The most adequate analytical expressions have been given for an initially coherent field [2] (see also Yoo and Eberly [16]). The usual way of solving the coherent JCM consists in the replacement of the summation over n by integration and in the use of a saddle-point technique which is equivalent to expansion of the Rabi frequency Ω_n (4) in a Taylor series about $\bar{n}$, i.e., about the point at which Poissonian distribution is peaked, within an accuracy of $(n-\bar{n})^2$. In the case of a thermal field the geometrical distribution function has its maximum at $n=0$ which is the point of the singularity of all the derivatives of the Rabi frequency. In particular, if the atom interacts with a classical chaotic field, then the result for the time evolution of the system may be written in terms of the so-called Dawson integral [10]. Here, we propose another solution which describes extremely well the time evolution of the system under consideration for small numbers of initially thermal photons $\bar{n}<1$, i.e., when the semiclassical approach fails completely. The interval of small $\bar{n}$ is especially intriguing because of the possibility of a reduction of the quantum fluctuations of the photon number below Poissonian.

III. ANALYTICAL SOLUTION

To calculate the functions (11) and (12) we use a linear approximation in n for the Rabi frequency:

$$2\Omega_n=A_0+(n-1)A_1 \tag{13}$$

and postulate that the above equation is exact in the first two points $n=1$ and $n=2$:

$$2\Omega_1=A_0 ,\quad 2\Omega_2=A_0+A_1 . \tag{14}$$

From the definition (4) we find that

$$A_0=2g ,\quad A_1=A_0(\sqrt{2}-1) . \tag{15}$$

The function $S_0(t)$ then takes the following form:

$$\begin{aligned}S_0(t)&=\frac{1-q}{2}q\sum_{k=0}^{\infty} q^k\{\exp[i(A_0+kA_1)t]+\text{c.c.}\}\\&=\frac{1-q}{2}q\left[\frac{e^{iT}}{1-qe^{i\tau}}+\text{c.c.}\right]\\&=(1-q)q\frac{\cos[T+\phi(\tau)]}{\sqrt{D(\tau)}} ,\end{aligned} \tag{16}$$

where

$$\begin{aligned}&D(\tau)=1+q^2-2\cos\tau ,\\&\cos\phi(\tau)=\frac{1-q\cos\tau}{\sqrt{D(\tau)}} ,\\&\sin\phi(\tau)=\frac{q\sin\tau}{\sqrt{D(\tau)}} ,\\&T=A_0t ,\\&\tau=A_1t .\end{aligned} \tag{17}$$

Hence

$$\langle n(t)\rangle=\bar{n}-\frac{1}{2}q\left[1-(1-q)\frac{\cos[T+\phi(\tau)]}{\sqrt{D(\tau)}}\right] . \tag{18}$$

T determines "fast oscillations" while τ describes a "slow envelope." Obviously, the difference between these two scales is not so significant. Nevertheless, T and τ intervene differently in the formula (18) and, in fact, $1/\sqrt{D}$ determines the envelope of the oscillations while $\phi(\tau)$ determines their phase:

$$\text{env}\langle n(\tau)\rangle=\bar{n}-\frac{1}{2}q\left[1\mp(1-q)\frac{1}{\sqrt{D}(\tau)}\right] . \tag{19}$$

In turn

$$\begin{aligned}S_1(t)&=\frac{(1-q)q}{2}\frac{\partial}{\partial q}\left[q\sum_{k=0}^{\infty}q^k\{\exp[i(A_0+kA_1)t]+\text{c.c.}\}\right]\\&=(1-q)q\frac{\partial}{\partial q}\left[\frac{q\cos[T+\phi(\tau)]}{\sqrt{D(\tau)}}\right] .\end{aligned} \tag{20}$$

Making use of the relations

$$\frac{\partial D}{\partial q}=2(q-\cos\tau),$$
$$\frac{\partial \cos\phi}{\partial q}=-\frac{\sin\phi\sin\tau}{D}, \tag{21}$$
$$\frac{\partial \sin\phi}{\partial q}=\frac{\cos\phi\sin\tau}{D},$$

we get

$$S_1(t)=(1-q)q\frac{\cos[T+2\phi(\tau)]}{D(\tau)}. \tag{22}$$

The normally ordered photon-number variance is, by definition,

$$V(t)=\langle n(t)^2\rangle-\langle n(t)\rangle^2-\langle n(t)\rangle, \tag{23}$$

and in our linear approximation reads as

$$V(t)=\bar{n}^2-q\left\{\frac{\cos[T+\phi(\tau)]}{\sqrt{D(\tau)}}-(1-q)\frac{\cos[T+2\phi(\tau)]}{D(\tau)}+\frac{1}{4}q\left[1-(1-q)\frac{\cos[T+\phi(\tau)]}{\sqrt{D(\tau)}}\right]^2\right\}. \tag{24}$$

$V=0$ indicates Poissonian; $0<V<\bar{n}^2$, super-Poissonian; $V=\bar{n}^2$, chaotic; $V>\bar{n}^2$, superchaotic; and $V<0$, sub-Poissonian light. The fields with $V<0$ have no classical counterparts.

Figure 1 shows the oscillations of the exact $\langle n(t)\rangle$ (8) and the envelope of the approximate $\langle n(t)\rangle$ (19). The exact curve demonstrates the modulated oscillations. For the initial photon number assumed, agreement between the exact oscillation amplitudes and the approximate ones is excellent. Had we plotted the graph of the oscillations of the approximate mean photon number $\langle n(t)\rangle$ (18), they would have coincided with the exact ones on thee scale of the graph. On the other hand, Eq. (18) approximates pretty well the real situation even for $\bar{n}>1$, at least for times comprising the initial collapse and the first revival (Fig. 2). Equation (18), being a periodical function, exhibits subsequent collapses and revivals. In fact, the second collapse never appears; the subsequent revivals strongly overlap giving rise to a very irregular time behavior of the system [10].

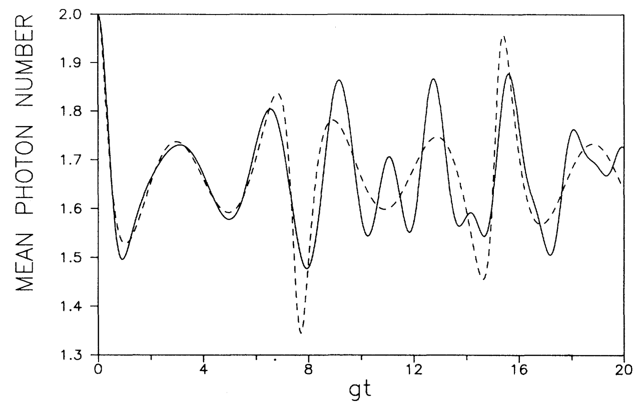

FIG. 2. Time evolution of the mean photon number for $\bar{n}=2$. The solid line corresponds to the exact solution (8) while the dashed line corresponds to the approximate result (18).

In Fig. 3 the oscillations of the exact variance (23) and the approximate variance (24) are presented. The subtle differences between the curves are already seen on the scale of the graph. Our approximate method gives a slightly worse description of the photon-number variance than the mean photon number, namely, the mean photon number depends only on the auxiliary function S_0 while the variance (24) depends additionally on the auxiliary function S_1 which contains the term proportional to the derivative of the function S_0 over q. The variance starts to decrease at the onset of the interaction, but the first minimum is positive pointing to super-Poissonian photon statistics. Later on, the photon statistics becomes even superchaotic and still later becomes sub-Poissonian.

Let us estimate the bound on sub-Poissonian photon statistics. The normally ordered variance takes its deepest negative minima for $\cos T\simeq 1$ (Fig. 3). On retaining only the terms linear in q in the braces of Eq. (24), we arrive at

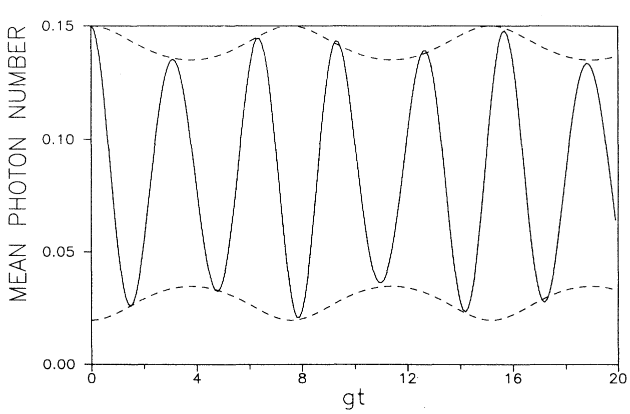

FIG. 1. Oscillations of the exact mean photon number (8) and the envelope of the approximate $\langle n(t)\rangle$ (19) for $\bar{n}=0.15$.

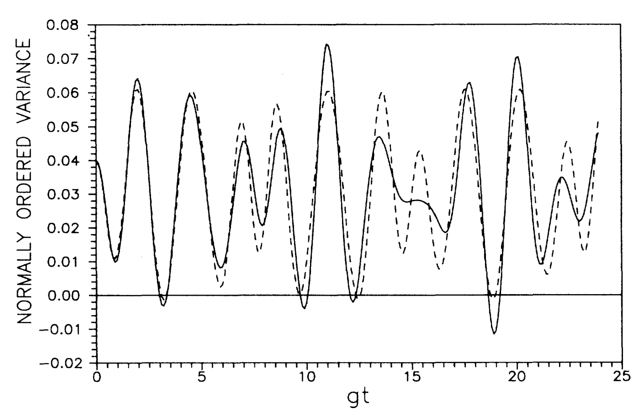

FIG. 3. Comparison of the exact normally ordered variance (23) and the approximate one (24) for $\bar{n}=0.2$.

$$V \simeq \bar{n}^2 - q^2(1-\cos\tau) . \quad (25)$$

Sub-Poissonian photon statistics occurs if

$$\bar{n}^2 + 2\bar{n} + \cos\tau < 0 . \quad (26)$$

Putting $\cos\tau \simeq -1$ we maximize the value of $\bar{n}$ at which sub-Poissonian photon statistics can still be observed. Then one obtains the following bound: $\bar{n} < 0.41$ in good agreement with the numerically found limit $\bar{n} < 0.32$ [12]. Strictly speaking, for $\bar{n} = 0.32$ sub-Poissonian photon statistics can still be noticed numerically, albeit in times unrealistic from the experimental point of view, namely, for $gt > 400$.

Recently, Rempe, Schmidt-Kaler, and Walther [19] have experimentally revealed the sub-Poissonian photon statistics of the micromaser field. The results presented in this paper correspond to a beam of atoms excited to their lower micromaser level and injected into the not totally cooled cavity.

The JCM is often referred to in the literature as "lying at the heart of quantum optics" or as "being the core or the heart of quantum optics." From our present considerations it results that this model, even with an initially unexcited atom, is also able to turn chaotic light into sub-Poissonian light which only confirms, to some extent, that these poetic terms are actually relevant.

Similar simple analytical calculations could be done for the multiphoton Jaynes-Cummings models described by the effective Hamiltonian [9]. Obviously, in the results based on this Hamiltonian the dynamic Stark shifts of the lower and upper atomic levels due to the transitions to the intermediate levels would be ignored. This problem for the two-photon model has been widely discussed in papers [20,21].

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