Results from the nonrelativistic dipole-approximation theory of two-photon electron bremsstrahlung in the Coulomb field

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We extend previous nonrelativistic dipole-approximation theoretical studies of two-photon electron bremsstrahlung in the Coulomb field by calculations exploring the cross section that describes the emitted photons, irrespective of the direction of the scattered electron, and its dependence on the charge Z of the target, for different detection geometries. A Born-approximation equation, valid for any emitted-photon configuration, is derived. Comparisons with the most recent experimental data are presented.

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I. INTRODUCTION

Although the process of simultaneous emission of two photons by an electron moving in the presence of an atomic field was described years ago [1], its quantitative treatment is limited to some relativistic Born-approximation calculations [2] and to exact nonrelativistic dipole-approximation calculations [3,4], all of them considering the Coulomb potential case. The soft-photon behavior of two-photon bremsstrahlung [5] and even-N-photon bremsstrahlung [6] have been investigated recently.

Altman and Quarles [7] have reported the first observation of two photons emitted simultaneously by electrons interacting with solid thin targets. New experimental data came out from the measurements made by Hippler [8] using gaseous targets, and by Kahler, Liu, and Quarles [9].

A reinterpretation of the first experiment [10] has reduced the disagreement between experiment and theory. The new experimental results of Hippler [8] and of Kahler, Liu, and Quarles [9] have been compared only partially with theory, because of lack of theoretical data.

It is the purpose of our paper to present new results from the nonrelativistic theory of two-photon bremsstrahlung in the Coulomb field of a fixed charge Z. This model could still serve as a useful guide in the interpretation of the complex experiments which detect photons in coincidence. In contrast to Refs. [3] and [4], where only one particular geometry was studied, now we consider the cross sections for more than one configuration of the experiments. In Sec. II we define the cross sections we are interested in. We present in Sec. III a formula, Eq. (6), which represents the result of an analytic nonrelativistic Born-approximation calculation, valid for any final photon configuration. Section IV is devoted to the comparison of both exact Coulomb and Born-approximation results with the most recent experimental data. In Sec. V we explore more some features of the two-photon bremsstrahlung in the Coulomb field, especially (a) the Z dependence of the cross sections beyond the Z^2 dependence predicted by the Born approximation; (b) the dependence on the geometry of the experiment.

II. DIFFERENTIAL CROSS SECTIONS FOR TWO-PHOTON BREMSSTRAHLUNG

The starting point for the description of electron two-photon bremsstrahlung in the field of a fixed charge Z is the Kramers-Heisenberg (KH) matrix element, denoted by M, between two continuum states of the electron in the Coulomb field [11,12]. The fivefold differential cross section is

$$\frac{d^5\sigma}{dk_1dk_2d\Omega_1d\Omega_2d\Omega_e}=\frac{1}{2}\frac{r_0^2}{E_1m_ec^2}k_1k_2|M|^2 , \qquad (1)$$

where $d\Omega_1$, $d\Omega_2$, and $d\Omega_e$ are solid angle elements referring, respectively, to the photons and electron directions; k_1 and k_2 are the energies of the two photons.

The quantity relevant for the experiments performed up to now, in which the electron direction and the polarizations of the photons have not been observed, is

$$\frac{d^4\sigma}{dk^2d\Omega^2}=\int_0^{\pi}\int_0^{2\pi}\frac{d^5\overline{\overline{\sigma}}}{dk_1dk_2d\Omega_1d\Omega_2d\Omega_e}\sin\theta_e d\theta_e d\phi_e , \qquad (2)$$

where $d^4\sigma/dk^2d\Omega^2$ is an abbreviation for the fourfold differential cross section $d^4\overline{\overline{\sigma}}/dk_1dk_2d\Omega_1d\Omega_2$. The double bar means summation over the polarization vectors $\mathbf{s}_1$ and $\mathbf{s}_2$ of the photons. The angles θ_e and ϕ_e describe the final electron direction in a reference frame with the Oz axis along the initial electron momentum, and with the xOz plane containing the photon momentum of frequency k_1 [13].

In the following, we shall denote by $\mathbf{n}_1$ and $\mathbf{n}_2$ the unit vectors along the photon directions, and by q_{12}, q_{01}, q_{02}, q_{e1}, and q_{e2} the following quantities:

$$q_{12}=\mathbf{n}_1\cdot\mathbf{n}_2=\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2\cos\Phi ,$$
$$q_{0i}=\mathbf{n}_i\cdot\hat{\mathbf{p}}_1=\cos\theta_i,\quad q_{ei}=\mathbf{n}_i\cdot\hat{\mathbf{p}}_2,\quad i=1,2 . \qquad (3)$$

θ_1 and θ_2 are, respectively, the polar angles of $\mathbf{n}_1$ and $\mathbf{n}_2$, and Φ is the angle between their azymuthal planes.

In Ref. [4] we have given cross sections valid only for the configuration of Altman and Quarles [7] and Hippler [8] experiments, i.e., for

$$q_{12}=-1\ ,\quad q_{01}=q_{02}=0\ . \tag{4}$$

The integration on θ_e in (2) cannot be performed analytically.

III. BORN NONRELATIVISTIC DIPOLE APPROXIMATION RESULTS

The first-order Born results can be obtained from the exact nonrelativistic results in the approximation $\eta_1 \ll 1$, $\eta_2 \ll 1$, with $\eta_{1,2}=\alpha Z m_e c/p_{1,2}$ (m_e is the electron mass; c is the velocity of light).

The KH matrix element has a very simple expression in the first-order Born approximation:

$$M^B(\Omega)=-\frac{\alpha Z c}{2\pi^2}\frac{(p_1p_2)^{1/2}}{k_1k_2}\frac{\mathbf{s}_1^*\boldsymbol{\Delta}\mathbf{s}_2^*\boldsymbol{\Delta}}{\Delta^2}\ . \tag{5}$$

$\boldsymbol{\Delta}$ is the momentum transfer. Based on Eq. (5), the integration in Eq. (2) leads to a simple expression for the cross section:

$$\left[\frac{d^4\overline{\overline{\sigma}}}{dk^2d\Omega^2}\right]^B=\frac{r_0^2}{64\pi^2}\frac{(\alpha Z)^2}{k_1k_2}\left[ax+bx^3+cx^5+(d+ex^2)\times(1-x^2)^2\ln\frac{1+x}{1-x}\right], \tag{6}$$

where $x=p_2/p_1$, and the coefficients have the expressions

$$a=2[25-21(q_{01}^2+q_{02}^2)+27q_{01}^2q_{02}^2-4q_{01}q_{02}q_{12}+2q_{12}^2],$$

$$b=-\tfrac{2}{3}[26-58(q_{01}^2+q_{02}^2)+190q_{01}^2q_{02}^2-88q_{01}q_{02}q_{12}+4q_{12}^2]\ ,$$

$$c=2[1-5(q_{01}^2+q_{02}^2)+35q_{01}^2q_{02}^2-20q_{01}q_{02}q_{12}+2q_{12}^2]\ , \tag{7}$$

$$d=7-11(q_{01}^2+q_{02}^2)+5q_{01}^2q_{02}^2-4q_{01}q_{02}q_{12}-6q_{12}^2\ ,$$

$$e=-c/2\ ,$$

where q_{01}, q_{02}, and q_{12} are the scalar products in (3). For the geometry (4) we reobtain our previous result [Eq. (21) of Ref. 4]. Our Eq. (6) has the same structure as the mentioned particular equation, in its dependence on the variable x. In the limit $x\to 1$ (both photons soft), the last term in (6) does not contribute.

One notices that the cross section (6) is invariant to the change $\mathbf{p}_1\to-\mathbf{p}_1$, i.e., xOy is a plane of symmetry for the angular distribution of the photons. This is not particular to the Born approximation; the exact cross section has the same symmetry. Very recent calculations performed by Quarles [14], based on the relativistic formula of Smirnov [2], indicate serious deviations from these symmetries. The Smirnov equation is the result of a relativistic Born-approximation calculation, automatically including retardation. We think that the retardation effects are responsible for these asymmetries. This aspect deserves further consideration.

IV. COMPARISON WITH RECENT EXPERIMENTAL DATA

We first refer to the comparison with the results of Hippler [8]. These are measurements for Ar, Kr, and Xe gaseous targets. We present Fig. 1, which corresponds to the conditions of Fig. 1 in Ref. 8, i.e., $E_1=8.82$ keV, $k_1=2.8$ keV, k_2 variable. Although the cross section is scaled (divided by Z^2), the nonrelativistic equations lead to three distinct curves, that we have drawn. This illustrates the fact that the cross section has a Z dependence beyond the Z^2 that is factored out in the Born approximation. We shall comment more on this in Sec. V. In Ref. 8 only one theoretical curve was shown (solid line in Fig. 1 of Ref. 8, corresponding to $Z=36$, based on the results in [3]). The points are the experimental data. The disagreement between experiment and theory is obvious; besides the differences in the order of the magnitude, the experimental data do not have the Z dependence predicted by theory.

Now we consider the new experimental results of the work of Kahler, Liu, and Quarles [9]. In this experiment the geometry is new: the two photon detectors do not see each other anymore, the recorded photon momenta make angles of $\pi/4$ with incident electron direction and are coplanar with it, i.e.,

$$q_{01}=q_{02}=\sqrt{2}/2\ ,\quad q_{12}=0\ . \tag{8}$$

In Fig. 2 we represent the Coulomb results for $E_1=70$ keV, $k_1=k_2=20$ keV as function of Z for both geometries (4) and (8). The experimental data (Ref. 9)

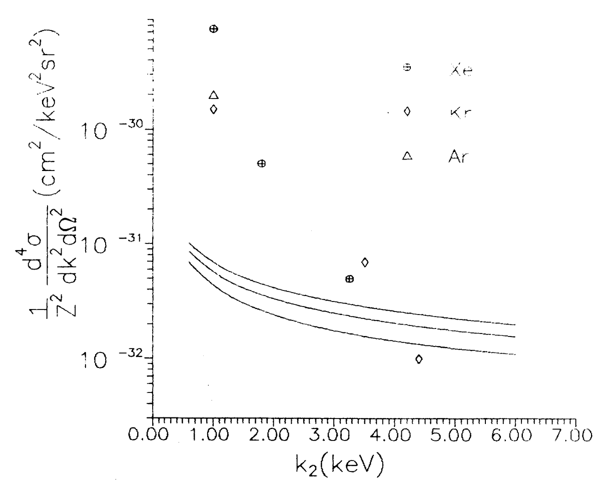

FIG. 1. Exact nonrelativistic Coulomb cross section $Z^{-2}d^4\sigma/dk^2d\Omega^2$ [Eq. (2)] as a function of the photon energy k_2 at the incident electron energy $E_1=8.82$ keV and photon energy $k_1=2.87$ keV for Ar, Kr, and Xe (in order from below), configuration described by Eq. (4). The points represent the experimental results of Hippler [8].

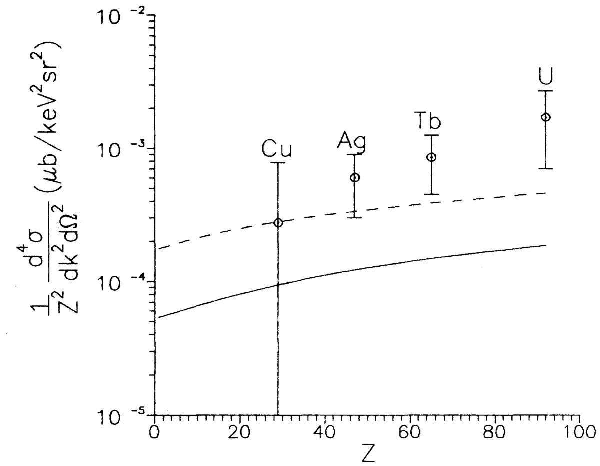

FIG. 2. Nonrelativistic exact Coulomb cross section $Z^{-2}d^4\sigma/dk^2d\Omega^2$ [Eq. (2)] as function of Z for $E_1=70$ keV, $k_1=k_2=20$ keV: the dashed line corresponds to the geometry (4), the full line and the experimental results (taken from Kahler *et al.* [9]) correspond to the geometry (8).

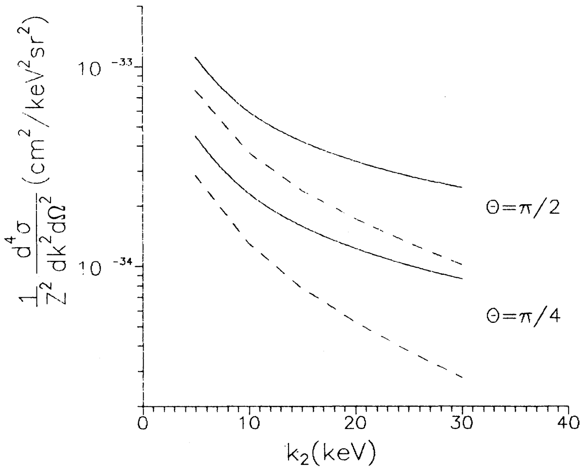

FIG. 3. Nonrelativistic exact Coulomb (full line) [Eq. (2)] and Born-approximation (dashed line) cross section $Z^{-2}d^4\sigma/dk^2d\Omega^2$ [Eq. (6)] as a function of k_2 for $Z=47$, $E_1=70$ keV, $k_1=20$ keV, for the two geometries (4) and (8).

correspond to the geometry (8). We see again the Z dependence beyond the Z^2 law of the Born approximation. We find a higher cross section for the geometry (4) than for the geometry (8).

In conclusion, we see that the discrepancy between the theoretical results and the experimental data exists in all studied cases, being larger in the case of Hippler's experiment than in that of Kahler *et al.* These experiments are done in different regions of electron and photon energies. Experimental measurements in similar conditions would be desirable.

We can ask ourselves how good should be the agreement between experiment and the present theory. We refer to an existing comparison [15] between nonrelativistic Coulomb field results and "exact" calculation for single-photon bremsstrahlung. In this case for low Z and low electron energies (tens of keV) the agreement is excellent, discrepancies develop with increasing Z and electron energy. We notice that (i) the discrepancies are not as large as those met in two-photon bremsstrahlung, and, (ii) screening and relativistic effects reduce the Coulomb cross sections. The effect of screening is expected to be similar in double bremsstrahlung and single bremsstrahlung, which is not the case in the situations we analyze here.

In connection with the experiment of Kahler *et al.* we present also Fig. 3, in which at fixed Z ($Z=47$), $E_1=70$ keV, $k_1=20$ keV, the cross section is represented as a function of k_2. Both geometries (4) and (8) are considered.

V. MORE COULOMB-FIELD RESULTS

We start by noticing that Ref. 3 contains, in its Figs. 1–3, in a compact representation, much numerical information about two-photon bremsstrahlung in the Coulomb field. Nevertheless, in comparing with experimental data, more explicit representations of the results are useful. At the same time, the results of Ref. 3 refer only to the configuration of the first experiment. So, we present our present data as a function of energies given in keV, and for specific values of Z. For the cross section we find it useful to represent $Z^{-2}d^4\sigma/dk^2d\Omega^2$, as was already done before [8,9].

In presenting our numbers we try to answer three different questions: (1) to what extent the scaled cross section depends on Z, (2) in which condition is the Born approximation reliable, and (3) which is the dependence of the results on the detection geometry.

In comparing exact and Born-approximation results we have found that the Z dependence of the scaled cross section persists even at low values of Z. Also, we have no-

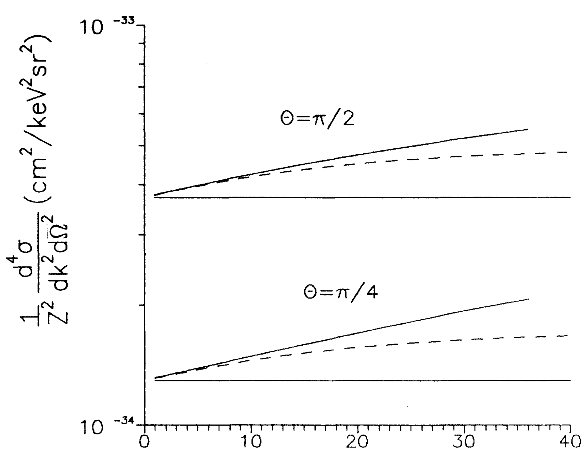

FIG. 4. Comparison between Born-approximation results (the horizontal line), the exact Coulomb results (full line), and the Born-Elwert approximation (dashed line), represented as a function of Z for $E_1=70$ keV, $k_1=10$ keV, $k_2=20$ keV, and the geometries (4) and (8).

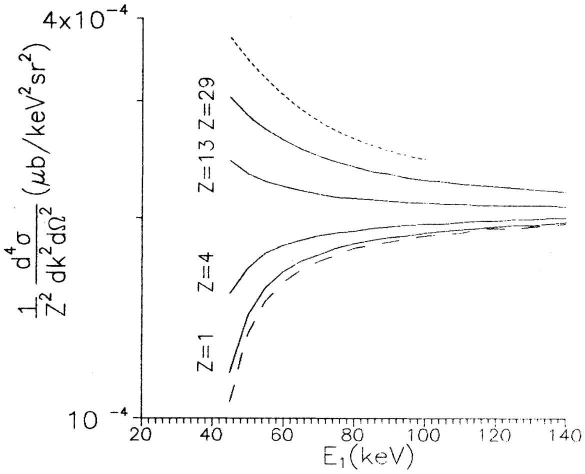

FIG. 5. The dependence on the incident electron energy E_1 of the cross section $Z^{-2}d^4\sigma/dk^2d\Omega^2$, in the Born approximation, modified by the Elwert factor, for several values of Z. The dashed curve represents the Born approximation. The dotted curve corresponds to the exact Coulomb results at $Z=29$.

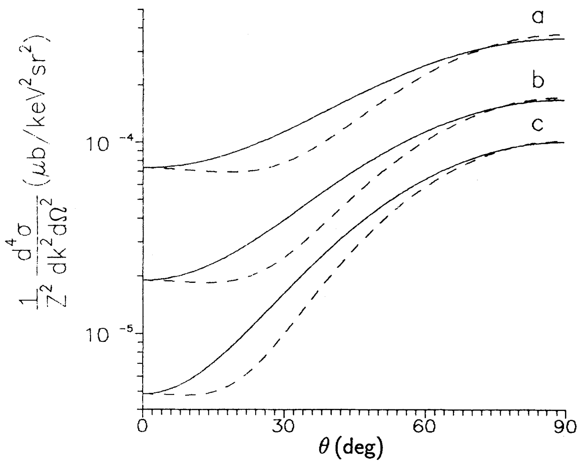

FIG. 6. The dependence on angle θ $(=\theta_1=\theta_2)$ of the cross section $Z^{-2}d^4\sigma/dk^2d\Omega^2$, in the Born approximation for $E_1=70$ keV, $k_2=20$ keV, and three values of k_1: (a) 10 keV; (b) 20 keV; (c) 30 keV. The full and dashed lines correspond, respectively, to $\Phi=\pi/2$ and π.

ticed that for low Z the Born-approximation results get closer to the exact ones if they are multiplied by the Elwert factor,

$$f_E=\frac{\eta_2}{\eta_1}\frac{1-\exp(-2\pi\eta_1)}{1-\exp(-2\pi\eta_2)} . \tag{9}$$

This correction factor is known from the study of single-photon bremsstrahlung [16]. The situation is illustrated in Fig. 4. The electron and photon energies are those of the experiment in Ref. 9, both geometries (4) and (8) are considered. The solid line represents the exact Coulomb results, the straight line the Born-approximation value, while the dashed line results by multiplication of the Born results with the Elwert factor (9).

In Fig. 5 the Born cross section corrected by the Elwert factor is represented as a function of the incident electron energy for different values of Z, for the geometry (8). The dashed curve does not include the Elwert factor. The upper dotted curve represents exact Coulomb results for $Z=29$. It is to be noted that the Elwert factor follows rather well the change from the Born approximation to the exact Coulomb approach.

Finally, we present some predictions based entirely on the Born approximation. In Fig. 6, for $E_1=70$ keV and $k_2=20$ keV, we have plotted the cross section (6) as a function of the angle θ $(=\theta_1=\theta_2)$ of the photons, for $\Phi=\pi$ (dashed lines) and $\Phi=\pi/2$ (solid lines). The cross section is unchanged if $\theta\rightarrow\pi-\theta$. The maximum is reached at $\theta=\pi/2$, in all cases, but the cross section changes with θ in a different way for $\Phi=\pi/2$ and π. It is to be noted that for $\theta=0$ the value is independent on Φ, and for $\theta=\pi/2$, the dependence on Φ is very weak. On the contrary, at $\theta=\pi/4$, there is a clear Φ dependence, and when Φ changes from 0 to π the cross section decreases. This situation might be of interest in choosing the experimental configuration.

In conclusion, in this paper we have compared the most recent experimental results for two-photon bremsstrahlung with the theoretical predictions coming from a nonrelativistic dipole-approximation evaluation of the process in the Coulomb field. A Z dependence beyond the Born approximation exists in the theoretical predictions, in qualitative agreement with that found by the work of Kahler, Liu, and Quarles [9]. The Born approximation is improved by the use of the Elwert factor. The disagreement between experiment and theory in the dependence of configuration seems to be mainly due to the use of the dipole approximation.

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