Weak and nucleoweak decays of muonic molecules

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Weak decay of the negative muon in hydrogenic media is generally routed through muonic molecular or atomic states. The distortions to the free-muon decay rate due to the muon's Coulomb dressing is computed for all isotopic combinations for muonic molecular states in hydrogen and compared with the atomic case in light media. The nucleoweak channels corresponding to nuclear fusion accompanying the weak decay of the muon from a molecular environment are also investigated.

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INTRODUCTION

Weak interactions of negative muons in low-Z matter are routed through muonic molecular or atomic states. These Coulomb dressings can modify the unperturbed decay rates to the order of the relevant Coulomb parameters. As experimental probes of muon physics improve in sophistication, these effects could vitiate precision measures of the weak characteristics of negative muons. We have earlier investigated the weak characteristics of the positive muon from the Coulomb-dressed state in muonium [1]. The presence and state of the muon's nuclear Coulomb partner are of vital importance for the weak capture reaction as the nucleus is an active reactant.

On the other hand, for the weak decay of the muon bound in an atom or molecule according to

$$(\mu^- N)\rightarrow e^- +\nu_\mu+\bar{\nu}_e+N \ , \tag{1}$$

$$(\mu^- NN')\rightarrow e^- +\nu_\mu+\bar{\nu}_e+N+N' \ , \tag{2}$$

where N,N' refers to the relevant nuclei, the Coulomb partner of the muon is not an essential ingredient of the reaction, but a spectator.

The bare weak decay (Fig. 1) therefore proceeds dressed in the Coulomb field of the nucleus, resulting in appropriate modifications of the rate and decay electron spectrum. Naturally these corrections increase in significance with the Z value of the nucleus. The spectating nucleus also participates in the exit channel kinematics through conservation constraints.

The Coulomb parameters ξ and η are of the general form $\xi=Z/(137V)$ and $\eta=Zm_R/137$, where V is the relative velocity of the exit channel particles connected by the Coulomb force, m_R is the reduced mass for the predecay Coulomb connection, and Z is the atomic number. The decay electron retains its Michel character for light elements, so that the spectrum is dominated by high electron energies. Since the maximum electron energies are $\sim m_\mu/2$, where m_μ is the mass of the muon, the Coulomb parameter ξ is necessarily small. For increasing Z, the spectrum deviates from its Michel nature, lower energies contribute more, and the Coulomb concentration of the wave function becomes increasingly effective. In most cases, the Coulomb parameters are of order $\xi\sim4\times10^{-4}$ and $\eta\sim m_R/137$.

The W boson terms that contribute to order $(m_\mu/m_W)^2$ in the bare decay can be neglected [1,2]. Collapsing the W boson propagators to point coupling, the Coulomb-dressed μ^- decay can be represented as in Fig. 2 for the atomic and molecular cases.

For heavy nuclei any molecular states would be extremely short lived, their lifetimes swamped by the cas-

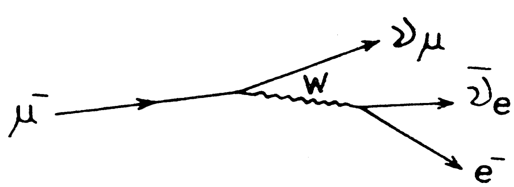

FIG. 1. Bare weak μ^- decay.

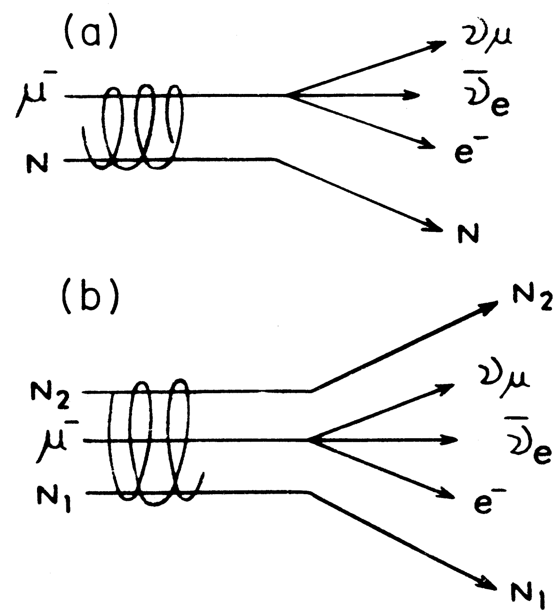

FIG. 2. (a) μ^- decay from muonic atom. (b) μ^- decay from muonic molecule.

cade rates which rapidly stabilize the muon in the ground orbits of the highest Z component. For hydrogenic systems, however, molecular states are known to be an integral part of the bound-state physics [3] and indeed acquire additional importance for their ability to host fusion reactions [4]. For the $dt\mu$ molecule, in fact, nuclear fusion of the d and t nuclei constitute the dominant channel for the decay of the muonic molecules, as the fusion rate in this system is 10^{-11} or^{-12} sec^{-1} [4]. For the nonresonant nuclear systems involved in $(pp\mu)$, $(pd\mu)$, and $(tt\mu)$, the fusion rate being slower, weak interaction forms a competitive channel to the fusion reaction for the molecular decay. We have therefore computed the muon decay from its hydrogenic molecular states for all the isotopic combinations. In the background of the high-precision experimental accuracy attainable in muon studies today, these modifications to the free-muon decay due to its Coulomb dressing may prove detectable. We have also investigated the possibility of fusion accompanying muon decay.

FORMALISM AND DECAY RATE

The Hamiltonian H_w for the bare weak interaction (Fig. 1), collapsed to point coupling, can be written in the $(V-A)$ form as customary for low-energy decay processes [1,2]. Thus

$$H_w=(G/\sqrt{2})[\bar{\psi}_\mu O^\alpha \psi_{\nu_\mu}\bar{\psi}_1 O_\alpha \psi_{\nu_e}] , \tag{3}$$

with

$$O_\alpha=\gamma_\alpha(1+\gamma_5) .$$

G is a weak-interaction constant. The matrix element for the decay of μ^- from the general bound state can be written (using plane waves for the two neutrinos and the electron)

$$|M_{fi}|=-(G/\sqrt{2})(\bar{U}_1 O_\beta U_0)(\bar{U}_{\nu_\mu} O_\beta U_{\nu_e})I , \tag{4}$$

where

$$I=\int \Phi_\mu(r)\exp(-i\mathbf{p}_s\cdot\mathbf{r})d^3\mathbf{r} . \tag{5}$$

U represents the spinor corresponding to the subscripted particles and U_0 represent the zero-momentum spinor for the bound muon. $\mathbf{p}_s$ is the momentum of the bound muon and $\Phi_\mu(r)$ is its initial bound-state wave function. For decay from muonic atoms,

$$I=\int B\exp(-r\eta_e)\exp(i\mathbf{p}_s\cdot\mathbf{r})d^3\mathbf{r} ,$$

where

$$\eta=m_R Z/137 \quad\text{and}\quad B=\eta^{3/2}/\sqrt{\pi} . \tag{6}$$

For light atoms, relativistic terms for the muon and finite size effects are very small and have been neglected, as the first-order dominant corrections are being probed and the atomic case is being used as a reference for comparing the molecular effects. Therefore we use nonrelativistic wave functions for the bound muon, but retain the spinors in principle to enable the spin sum evaluations required.

For the molecular case we take the wave function in the Born-Oppenheimer approximation [5] for a molecular ion,

$$\psi_\mu(r_a,r_b,R)=\psi_{\text{vib}}(R,R_0)\psi_\mu(r) , \tag{7a}$$

with

$$\psi_\mu(r)=A_m[\eta_a^{3/2}\exp(-\eta_a r_a)+\eta_b^{3/2}\exp(-\eta_b r_b)]$$

and

$$\psi_{\text{vib}}(R,R_0)=A_v\exp[(-B_1/2)(R-R_0)^2]/R , \tag{7b}$$

where

$$\eta_a=Zm_a/137 \quad\text{and}\quad \eta_b=Zm_b/137 .$$

m_a and m_b are the reduced masses of the muon and the nuclei a and b, respectively. A_v and B_1 are relevant constants.

$$\begin{aligned} A_m&=1/\sqrt{2\pi(1+S_1)} ,\\ S_1&=(\eta_a^{3/2}\eta_b^{3/2}/\pi)\{8\pi/[R(\eta_a^2-\eta_b^2)^2]\}\\ &\quad\times\{R[\eta_a\exp(-\eta_b R)+\eta_b\exp(-\eta_a R)]\\ &\quad+[4\eta_a\eta_b/(\eta_a^2-\eta_b^2)]\\ &\quad\times[\exp(-\eta_a R)-\exp(-\eta_b R)]\} \end{aligned} \tag{7c}$$

for the general heteronuclear case. $R=aR+bR$ is the intermolecular separation and $\mathbf{r}_a$ and $\mathbf{r}_b$ are the distances of the muon from the two nuclei. The expression for S_1 acquires a much simpler form for the homonuclear case but cannot be recovered from (7c) by substituting $\eta_a=\eta_b$, but has to be derived independently.

Breaking into variables $\mathbf{r}_a,\mathbf{r}_b$ and using $\mathbf{R}=\mathbf{r}_a-a\mathbf{R}$ and $\mathbf{R}=\mathbf{r}_b+b\mathbf{R}$, we have finally, after integrating the space integral,

$$I=8\pi\left[e^{ia\mathbf{p}_s\cdot\mathbf{R}}\frac{\eta_a A_{m_1}}{(\eta_a^2+p_s^2)^2}+A_{m_2}e^{-ib\mathbf{p}_s\cdot\mathbf{R}}\frac{\eta_b}{(\eta_b^2+p_s^2)^2}\right] , \tag{8}$$

where

$$A_{m_1}=\eta_a^{3/2}A_m \quad\text{and}\quad A_{m_2}=\eta_b^{3/2}A_m .$$

For accurate calculation of the μ atom energy levels it is important to use accurate wave functions probing beyond the Born-Oppenheimer approximation limitations—as it is the Coulomb field that is of importance. For the case of muon decay, and indeed for the fusion potential as well, the Coulomb field provides only the dressing for the main reaction, which is weak or nuclear in character. As such it is adequate in these cases to use the Born approximations for the wave functions combined with the accurate energy levels obtained by variational methods for the bound-state energies. We treat the exiting electron as a plane wave in the main calculation, as this is adequate for light atoms.

The spin sums, etc., are carried out as for free-muon decay, but replacing the bound muon momentum by a zero-momentum spinor as is customary for bound states.

The spectator momentum p_s has been introduced in Eq. (5) using momentum conservation according to $\mathbf{p}_1+\mathbf{p}_{\nu_\mu}+\mathbf{p}_{\nu_e}=-\mathbf{p}_s$ in the rest frame of the muon.

The bound muon decay rate is obtained by integrating the square of the spin-averaged matrix element over the phase space of the final particles. Thus

$$R_b=[(2\pi)^4/(2\pi)^{12}]\times\int\int\int d^3\mathbf{p}_s d^3\mathbf{p}_1 d^3\mathbf{p}_{\nu_\mu} d^3\mathbf{p}_{\nu_e}\times\delta^4(P_0-P_1-P_{\nu_\mu}-P_{\nu_e})\tfrac{1}{2}|M_{fi}|^2 . \quad (9)$$

s refers to the spectating nucleus for the bound atomic case. In the case of the molecule we take p_s to represent the center-of-mass momentum of the nuclei. As the spectator spectra are dominated by low momenta and energy, the individual distributions can be ignored and their combined effect considered.

The neutrino integral is evaluated using the four-component δ function in the center-of-mass frame of the two neutrinos as for free-muon or τ decay [2] and muon decay from muonium [1]. Then reverting to the rest frame of the muonic molecule, which is assumed coincident with the rest frame of the muon, we have

$$R_b=\frac{4G^2}{(2\pi)^8}\times\int d^3\mathbf{p}_1 d^3\mathbf{p}_s P_{1\alpha}Q_\beta/[E_l Q_4]I_{\alpha\beta}\times\{A_1^2+B_1^2+2A_1B_1\cos[(a+b)(\mathbf{p}_s\cdot\mathbf{R})]\} , \quad (10)$$

where

$$I_{\alpha\beta}=(\pi/6)(p_\nu^2\delta_{\alpha\beta}+2P_{\nu_\alpha}P_{\nu_\beta}) ,$$

$$A_1=\frac{8\eta_a\pi}{(\eta_a^2+p_s^2)^2}A_{m_1} ,$$

$$B_1=\frac{8\eta_b\pi}{(\eta_b^2+p_s^2)^2}A_{m_2} .$$

$\mathbf{p}_1$ and Q are four-vectors of e^- and μ^-, while P is the total four-momentum carried by the neutrinos. E_1 is the energy of e^-.

The constraints on the decay electron-spectator phase space follow the δ-function condition as

$$2E_1W+2p_1p_su-m_1^2=S-WE_s-p_s^2-\sqrt{S}E_s , \quad (11a)$$

where S is the total entrance channel four-momentum given by

$$S=P_0^2=(P_\mu+P_s-\epsilon)^2 , \quad u=\cos(\mathbf{p}_1,\mathbf{p}_s) ,$$

where ϵ is the magnitude of the binding energy of the relevant system and

$$W=\sqrt{S}-E_s . \quad (11b)$$

This gives the kinematic limits for the decay electron momenta and have been used to evaluate the phase-space integration. For the spectator momentum the opposite approximation of treating it nonrelativistically has been made as even at maximum p_s; the kinetic energy is much less than the nuclear mass.

The decay electron is next integrated over kinematic limits, so that we are left with the final spectator distribution.

$$R_{\mathrm{mol}}=\frac{R_b}{R_f}=\frac{32}{W_0^5m_\mu^5}\int_0^1 Wx^2(W^4-2x^2W_0^2W^2+x^4W_0^4)\times\left[\frac{\eta_a^2A_{m_1}^2}{\left[x^2+\frac{\eta_a^2}{W_0^2}\right]^4}+\frac{\eta_b^2A_{m_2}^2}{\left[x^2+\frac{\eta_b^2}{W_0^2}\right]^4}+\frac{2\eta_a\eta_bA_{m_1}A_{m_2}}{\left[x^2+\frac{\eta_a^2}{W_0^2}\right]^2\left[x^2+\frac{\eta_b^2}{W_0^2}\right]^2}\frac{\sin[(a+b)xW_0R]}{(a+b)xW_0R}\right]dx, \quad (12)$$

where R_f is the free muon decay rate,

$$R_f=\left[\frac{m_\mu^5G^2}{192\eta^3}\right] ,$$

$x=P_s/W_0$, and W_0 is the maximum spectator momentum.

For light atoms the unicenter form of the space integrals yields a simple form for the decay rate, which is

$$R_{\mathrm{atom}}=\frac{32B^2\eta^2}{m_\mu^5W_0}\int_0^1\left[\frac{Wx^2}{\left[x^2+\frac{\eta^2}{W_0^2}\right]^4}\times\left[\frac{W^4}{W_0^4}-2\frac{W^2}{W_0^2}x^2+x^4\right]\right]dx , \quad (13)$$

where $B^2=\eta^3/\pi$, W_0 is the maximum spectator momentum, $W_0=m_\mu-\epsilon$, $W=\sqrt{S}-E_s$, $\sqrt{S}=m_\mu+m_s-\epsilon$, $\epsilon=m_R Z^2/[2/(137)^2]$, $E_s=(m_s^2+x^2W_0^2)$, m_s is the mass of the spectator, $\eta=m_R Z/137$, and $m_R=m_\mu/(1+m_\mu/m_s)$. R_{atom} is in units of the free-muon decay rate. $R_{\text{mol}}/R_{\text{atom}}$ can also be obtained for different systems.

NUCLEOWEAK DECAY CHANNELS OF THE MUONIC MOLECULE

In most general terms, the decay channels of the muonic molecule include, apart from the weak decay of the muon [Eq. (2)], the weak capture channels, as well as electromagnetic and nuclear transformations. Thus

$$(\mu N_1 N_2)\to(\nu_\mu N_1' N_2) \text{ or } (\nu_\mu N_1 N_2') , \tag{14}$$

corresponding to capture of the muon by either nuclei, also leads to disappearance of the molecule.

An interesting channel for muonic molecule decay is provided by the nucleoweak process.

$$(\mu^- N_1 N_2)\to N_3+N_4+\nu_\mu+e^-+\bar{\nu}_e , \tag{15}$$

where fusion of the nuclei accompanies the muon decay. Similar reactions are available for the capture channels.

In this work we do not study the weak capture of nucleoweak capture channels but concentrate on reactions of type (2) as discussed above and of type (15).

The nucleoweak channels can be written for some of the isotopic combination as in hydrogen,

$$(\mu^- pd)\to\nu_\mu+e^-+\bar{\nu}_e+{}^3\text{He} , \tag{16a}$$

$$(\mu^- dd)\to\begin{cases}\nu_\mu+e^-+\bar{\nu}_e+t+p & \text{(16b)}\\ \nu_\mu+e^-+\bar{\nu}_e+{}^3\text{He}+n , & \text{(16c)}\end{cases}$$

$$(\mu^- dt)\to\nu_\mu+e^-+\bar{\nu}_e+\alpha+n , \tag{16d}$$

and

$$(\mu^- pp)\to\nu_\mu+e^-+\bar{\nu}_e+d+e^++\nu_e , \tag{16e}$$

and are described by the typical diagram of Fig. 3.

The causal structure of this combined process depends on the propensity of the individual weak and nuclear channels. For the $dd\mu$ and $dt\mu$ systems, the ground-state fusion potential is very high, giving the pure fusion decay channel a rate $\sim 10^{11}-10^{12}$ sec.$^{-1}$. The probability of the muon's weak decay is to be understood as an appendage to the main process, i.e., as an alternate loss channel for the muon, somewhat analogous to the sticking loss. However, for the $pd\mu$ system the fusion process requiring an electronmagnetic exit channel is much slower, $\sim 10^6$ sec^{-1}, so that it is comparable to the weak decay of the muon, and it becomes difficult to discuss which process, the weak or nuclear, occurs first, or if they occur together.

This is reminiscent of the primal nucleoweak reaction that initiates the proton chain in steller fusion, yet it is greatly suppressed due to the low probability of its weak sector in

$$p+p\to d+e^++\nu_e . \tag{17}$$

The weak decay of one of the protons yields a neutron that fuses with the other proton. The weak and nuclear sectors of the reaction are completely dependent on each other so that a causal separation of the vertex cannot be made.

The p-p nucleoweak fusion can also occur from its dressed state in the muonic molecule according to Eq. (2). This weak capture channel would be more effective as it could provide the neutron required for the fusion channel, so we have not investigated channel (16e) at present.

NUCLEOWEAK DECAY RATES

Since our starting point is the muonic molecule, we take as the initial state the complete muon-molecular wave function. Again, as for the aspects studied the Born-Oppenheimer description suffices, we use (7b) and write its dimensionless form

$$\psi(x,x_0,R)=(\alpha/\pi)^{1/4}e^{-(\alpha/2)(x-x_0)^2}(1/C_1 x) , \tag{18}$$

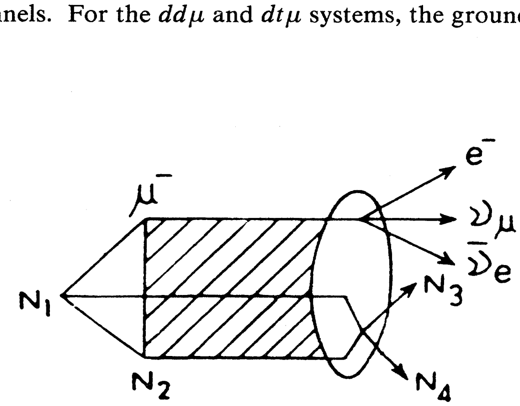

FIG. 3. The nucleoweak channel for muonic molecule decay.

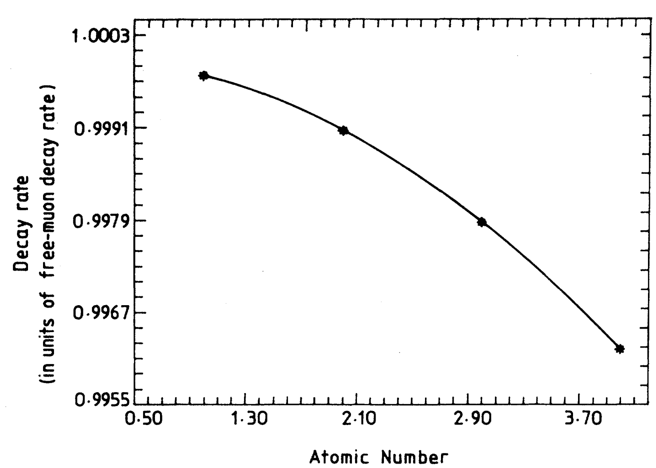

FIG. 4. The μ^- decay rates from muonic atoms in units of free-muon decay rate as a function of atomic number of binding nucleus.

TABLE I. The μ^- decay rate in units of free-muon decay rate from different hydrogenic molecules.

μ^--hydrogenic molecule	$pp\mu$	$dd\mu$	$tt\mu$	$pd\mu$	$pt\mu$	$dt\mu$
Decay rate in units of free-muon decay rate	0.999 917	0.999 908 5	0.999 904 8	0.999 790 5	0.999 788 0	0.999 779 7

where

$$C_1=\sqrt{4\pi a_1^3}\ ,$$

$$a_1=(\text{Bohr radius})\ ,$$

$$\alpha=M\hbar\omega/\mu\ ,$$

M and μ are the reduced mass of the nuclei and the mass of binding particle, respectively. W is the angular frequency of the nuclei, x is the internuclear separation, and x_0 is its equilibrium value. We assume the combined nucleoweak channels are given by the simultaneous probability of the muon decay and nuclear collapse to fusion distance. To estimate the effect in terms of the probability of pure decay without fusion, one can factor out the weak decay sector so that the probability of fusion accompanying muon decay can be taken as

$$P=M_{N_1}/M_{N_2}\ , \tag{19}$$

with

$$M_{N_1}=\int_{x_0}^{\infty}\psi_{\text{vib}}^*(x)\psi_{\text{vib}}(x\to 0)d^3\mathbf{x}\ ,$$

$$M_{N_2}=\int_{x_0}^{\infty}\psi_{\text{vib}}^*(x)\psi_{\text{vib}}(x)d^3\mathbf{x}\ .$$

M_{N_1} therefore describes the sudden nuclear collapse from its finite separation in the predecay muonic molecule to zero separation. This gives the fusion probability as fusion can be considered instantaneous at nuclear collapse. M_{N_2} normalizes this to the case of noncollapse that pertains to muon decay without fusion. The nuclear reaction constants and related effects that selectively favor different reaction rates are not introduced here, as we are not computing fusion rates but only probabilities for nuclear collapse.

We do not take the fusion probability to be given by the sector of the wave function that corresponds to zero nuclear separation as is customary for computing the fusion probability. Instead we overlap the initial nuclear state with its state at zero separation, as this is a better representation of the sudden collapse of the nuclear wave function from its stable molecular separation at the instant of muon decay for this channel. Since the collapsed wave function is not a true eigenfunction of the initial vibrational function, this does not introduce any orthogonality problems.

In Eq. (1) the collapsed wave function is taken as the molecular vibrational wave function extended into barrier suppressed distances by the WKB approximation as used by Jackson [6]. Thus

$$\psi_{\text{vib}}(x\to 0)=(\alpha/2\pi)^{1/2}(x/C_1x)e^{-\lambda(x)/2}\ . \tag{20}$$

$\lambda(x)$ is the barrier penetration factor for the muonic hydrogen molecule. However, we reiterate that our approach differs from that of Jackson in that we retain the vibrational wave function in the initial state and overlap it with the collapsed wave function. On integrating M_{N_1} analytically we have

$$\begin{aligned}M_{N_1}=Ae^{-\lambda}\{&-(b/2a)\sqrt{\pi/a}\,\text{erf}(\sqrt{a}x+b/\sqrt{a}\,)\\&\times e^{(b^2-ac)/a}\\&-\tfrac{1}{2}a\exp[(b^2-ac)/a\\&\qquad-(\sqrt{a}x+b/\sqrt{a}\,)^2]\}_{x_0}^{\infty}\ ,\end{aligned} \tag{21}$$

where

$$A=(\alpha/\pi)^{3/4}(\sqrt{2}a_1^3)\ ,$$

$$a=\alpha/2\ ,\quad b=-(\alpha/2)x_0\ ,$$

and

$$c=(\alpha/2)x_0^2\ .$$

On introducing the correct limits and simplifying, we have finally

$$M_N=\sqrt{2}e^{-\lambda}(\alpha/\pi)^{3/4}[(1/\alpha)+\sqrt{\pi/(2\alpha)}x_0]\ . \tag{22}$$

RESULTS AND DISCUSSION

This work reports a calculation of muon decay from all the hydrogenic muon-molecular systems. Tables I and II display μ^- decay rates from muonic molecules and atoms for different hydrogenic combinations, respectively. The difference between the atomic and molecular cases is clearly visible in Tables II and I and is a natural consequence of the two-center nature of the Coulomb field for muonic molecules in contrast to the unicenter atomic case. The hetero- and homonuclear systems are distinguished by the asymmetric and symmetric forms of the Coulomb forces in the two cases. The splitting of the Coulomb parameter through the reduced mass is also felt

TABLE II. The μ^- decay rate in units of free-muon decay rate from light muonic atoms.

Muonic atom	$p\mu$	$d\mu$	$t\mu$
Decay rate in units of free-muon decay rate	0.999 789 3	0.999 783 9	0.999 777 8

TABLE III. Probability of fusion accompanying muon decay from molecules.

Molecular system	$pd\mu$	$dd\mu$	$dt\mu$
Probability	$5.552\,55\times10^{-3}$	$5.674\,78\times10^{-4}$	$1.735\,83\times10^{-4}$

in the rates for the various systems. It is noted that the decay rate is uniformly higher for the homonuclear system as compared to the heteronuclear ones. Values of ϵ for the molecular case has been taken from Ref. [4].

The variation in decay rate for light muonic atoms is displayed in Fig. 4 for hydrogen, helium, lithium, and beryllium. We note from Fig. 4 that the decay rate gradually falls with increasing Z. The modification of the bound-muon decay rate from the free one is a result of several factors, such as the reduction in available energy due to binding, and the recoil spectator phase space. The case of heavy muonic atoms has been extensively studied in the past, taking relevant finite-size, relativistic, and Coulomb effects into account [7]. These have demonstrated that the decay rate falls rapidly for high Z.

We have studied only the light atoms where the finite size and Coulomb corrections are unimportant, as our main motivation has been to study the molecular cases and use the atomic systems as a reference.

The probability of the nucleoweak channels are shown in Table III. We have taken the values of $\lambda(x_N)$ from Ref. [6] for $x_N=0$. Values of λ were separately evaluated in an earlier work [8] and the use of $\lambda(x_N)$ here could therefore be justified.

The advent of pulsed beams at various muon factories has generated a new generation of high-precision experiments for low-energy muon physics. Muon capture in hydrogen has been measured with a high degree of accuracy [9]. The correction to free-muon decay due to molecular binding as determined by us may be detectable in future experiments. Pure and various mixed isotopic combinations of hydrogen have been used for μ-catalyzed fusion experiments [4]. Time-of-flight measurement of muon decay in these media or direct count of the decay electrons could permit direct determination of the decay rate from molecules. Our primary motivation was to see if the correction due to the molecular binding could influence the μ-catalyzed fusion parameters as discussed in the Introduction. The probability of the nucleoweak channels and, in fact, these precision calculations of the decay rate are most important for the molecular systems that do not sport a resonant fusion channel. They could also prove useful in additional decoupling of precision values of standard electroweak parameters from Coulomb mixing when they are measured in hydrogenic media.

ACKNOWLEDGMENTS

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[1] L. Chatterjee, A. Chakrabarty, G. Das, and S. Mondal, Phys. Rev. D **46**, 5200 (1992).

[2] L. Chatterjee and V. P. Gautam, Phys. Rev. D **41**, 1698 (1990).

[3] V. P. Dzhelepov *et al.*, Zh. Eksp. Teor. Fiz. **50**, 1235 (1966) [Sov. Phys. JETP **23**, 820 (1966)]; E. A. Vesmen, Pis'ma Zh. Eksp. Teor. Fiz. **5**, 113 (1967) [JETP Lett. **5**, 91 (1967)].

[4] L. Chatterjee, Indian J. Phys. A **65** (3), 175 (1991); Phys. Lett. A **137**, 4 (1989).

[5] B. H. Bransden and C. J. Joachain, *Physics of Atoms and Molecules* (Halsted, London, 1983), pp. 401 and 646.

[6] J. D. Jackson, Phys. Rev. **106**, 330 (1957).

[7] D. D. Yovanovitch, Phys. Rev. **117**, 1580 (1960).

[8] L. Chatterjee and G. Das, Phys. Lett. A **154**, 5 (1991).

[9] G. Bardin *et al.*, Nucl. Phys. A **352**, 365 (1981).