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Probe absorption spectrum of a laser-driven atom near a phase conjugator

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The absorption spectrum for weak radiation of a two-state atom near the surface of a phase-conjugating four-wave mixer has been evaluated. Also included is the option that the atom is illuminated by a near-resonant laser field. In the secular limit with respect to the laser-dressed atomic states the absorption spectrum has two lines, just as for a free atom. The strengths and the widths of these lines depend on the phase-conjugate reflectivity, as well as on the optical parameters. It is shown that increasing the reflectivity drives the atom towards saturation through three-photon processes. Also the driving laser tends to saturate the transition, but it is shown that both mechanisms have a different effect on the absorption profile, and can hence be distinguished experimentally.

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I. INTRODUCTION

Interactions between an atom and its environment can be monitored by observing either the emission or absorption of radiation. Spectral line shapes are determined by the time-evolution operator for the atomic density operator, and hence the matrix elements of this evolution (or regression) operator, containing information about interactions, are revealed in the various details of the emission and absorption spectra. Consider a two-state atom with an excited state $|e\rangle$ and ground state $|g\rangle$, separated by $\hbar\omega_0$. Its dipole moment $\boldsymbol{\mu}$ interacts with the electric field of the vacuum, and this gives rise to spontaneous decay from the excited state to the ground state. The transition rate is equal to the Einstein A coefficient, given by

$$A=\frac{\omega_0^3}{3\pi\varepsilon_0\hbar c^3}|\langle e|\boldsymbol{\mu}|g\rangle|^2 . \tag{1.1}$$

When a weak field with angular frequency ω and polarization $\mathbf{e}_p$ probes the atom, then stimulated excitations will occur at a rate determined by the Einstein B coefficient, given by

$$B=\frac{\pi}{\varepsilon_0\hbar^2 c}|\langle e|\boldsymbol{\mu}\cdot\mathbf{e}_p|g\rangle|^2 . \tag{1.2}$$

With I_p the intensity of the probe beam (energy per unit of time through a unit cross section), the transition rate equals BI_p, and every excitation is brought about by a photon absorption. The absorbed energy per unit of time, $I(\omega)$, assumes the form

$$I(\omega)=\hbar\omega BI_p\,\mathrm{Re}\frac{1/\pi}{\frac{1}{2}A-i(\omega-\omega_0)} . \tag{1.3}$$

As a function of ω, this is a Lorentzian with half width at half maximum (HWHM) equal to $A/2$ and a strength of

$$\int_{\text{line}} d\omega\, I(\omega)=\hbar\omega BI_p , \tag{1.4}$$

which is just the transition rate times the energy of a photon.

When the atom is, in addition, irradiated by a strong, nearly monochromatic laser beam with frequency ω_L, then the absorption profile is determined by transitions between laser-dressed atomic states [1], rather than between the bare atomic ground state and excited state. The line around the atomic resonance ω_0 remains, and a second line appears [2] around $2\omega_L-\omega_0$. These lines are located at the same positions as the fluorescence (F) line and the three-photon (T) line, respectively, in the emission spectrum (the Mollow triplet) [3], and therefore we shall refer to these lines as the F and the T line in the probe absorption spectrum. Both the F and the T line have contributions from stimulated absorption and stimulated emission, and the net absorption is the balance between the two processes. It turns out that the F line always has a positive strength, indicating that absorption dominates, but the T line has a negative strength, corresponding to amplification of the probe beam. The emission spectrum has a third line (the Rayleigh line) at the laser frequency ω_L, but in the absorption profile this line disappears due to an exact cancellation of stimulated absorption and emission. This two-line spectrum has been observed experimentally [4].

Another influence on atomic spectra is the presence of a dielectric or metal surface. When the atom is close to a medium, then its Einstein A coefficient changes, as has been predicted theoretically [5–12], and verified experimentally [13–15]. This modification is due to the fact that the vacuum field near a medium is different from the vacuum in empty space. On one hand, this affects the populations of the atomic levels and thereby the line strengths in absorption and emission spectra (a transition rate is proportional to the population of the initial level). On the other hand, the Einstein A coefficient determines the relaxation rate in the time evolution of the atomic-density operator, and therefore also the damping in the time regression of atomic correlation functions. The time regression is responsible for the details of the shape of a spectral line. In this paper we consider a two-state atom near the surface of a four-wave-mixing phase conjugator (PC). Just as for a dielectric, the Einstein A coefficient for spontaneous decay from the excited state changes, and for a PC its value is found to be [16–18]

$$A_e = A(1+\tfrac{1}{2}R) \ , \tag{1.5}$$

where R is the (intensity) reflection coefficient for a plane wave, averaged over the angle of incidence. Furthermore, an atom near a PC can make stimulated transitions from the ground state to the excited state in three-photon events, which leads to a population of the excited state even in the absence of any driving field. This process is sometimes called spontaneous excitation [19], and the mechanism for it is illustrated in Fig. 1. The rate constant for stimulated excitation is given by

$$A_g = \tfrac{1}{2}AR \ , \tag{1.6}$$

and the transition rate equals A_g times the population of the ground state. It turns out that the relaxation constants are (as good as) independent of the distance between the atom and the medium [20], as can be understood from the mechanism shown in Fig. 1. The spontaneously emitted photons by the PC are focused on the atom, regardless of the distance between the atom and the four-wave mixer.

II. ABSORPTION PROFILE

The atom near the surface of the PC is irradiated by a strong laser field, propagating parallel to the surface of the medium, and at the position of the atom it is given by

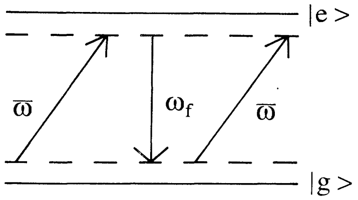

FIG. 1. An atomic dipole near a four-wave mixer polarizes the medium, giving rise to spontaneous emission of photons by the medium in a four-wave mixing interaction. Let $\bar{\omega}$ be the frequency of the pump photons in the crystal. In the phase conjugation geometry, these emitted photons with frequency $\bar{\omega}$ are focused on the atom. An absorption of two of these photons and the emission of a fluorescent photon in between, then brings the atom to the excited state, as shown in the diagram. It then follows that the fluorescent photon must have a frequency $\omega_f = 2\bar{\omega} - \omega_0$. After this three-photon event, the atom decays spontaneously back to the ground state, accompanied by emission of a photon with frequency ω_0.

$$\mathbf{E}_L = E_0 \mathrm{Re}\,\mathbf{e}_L e^{-i[\omega_L t + \phi(t)]} \ , \tag{2.1}$$

with E_0 the amplitude and $\mathbf{e}_L$ a unit polarization vector. The phase $\phi(t)$ is a random process which accounts for the laser linewidth, and we shall take this to be the independent-increment process [21], which has the phase-diffusion model as its Gaussian limit. The laser profile is then a Lorentzian with HWHM equal to λ_1 (a parameter of the stochastic process).

The absorbed energy per unit of time from a probe beam with frequency ω can then be expressed as [22]

$$I(\omega) = \frac{\omega I_p}{\varepsilon_0 \hbar c} \lim_{t\to\infty} \mathrm{Re} \int_0^\infty d\tau\, e^{i\omega\tau} < \langle [\boldsymbol{\mu}(t+\tau)\cdot\mathbf{e}_p^*, \boldsymbol{\mu}(t)\cdot\mathbf{e}_p] \rangle > \ , \tag{2.2}$$

where the time dependence of an operator signifies the Heisenberg picture. The notation $< >$ indicates an average over the laser fluctuations, and $\langle \ \rangle$ represents a quantum average. The limit $t \to \infty$ implies that the atom has reached its steady state. The two terms in the commutator correspond to stimulated absorption (positive term) and stimulated emission (negative term), and the net absorption is the difference between the two. In terms of the atomic raising operator $d = |e\rangle\langle g|$, the dipole-moment operator can be represented as

$$\boldsymbol{\mu} = \langle e|\boldsymbol{\mu}|g\rangle d + \mathrm{H.c.} \tag{2.3}$$

Then the absorption profile becomes

$$I(\omega) = \hbar\omega B I_p \lim_{t\to\infty} \mathrm{Re} \frac{1}{\pi} \int_0^\infty d\tau\, e^{i\omega\tau} < \langle [d^\dagger(t+\tau), d(t)] \rangle > \ , \tag{2.4}$$

where we have dropped the terms that oscillate with twice the optical frequency.

The total strength of the spectrum is

$$\int_{\mathrm{line}} d\omega\, I(\omega) = \hbar\omega_0 B I_p \lim_{t\to\infty} < \langle P_g(t) - P_e(t) \rangle > \ , \tag{2.5}$$

where we have set $\omega \approx \omega_0$ in the overall factor. The projectors onto the atomic states are defined as

$$P_g = |g\rangle\langle g|, \quad P_e = |e\rangle\langle e| \ , \tag{2.6}$$

in the Schrödinger picture, and their expectation values are the populations $n_g(t)$ and $n_e(t)$ of the corresponding levels. If we indicate by an overbar a steady-state value, after an average over the laser fluctuations, then the strength becomes

$$\int_{\mathrm{line}} d\omega\, I(\omega) = \hbar\omega_0 B I_p (\bar{n}_g - \bar{n}_e) \ . \tag{2.7}$$

It is convenient to introduce the normalized spectral profile by

$$J(\omega)=\frac{1}{\hbar\omega_0 B I_p}I(\omega)\ , \tag{2.8}$$

which has a strength of

$$\int_{\text{line}} d\omega\, J(\omega)=\bar{n}_g-\bar{n}_e\ . \tag{2.9}$$

The term $\bar{n}_g$ corresponds to stimulated absorptions and the term $\bar{n}_e$ represents stimulated emissions.

III. EQUATION OF MOTION

The Hamiltonian of the free atom is given by

$$H_a=\hbar\omega_g P_g+\hbar\omega_e P_e\ , \tag{3.1}$$

and the transition frequency is $\omega_0=\omega_e-\omega_g$. The interaction between the atom and the driving laser, in the rotating-wave approximation, is

$$H_{ar}(t)=-\tfrac{1}{2}\hbar\Omega d e^{-i[\omega_L t+\phi(t)]}+\text{H.c.} \tag{3.2}$$

where

$$\Omega=\frac{E_0}{\hbar}\langle e|\boldsymbol{\mu}\cdot\mathbf{e}_L|g\rangle \tag{3.3}$$

is the (complex-valued) Rabi frequency. Relaxation due to the coupling of the atomic dipole to the vacuum gives rise to spontaneous decay and stimulated excitations, and this can be accounted for by a Liouville operator Γ, defined as

$$\Gamma(\Pi)=\tfrac{1}{2}A_e\{P_e\Pi+\Pi P_e-2d^\dagger\Pi d\}+\tfrac{1}{2}A_g\{P_g\Pi+\Pi P_g-2d\Pi d^\dagger\}\ . \tag{3.4}$$

This gives the effect of Γ on an arbitrary operator Π in Hilbert space. Then the equation of motion for the atomic density operator $\rho(t)$ becomes

$$i\hbar\frac{d\rho}{dt}=[H_a+H_{ar}(t),\rho]-i\hbar\Gamma\rho\ . \tag{3.5}$$

The time dependence in Eq. (3.5) can be simplified considerably by a transformation. Let us introduce the Liouville operator L_g by

$$L_g(\Pi)=[P_g,\Pi] \tag{3.6}$$

for Π arbitrary. Then the transformed density operator $\sigma(t)$ is defined as

$$\sigma(t)=e^{-i[\omega_L t+\phi(t)]L_g}\rho(t)\ , \tag{3.7}$$

and with Eq. (3.5) we find the equation of motion for $\sigma(t)$ to be

$$i\frac{d\sigma}{dt}=[L_d+\dot{\phi}(t)L_g-i\Gamma]\sigma\ . \tag{3.8}$$

Here we have used a few times the closure relation $P_g+P_e=1$. In terms of the detuning $\Delta=\omega_L-\omega_0$, the dressed atom Liouvillian L_d is defined as

$$L_d(\Pi)=\Delta L_g\Pi-\tfrac{1}{2}[\Omega d+\Omega^* d^\dagger,\Pi]\ . \tag{3.9}$$

The only time dependence left in Eq. (3.8) is through $\dot{\phi}(t)$.

The equation of motion (3.8) is a multiplicative stochastic differential equation, which has to be solved for the average $<\sigma(t)>$. This average obeys the equation of motion [23]

$$i\frac{d<\sigma>}{dt}=(L_d-iW_0-i\Gamma)<\sigma>\ . \tag{3.10}$$

The Liouville operator W_0 accounts for the laser-phase fluctuations, and is given by

$$W_0=\lambda_1 L_g^2\ , \tag{3.11}$$

with λ_1 the laser linewidth.

IV. STEADY-STATE SOLUTION

In the steady state, the time derivative of $<\sigma(t)>$ vanishes, and the equation of motion becomes

$$(L_d-iW_0-i\Gamma)\bar{\sigma}=0\ , \tag{4.1}$$

with

$$\bar{\sigma}=\lim_{t\to\infty}<\sigma(t)> \tag{4.2}$$

the steady-state transformed density operator. The original density operator $\rho(t)$, averaged over the laser fluctuations, has coherences which oscillate with ω_L in the long-time limit. It follows from the transformation (3.7) that the populations of ρ and σ are the same, and therefore the steady-state level populations follow from the solution of (4.1). This equation is subject to the constraints

$$\text{Tr}\bar{\sigma}=1,\quad \bar{\sigma}^\dagger=\bar{\sigma}\ . \tag{4.3}$$

The operator $L_d-iW_0-i\Gamma$ can be represented by a 4×4 matrix with respect to the bare atomic states (in Liouville space), and Eq. (4.1) can readily be solved. For the coherence we obtain

$$\langle e|\bar{\sigma}|g\rangle=-\frac{1}{2}\Omega\frac{A(\Delta-i\eta)}{\Omega_0^2\eta+A(1+R)(\Delta^2+\eta^2)}\ , \tag{4.4}$$

where we have set

$$\Omega_0=|\Omega|,\quad \eta=\lambda_1+\tfrac{1}{2}A(1+R)\ . \tag{4.5}$$

The population difference between the ground state and the excited state, which equals the total strength of the spectrum, according to Eq. (2.9), is found to be

$$\bar{n}_g-\bar{n}_e=\frac{A(\Delta^2+\eta^2)}{\Omega_0^2\eta+A(1+R)(\Delta^2+\eta^2)}\ . \tag{4.6}$$

From $\text{Tr}\bar{\sigma}=1$ it follows that

$$\bar{n}_g+\bar{n}_e=1\ , \tag{4.7}$$

and Eqs. (4.6) and (4.7) can then be combined to give the populations of both levels. Equation (4.6) shows that increasing the phase-conjugate reflectivity R tends to reduce $\bar{n}_g-\bar{n}_e$. Then stimulated transitions followed by enhanced spontaneous decay, as in Fig. 1, dominate laser-induced transitions combined with ordinary decay. Both mechanisms tend to saturate the upper state ($\bar{n}_e\approx\frac{1}{2}$), but in a different way. This leads to different absorption profiles, as will be shown later.

V. ATOMIC CORRELATION FUNCTION

For the evaluation of the absorption profile we need the correlation function $<\langle[d^\dagger(t'),d(t)]\rangle>$ for $t'\geq t$. With the evolution operator $X(t',t)$ for the atomic density operator $\rho(t)$, defined by

$$\rho(t')=X(t',t)\rho(t),\quad t'\geq t\ , \tag{5.1}$$

we can transform the expression for the correlation function to the Schrödinger picture. This yields

$$\langle[d^\dagger(t'),d(t)]\rangle=\mathrm{Tr}d^\dagger X(t',t)[d,\rho(t)],\quad t'\geq t\ , \tag{5.2}$$

and transforming to the σ representation gives

$$\langle[d^\dagger(t'),d(t)]\rangle=e^{-i\{\omega_L(t'-t)+\phi(t')-\phi(t)\}}\times\mathrm{Tr}d^\dagger Y(t',t)[d,\sigma(t)] \tag{5.3}$$

for $t'\geq t$. Here, $Y(t',t)$ is the time evolution operator for $\sigma(t)$. Then we let $t'=t+\tau$ and take the average over the laser fluctuations, with the result [23]

$$<\langle[d^\dagger(t+\tau),d(t)]\rangle>=e^{-i\omega_L\tau}\mathrm{Tr}d^\dagger e^{-i(L_d-iW_+-i\Gamma)\tau}\times[d,<\sigma(t)>] \tag{5.4}$$

for $\tau\geq 0$. The operator W_+ accounts for the laser linewidth and is found to be

$$W_+(\Pi)=\lambda_1\{P_e\Pi P_e+P_g\Pi P_g\}+\lambda_2 P_g\Pi P_e \tag{5.5}$$

for Π arbitrary, and with λ_2 a second parameter for the independent-increment process. For the phase-diffusion model we have $\lambda_2=4\lambda_1$, but in general these parameters are independent, with the restriction

$$0\leq\lambda_2\leq 4\lambda_1\ . \tag{5.6}$$

For the steady state, $t\to\infty$, we replace $<\sigma(t)>$ by $\bar{\sigma}$.

VI. ABSORPTION PROFILE

With Eq. (5.4), the normalized steady-state absorption profile becomes

$$J(\omega)=\frac{1}{\pi}\mathrm{Re}\int_0^\infty d\tau\, e^{i(\omega-\omega_L)\tau}\mathrm{Tr}d^\dagger\times e^{-i(L_d-iW_+-i\Gamma)\tau}[d,\bar{\sigma}]\ . \tag{6.1}$$

Then we introduce the frequency relative to ω_L,

$$\Lambda=\omega-\omega_L\ , \tag{6.2}$$

and evaluate the integral. This gives the formal result

$$J(\omega)=\frac{1}{\pi}\mathrm{Re}\,\mathrm{Tr}d^\dagger\frac{i}{\Lambda-L_d+iW_++i\Gamma}[d,\bar{\sigma}] \tag{6.3}$$

in terms of an operator inversion.

In a matrix representation with respect to the atomic states, expression (6.3) can easily be evaluated. With the results from Sec. IV for $\bar{\sigma}$, we then obtain

$$\begin{aligned}J(\omega)=(\bar{n}_g-\bar{n}_e)\frac{1}{\pi}\mathrm{Re}\{\,&\Omega_0^2[\tfrac{1}{2}A(1+R)+\tfrac{1}{2}\lambda_2-i\Lambda]\\&+[A(1+R)+\lambda_1-i\Lambda][\tfrac{1}{2}A(1+R)-i(\Delta+\Lambda)][\tfrac{1}{2}A(1+R)+\lambda_2+i(\Delta-\Lambda)]\}^{-1}\\&\times\left[\tfrac{1}{2}\Omega_0^2+[\tfrac{1}{2}A(1+R)+\lambda_2+i(\Delta-\Lambda)]\left[\tfrac{1}{2}i\frac{\Omega_0^2}{\Delta-i\eta}+A(1+R)+\lambda_1-i\Lambda\right]\right]\ .\end{aligned} \tag{6.4}$$

The factor $\bar{n}_g-\bar{n}_e$ equals the total strength, according to Eq. (2.9), and therefore the factor $\pi^{-1}\mathrm{Re}(\ )$ represents the frequency distribution, normalized to unity, of the absorption profile.

Without the driving laser ($\Omega_0\to 0$), expression (6.4) reduces to

$$J(\omega)=\frac{1}{1+R}\frac{1}{\pi}\mathrm{Re}\frac{1}{\frac{1}{2}A(1+R)-i(\omega-\omega_0)}\ , \tag{6.5}$$

which is a Lorentzian with a HWHM equal to $(1+R)A/2$ and a strength of $1/(1+R)$. For $R=0$ this reduces to the absorption profile of an atom in empty space, as is given by Eq. (1.3). Consequently, the presence of the PC broadens the line by a factor of $(1+R)$ and diminishes the strength by the same factor. Stimulated transitions from the ground state to the excited state, as in Fig. 1, give $|e\rangle$ a finite population, and probe-photon transitions from this level are stimulated emissions which diminish the net absorption. Hence the presence of the PC tends to wash out the absorption line, and it has been suggested that this might be a way to observe the occurrence of three-photon events [24]. Alternatively, the population of the excited state could be measured directly, or the occurrence of spontaneous fluorescence with a spectral line at frequency $2\bar{\omega}-\omega_0$ (see Fig. 1) could be observed.

VII. SECULAR LIMIT

Another limit of interest arises when either the Rabi frequency Ω_0 or the absolute value of the detuning, $|\Delta|$, is large compared to A, λ_1, and λ_2. Then the operator L_d dominates the time evolution of the density operator in Eq. (3.10) and the time regression of the correlation function in Eq. (5.4). In this limit, the coupling between eigenvectors of L_d with different eigenvalues can be neglected. This is called the secular approximation [1].

The laser-dressed atomic states in Hilbert space are

$$|+\rangle=|e\rangle\cos\tfrac{1}{2}\theta+|g\rangle e^{-i\psi}\sin\tfrac{1}{2}\theta\ , \tag{7.1}$$

$$|-\rangle=|g\rangle\cos\tfrac{1}{2}\theta-|e\rangle e^{i\psi}\sin\tfrac{1}{2}\theta\ , \tag{7.2}$$

with

$$\theta=\arctan(\Omega_0/\Delta),\quad \psi=\arg(\Omega)\ . \tag{7.3}$$

Then the eigenvalue equations for L_d are

$$L_d|+\rangle\langle+|=L_d|-\rangle\langle-|=0\ , \tag{7.4}$$

$$L_d|+\rangle\langle-|=-\Omega'|+\rangle\langle-|\ , \tag{7.5}$$

$$L_d|-\rangle\langle+|=\Omega'|-\rangle\langle+|\ , \tag{7.6}$$

where

$$\Omega'=\mathrm{sgn}(\Delta)\sqrt{\Delta^2+\Omega_0^2}\ . \tag{7.7}$$

The dressed states $|+\rangle$ and $|-\rangle$ are the eigenstates of the Hamiltonian H_d corresponding to the Liouvillian L_d, and $\hbar\Omega'$ equals the separation between the eigenvalues of H_d.

The steady-state density operator becomes in the secular limit

$$\bar{\sigma}=|+\rangle\bar{n}_+\langle+|+|-\rangle\bar{n}_-\langle-|\ , \tag{7.8}$$

as can be found by solving Eq. (4.1) in the secular approximation. It is diagonal with respect to the dressed states, and the populations can be parametrized as

$$\bar{n}_+=\frac{v}{u+v},\quad \bar{n}_-=\frac{u}{u+v}\ , \tag{7.9}$$

where

$$u=A_e g_-^2+A_g g_+^2+2\lambda_1 g_0^2\ , \tag{7.10}$$

$$v=A_e g_+^2+A_g g_-^2+2\lambda_1 g_0^2\ . \tag{7.11}$$

Here we have introduced the optical parameters

$$g_\pm=\frac{\Omega'\mp\Delta}{2\Omega'},\quad g_0=\frac{\Omega_0}{2\Omega'}\ , \tag{7.12}$$

which only depend on the Rabi frequency and the detuning. It then follows that

$$u-v=A\cos\theta>0\ , \tag{7.13}$$

and therefore the population of $|-\rangle$ is larger than the population of $|+\rangle$.

The total strength of the absorption profile is

$$\bar{n}_g-\bar{n}_e=\frac{A\cos^2\theta}{u+v} \tag{7.14}$$

in the secular limit. For a strong field we have $\theta=\pi/2$, and this strength goes to zero. This is consistent with the fact that $|e\rangle$ and $|g\rangle$ become populated equally, so that the rate of stimulated emission equals the rate of stimulated absorption.

VIII. SPECTRUM OF SEPARATED LINES

Expression (6.3) for $J(\omega)$ can be evaluated in the secular limit, and the result is

$$J(\omega)=\frac{1}{\pi}\mathrm{Re}\left\{\frac{S_F}{y_F-i(\Lambda+\Omega')}-\frac{S_T}{y_T-i(\Lambda-\Omega')}\right\}\ . \tag{8.1}$$

The F line is a Lorentzian with strength S_F, HWHM equal to y_F, and is located at $\Lambda=-\Omega'$, which is near the atomic resonance ω_0. The T line has a width of y_T and is positioned around $\Lambda=\Omega'$. Its strength $-S_T$ is negative, indicating stimulated emission. The expressions for the strengths and the widths are found to be

$$S_F=\frac{A\cos\theta}{u+v}g_-^2\ , \tag{8.2}$$

$$S_T=\frac{A\cos\theta}{u+v}g_+^2\ , \tag{8.3}$$

$$y_F=\tfrac{1}{2}A(1+R)(1+2g_0^2)+2\lambda_1 g_0^2+\lambda_2 g_+^2\ , \tag{8.4}$$

$$y_T=\tfrac{1}{2}A(1+R)(1+2g_0^2)+2\lambda_1 g_0^2+\lambda_2 g_-^2\ . \tag{8.5}$$

The total strength is

$$S_F-S_T=\frac{A\cos^2\theta}{u+v}\ , \tag{8.6}$$

which is consistent with Eq. (7.14). This shows that S_F is always larger than S_T, which can also be seen from their ratio

$$\frac{S_T}{S_F}=(\tan\tfrac{1}{2}\theta)^4<1\ . \tag{8.7}$$

Notice that this ratio is independent of A, R, λ_1, and λ_2. The widths of the lines are related by

$$y_T-y_F=\lambda_2\cos\theta\geq 0\ , \tag{8.8}$$

showing that the T line is always wider than the F line. This difference is determined by the second parameter of the independent-increment process for the laser phase, which suggests that this parameter could be determined experimentally by measuring the widths of the F and T lines.

IX. STRONG DRIVING FIELD

If the laser field is sufficiently strong, the secular approximation applies. In addition we shall assume $\Omega_0\gg|\Delta|$. Then the strengths of the lines become

$$S_F=S_T=\frac{A\cos\theta}{2A(1+R)+4\lambda_1}\ , \tag{9.1}$$

which goes to zero because $\theta\to\pi/2$. Nevertheless, the ratio of the strengths approaches unity, according to Eq. (8.7). The widths of the lines are

$$y_F=y_T=\tfrac{3}{4}A(1+R)+\tfrac{1}{2}\lambda_1+\tfrac{1}{4}\lambda_2\ . \tag{9.2}$$

In this saturation limit the spectrum has two lines of equal but opposite strengths, located at $\Lambda=-\Omega'$, and $\Lambda=\Omega'$, and they both have a width of $\frac{3}{4}A(1+R)$, plus corrections due to the finite laser linewidth. Saturation can alternatively be achieved by increasing the phase-conjugate reflectivity R (this parameter can have any positive value, and can easily exceed unity), and without a driving laser. Then the absorption profile is given by Eq.

(6.5), with R large. In this case the spectrum consists of only the F line, which has a width of $\frac{1}{2}A(1+R)$ and a strength of $1/(1+R)$. The strength of this line also tends to zero. Apparently, here we have two mechanisms for obtaining saturation, but in the saturation limit both systems have a different absorption profile.

X. CONCLUSIONS

We have studied the probe absorption spectrum of a laser-driven two-state atom near the surface of a PC. Without the laser field, the absorption spectrum is a single line at the position of the atomic resonance ω_0. The line is broadened by a factor of $1+R$, and the strength is diminished by the same factor. The latter is the result of the fact that the upper level becomes populated, thereby changing stimulated absorptions of probe photons into stimulated emissions. Since the reflectivity R of the PC varies with the intensity and frequency of the pump beams of the nonlinear crystal (in a complicated way), this parameter R can be controlled in an experiment. This opens the possibility of measuring indirectly the occurrence of the three-photon events from Fig. 1.

When the atom is irradiated by a laser beam near resonance, the absorption line shifts to $\omega=\omega_L-\Omega'$, which is the position of the F line in the emission spectrum. In addition, a second line appears at $\omega=\omega_L+\Omega'$, the position of the T line. At $\omega=\omega_L$ the emission spectrum has a third line, but at this frequency stimulated emission cancels stimulated absorption, so that there is no line at $\omega=\omega_L$ in the probe spectrum. Both the F and T line broaden linearly with R, as follows from Eqs. (8.4) and (8.5). The line strengths only depend on R through $u+v$, as seen from Eqs. (8.2) and (8.3), and this factor can be written as

$$u+v=A(1+R)(g_+^2+g_-^2)+4\lambda_1 g_0^2\ . \tag{10.1}$$

Therefore, both lines weaken with R, and both at the same rate.

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