Index of refraction of a system of strongly driven two-level atoms

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We study the refractive index of a system of two-level atoms driven by a strong coherent external field. We show that this system can produce a large index of refraction accompanied by vanishing absorption.

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The radiative properties of a strongly driven system of atoms have long been a subject of extensive research in quantum optics [1]. In an early theoretical study, Mollow predicted that strongly driven atoms would display a three-peaked fluorescence spectrum, quite different from that observed for weak-field resonance fluorescence [2]. This spectrum was soon observed by a number of experimental groups [3]. Resonance fluorescence is a "physical laboratory" in which many quantum phenomena can be detected such as photon antibunching [4], sub-Poissonian photon statistics [5], and squeezing [6,7]. The absorption spectrum of a probe field by the driven atoms is also dramatically different from that by the bare atom [8]. Innumerable articles have been written over the past 20 years on the subjects of resonance fluorescence and absorption by a strongly driven atom in a monochromatic [9] and more recently bichromatic field [10]. In addition, the technique of high-resolution saturation spectroscopy [11] is based on the absorption properties of driven atoms.

On the other hand, the dispersion of radiation by matter is certainly one of its most fundamental physical properties [11–13]. The usual dispersion-absorption relations tell us that at frequencies at which the index of refraction is large, absorption by the medium is also very large. Near the resonance frequency, the index of refraction of a gaseous medium at ≈ 1 Torr can reach values as high as 10 or 100, but at these frequencies the absorption is very high as well, and an incident light beam would be absorbed within a distance of a small fraction of a wavelength. Far from resonance, where the gas becomes transparent, the index of refraction is almost the same as in a vacuum. Recently, schemes aimed at achieving a medium having a high index of refraction accompanied by vanishing absorption via atomic coherence in systems of three-level [14] and multilevel [15] atoms have become an area of active interest.

There is a natural fundamental question: What are the refractive index and dispersion-absorption relations of a system of two-level atoms driven by a strong, near-resonant external field? In this paper we answer this question. In particular, we show that in a range of frequencies the driven atoms produce a high index of refraction accompanied by vanishing absorption. The two-level system is simpler than the three- or four-level atomic systems suggested by Scully and co-workers [14,15] and an optical material having a high index of refraction along with vanishing absorption has potential applications in fundamental [14,16] and applied physics [17].

We consider a two-level atom, with excited state $|2\rangle$, ground state $|1\rangle$, and transition frequency ω_a. The atom is driven by a coherent external field at a frequency ω_L with resonant Rabi frequency Ω, and damped at the rate γ by spontaneous emission. In the interaction picture (with $\hbar=1$), the master equation for the atomic system has the form

$$\frac{\partial}{\partial t}\rho=-i[H,\rho]+\tfrac{1}{2}\gamma L\rho \ , \tag{1}$$

where

$$H=\tfrac{1}{2}\Delta_a\sigma_z+\frac{\Omega}{2}(\sigma_{21}-\sigma_{12}) \ , \tag{2}$$

$$L\rho=2\sigma_{12}\rho\sigma_{21}-\sigma_{21}\sigma_{12}\rho-\rho\sigma_{21}\sigma_{12} \ , \tag{3}$$

$$\sigma_z=\sigma_{22}-\sigma_{11} \ . \tag{4}$$

Here L is the operator representing the damping of the atom via spontaneous emission; σ_{ij} are the atomic operators $\sigma_{ij}=|i\rangle\langle j|$ $(i,j=1,2)$; and $\Delta_a=\omega_a-\omega_L$ is the detuning of the atomic resonance frequency ω_a from the driving field frequency ω_L.

We now suppose that the system of driven atoms is weakly perturbed by a monochromatic probe field at frequency ν, in whose dispersion and absorption we are interested. The linear susceptibility $\chi(\nu)$ of the probe field at frequency ν is given in terms of the Fourier transform of the average value of the two-time commutator of the atomic operator as [8,18]

$$\chi(\nu)=iNp^2\int_0^\infty\langle[\sigma_{12}(\tau),\sigma_{21}]\rangle_s e^{i\nu t}d\tau \ , \tag{5}$$

where the index s indicates the steady state of the atomic system, p is the transition dipole matrix element of the atom, and N is the atomic density.

The equations of motion for the mean atomic values $\langle\sigma_{ij}(t)\rangle$ $(i,j=1,2)$ can be derived from the master equation (1) as

$$\frac{\partial}{\partial t}\langle\sigma_{12}(t)\rangle^*=\frac{\partial}{\partial t}\langle\sigma_{21}(t)\rangle = \left[-\frac{\gamma}{2}+i\Delta_a\right]\langle\sigma_{21}(t)\rangle-\frac{i}{2}\Omega\langle\sigma_z(t)\rangle \ , \tag{6}$$

$$\frac{\partial}{\partial t}\langle\sigma_z(t)\rangle=-\gamma[1+\langle\sigma_z(t)\rangle]-i\Omega[\langle\sigma_{21}(t)\rangle-\langle\sigma_{12}(t)\rangle] \ . \qquad (7)$$

The steady-state solution of Eqs. (6) and (7) is

$$\langle\sigma_z\rangle_s=-\frac{\gamma^2+4\Delta_a^2}{\gamma^2+4\Delta_a^2+2\Omega^2} , \qquad (8)$$

$$\langle\sigma_{21}\rangle_s=\langle\sigma_{12}\rangle_s^*=\frac{i\Omega}{-\gamma+2\Delta_a}\langle\sigma_z\rangle_s \ . \qquad (9)$$

From the quantum regression theorem [19], we know that for $\tau\geq 0$ the two-time average $\langle\sigma_{ij}(\tau)\sigma_{kl}\rangle_s$ $(i,j,k,l=1,2)$ satisfies the same equation of motion as the one-time average $\langle\sigma_{ij}(\tau)\rangle$. The equations of motion for $\Phi_1(\tau)=\langle[\sigma_{12}(\tau),\sigma_{21}]\rangle_s$, $\Phi_2(\tau)=\langle[\sigma_{21}(\tau),\sigma_{21}]\rangle_s$, and $\Phi_3(\tau)=\langle[\sigma_z(\tau),\sigma_{21}]\rangle_s$ thus take the form

$$\frac{\partial}{\partial\tau}\Phi_1(\tau)=\left[-\frac{\gamma}{2}-i\Delta_a\right]\Phi_1(\tau)+\frac{i}{2}\Omega\Phi_3(\tau) , \qquad (10)$$

$$\frac{\partial}{\partial\tau}\Phi_2(\tau)=\left[-\frac{\gamma}{2}+i\Delta_a\right]\Phi_2(\tau)-\frac{i}{2}\Omega\Phi_3(\tau), \qquad (11)$$

$$\frac{\partial}{\partial\tau}\Phi_3(\tau)=i\Omega[\Phi_1(\tau)-\Phi_2(\tau)]-\gamma\Phi_3(\tau) , \qquad (12)$$

subject to the initial conditions

$$\Phi_1(0)=-\langle\sigma_z\rangle_s , \qquad (13)$$

$$\Phi_2(0)=0 , \qquad (14)$$

$$\Phi_3(0)=2\langle\sigma_{21}\rangle_s , \qquad (15)$$

where $\langle\sigma_z\rangle_s$ and $\langle\sigma_{21}\rangle_s$ are given in Eqs. (8) and (9).

The above equations are simple in form and can be solved exactly. The linear susceptibility $\chi(\nu)$ of the probe field is then given as

$$\chi(\nu)=\tfrac{1}{2}Np^2\langle\sigma_z\rangle_s\frac{\Omega^2\Delta_\nu-2iD(\nu)}{(\gamma/2+i\Delta_a-i\Delta_\nu)D(\nu)+\Omega^2(\gamma/2+i\Delta_a)(\gamma/2-i\Delta_\nu)} , \qquad (16)$$

where

$$D(\nu)=\left[\frac{\gamma}{2}-i\Delta_a\right]\left[\frac{\gamma}{2}-i\Delta_a-i\Delta_\nu\right](\gamma-i\Delta_\nu) \ . \qquad (17)$$

Here $\Delta_\nu=\nu-\omega_L$ is the detuning of the probe field from the driving frequency. The linear susceptibility (16) can be written as

$$\chi(\nu)=\chi'(\nu)+i\chi''(\nu) , \qquad (18)$$

where the real (χ') and imaginary (χ'') parts of χ determine the index of refraction and the absorption coefficient, respectively, of the probe field. The imaginary part $\chi''(\nu)$ has been evaluated and discussed by Mollow [8] and many others. However, $\chi'(\nu)$ has not previously been explicitly presented (to our knowledge).

In Fig. 1, χ' (solid curve) and χ'' (dashed curve) are displayed (in units of $Np^2/\gamma=1$) as a function of Δ_ν/γ for the case of $\Delta_a=0$ and $\Omega/\gamma=2$. Clearly, near the fluorescence sideband frequencies $\nu=\omega_L\pm\Omega$ the index of refraction (χ') reaches a maximum value, while the absorption vanishes. This property is dramatically different from that of the bare atom, Fig. 2, where χ' is large at the same near-resonance frequencies at which χ'' is large. In Fig. 3, we plot χ' and χ'' for the case of $\Delta_a/\gamma=0.5$ and $\Omega/\gamma=2$. As in the case of exact resonance ($\Delta_a=0$), the

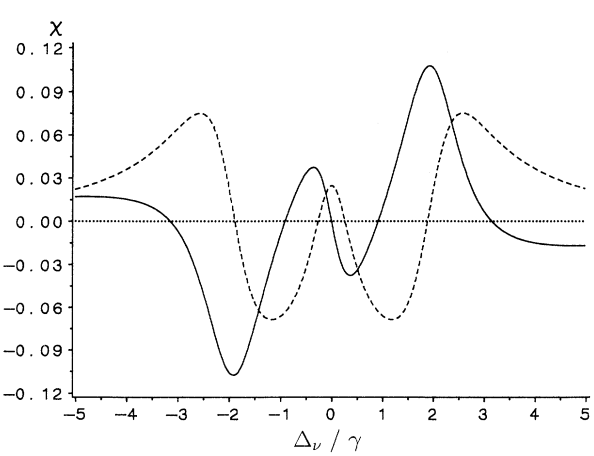

FIG. 1. Real χ' (solid) and imaginary χ'' (dashed) parts of the linear susceptibility (in units of $Np^2/\gamma=1$) of a probe field in a system of driven atoms as a function of Δ_ν/γ for $\Delta_a/\gamma=0$ and $\Omega/\gamma=2$.

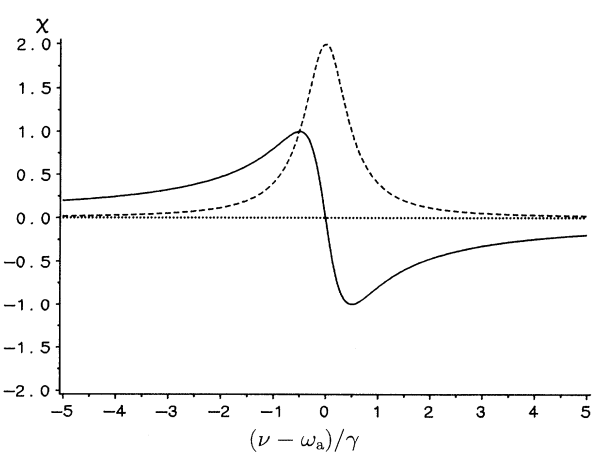

FIG. 2. Real χ' (solid) and imaginary χ'' (dashed) parts of the linear susceptibility (in units of $Np^2/\gamma=1$) in a system of the bare atoms ($\Omega/\gamma=0$) as a function of $(\nu-\omega_a)/\gamma$.

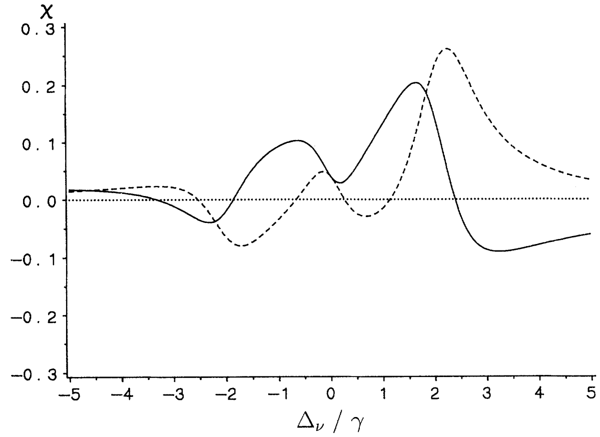

FIG. 3. Real χ' (solid) and imaginary χ'' (dashed) parts of the linear susceptibility (in units of $Np^2/\gamma=1$) in a system of the driven atoms as a function of Δ_ν/γ for $\Delta_a/\gamma=0.5$ and $\Omega/\gamma=2$.

index of refraction remains large while the absorption vanishes. Comparing Figs. 1 and 3 with Fig. 2 we see that in the system of driven atoms, the index of refraction is of the same order of magnitude as in that of the bare atoms. Thus the refractive index of a gaseous medium of driven atoms at ≈ 1 Torr can reach a very high value, while the absorption vanishes.

In conclusion, we have calculated the index of refraction and the dispersion-absorption relation of a system of driven two-level atoms and have shown that the driven atoms can produce a high index of refraction accompanied by vanishing absorption. A survey of existing schemes to produce such a transparent, high-index material has been given by Scully and co-workers ([14,15], and references therein). All of these schemes involve three or more atomic levels, with a pair of these levels coherently coupled by a strong auxiliary field either before atomic injection or simultaneously with the test field; clearly, a two-level atom allows far more freedom both in atomic selection and in the operation of the device. Some potential applications of a transparent, high-index optical material have been suggested in Refs. [14,16,17]; e.g., an optical interferometer based on a highly dispersive material [17] is shown to have potentially superior sensitivity to that of present devices.

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