Spontaneous emission into an electromagnetically induced transparency

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We investigate spontaneous emission into an electromagnetically produced transparency of the form recently proposed [A. Imamoğlu and S. E. Harris, Opt. Lett. **14**, 1344 (1989)]. We show that the achievable radiation temperature (or brightness) at the transparency is much greater than the atomic temperature.

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In this Brief Report we investigate the spontaneous emission from an electromagnetically produced transparency of the form recently discussed by Harris, Field, and Imamoğlu [1], and Imamoğlu and Harris [2]. The difference between the absorption and emission profiles of a three-state atom in the presence of a strong-coupling field was first studied by Mollow [3]. Transparencies in these systems have been demonstrated by Stroud and co-workers [4] and Orriols and co-workers [5]. The brightness of a spontaneous radiator is determined by the emissivity divided by the absorption coefficient. Harris [6] first noted that because the optical depth is increased at the frequency of the transparency and the spontaneous-emission rate is not decreased proportionately, the brightness that can be achieved at an optical depth is greater than that which would be predicted based on the Planck blackbody formula [7]. This is not a hidden atomic basis-set effect, since the total population in all excited states even if coherently phased and radiating from a single state would not produce the large brightness predicted.

The prototype system that we consider is shown in Fig. 1. The basic system is a three-state atom coupled to three thermal reservoirs at the three transition frequencies of the atom. Additionally, states $|2\rangle$ and $|3\rangle$ are strongly coupled by a laser field, creating a Rabi frequency Ω_c. We consider weak spontaneous emission from state $|3\rangle$ to state $|1\rangle$. This system is similar to a system that we recently proposed for a laser without inversion [8]. We have in the present case included the possibility of a pumping rate from state $|1\rangle$ directly to state $|2\rangle$. In the limit of this additional rate being small, we will obtain the lasing threshold condition given in Ref. [8].

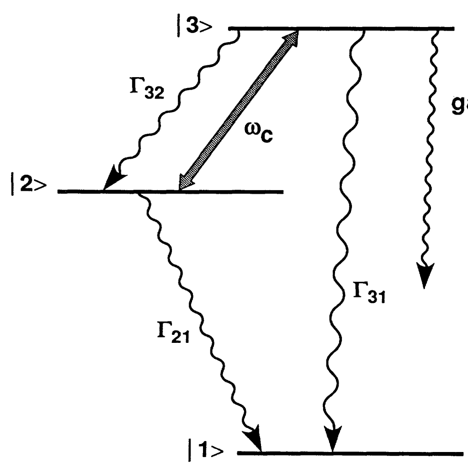

FIG. 1. Prototype energy-level diagram for a spontaneously radiating system. Γ_{21}, Γ_{31}, and Γ_{32} represent couplings to thermal reservoirs (i.e., spontaneous-emission rates and thermal pumping) and $g\hat{a}^{\dagger}$ represents the spontaneous mode that we are considering. ω_c is the coupling field between states $|2\rangle$ and $|3\rangle$.

Our calculation is based on the density matrix. For this calculation, we neglect any collisional dephasing. The $|2\rangle$-$|3\rangle$ coupling laser, Ω_c, is assumed to be in a coherent state and single mode. The thermal reservoirs at the $|2\rangle-|1\rangle$, $|3\rangle-|1\rangle$, and $|3\rangle-|2\rangle$ transition frequencies are assumed to be uncoupled and at independent temperatures T_{21}, T_{31}, and T_{32}, respectively. The mode into which we consider spontaneous emission to occur, $\hat{a}$, is at frequency ω_p and is assumed to be very weak. The Hamiltonian is

$$H=E_{21}|2\rangle\langle 2|+E_{31}|3\rangle\langle 3| +(\hbar\Omega_c/2)\exp(i\omega_{32}t)|3\rangle\langle 2|+\text{c.c.}] +\hbar\omega_p\hat{a}^{\dagger}\hat{a}+(\hbar g\hat{a}^{\dagger}|1\rangle\langle 3|+\text{c.c.}) +\sum_k \hbar\omega_k\hat{b}_k^{\dagger}\hat{b}_k+\left[\sum_{ij}\hbar g_{ij}(k)\hat{b}_k^{\dagger}|i\rangle\langle j|+\text{c.c.}\right], \tag{1}$$

where g is the coupling constant [9] between the spontaneous mode and the $|1\rangle-|3\rangle$ transition of the atom; $g_{ij}(k)$ is the atomic coupling to the normal vacuum modes, $\hat{b}_k$, resulting in spontaneous emission and thermal pumping. We trace over the thermal reservoirs leaving only the atomic density matrix and the weak spontaneous field mode [9]. In this trace, the thermal reservoirs are assumed to have a flat photon occupation number in frequency. We can now write the coupled density matrix equations for this reduced system:

$$\hat{\rho}_{11}+\hat{\rho}_{22}+\hat{\rho}_{33}=\hat{\rho}^{f}, \tag{2a}$$

$$\frac{d\hat{\rho}_{11}}{dt}=-ig\hat{a}^{\dagger}\rho_{31}+ig^{*}\hat{\rho}_{13}\hat{a}+\Gamma_{21}(1+n_{21})\hat{\rho}_{22} +\Gamma_{31}(1+n_{31})\hat{\rho}_{33}-(\Gamma_{21}n_{21}+\Gamma_{31}n_{31})\hat{\rho}_{11}, \tag{2b}$$

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$$\frac{d\hat{\rho}_{22}}{dt} = -i\Omega_c/2(\hat{\rho}_{32}-\hat{\rho}_{23}) - \Gamma_{21}(1+n_{21})\hat{\rho}_{22} + \Gamma_{32}(1+n_{32})\rho_{33} + \Gamma_{21}n_{21}\hat{\rho}_{11} - \Gamma_{32}n_{32}\hat{\rho}_{22} , \tag{2c}$$

$$\frac{d\hat{\rho}_{33}}{dt} = i\Omega_c/2(\hat{\rho}_{32}-\hat{\rho}_{23}) + ig\hat{\rho}_{31}\hat{a}^\dagger - ig^*\hat{a}\hat{\rho}_{13} + \Gamma_{31}n_{31}\hat{\rho}_{11} + \Gamma_{32}n_{32}\hat{\rho}_{22} - [\Gamma_{31}(1+n_{31}) + \Gamma_{32}(1+n_{32})]\hat{\rho}_{33} , \tag{2d}$$

$$\frac{d\hat{\rho}_{13}}{dt} = -ig\hat{a}^\dagger\hat{\rho}_{33} + ig\hat{\rho}_{11}\hat{a}^\dagger + i\Omega_c\hat{\rho}_{12}/2 - [\Gamma_{21}n_{21} + \Gamma_{31}(1+2n_{31})]\hat{\rho}_{13}/2 + i\Delta\omega_p\hat{\rho}_{13} , \tag{2e}$$

$$\frac{d\hat{\rho}_{32}}{dt} = -i\Omega_c(\hat{\rho}_{22}-\hat{\rho}_{33})/2 - ig^*\hat{a}\hat{\rho}_{12} - [\Gamma_{21}(1+n_{21}) + \Gamma_{31}(1+n_{31}) + \Gamma_{32}(1+2n_{32})]\hat{\rho}_{32}/2 + i\Delta\omega_c\hat{\rho}_{32} , \tag{2f}$$

$$\frac{d\hat{\rho}_{12}}{dt} = -ig\hat{a}^\dagger\hat{\rho}_{32} + i\Omega_c\hat{\rho}_{13}/2 - [\Gamma_{31}n_{31} + \Gamma_{21}(1+2n_{21}) + \Gamma_{32}n_{32}]\hat{\rho}_{12}/2 + i(\Delta\omega_p - \Delta\omega_c)\hat{\rho}_{12} , \tag{2g}$$

where Γ_{kl} and n_{kl} represent the spontaneous-emission rate and the average number of photons per mode at frequency ω_{kl}, respectively. $\Delta\omega_p = E_{31}/\hbar - \omega_p$ and $\Delta\omega_c = E_{31}/\hbar - E_{21}/\hbar - \omega_c$ are the detunings of the probe and coupling field frequencies. Note that these density equations are operator equations for the photon field described by $\hat{a}$ and $\hat{a}^\dagger$ [10]. To find a steady-state solution, we drop the time derivatives. To calculate the spontaneous emission, we develop a perturbation sequence in powers of the spontaneous mode coupling constant. First, we find the solution with $g=0$. With this steady-state solution in hand, we may reintroduce g as a small parameter and find the first-order correction to this solution. This correction term allows us to determine the steady-state photon occupation number of the spontaneous mode. In terms of the steady-state populations with all detunings set to zero ($\Delta\omega_p = \Delta\omega_c = 0$), this is

$$\frac{n_p}{1+n_p} = \frac{\rho_{33}}{\rho_{11}} + \frac{\Omega_c^2/4}{\gamma_2\gamma_3}\left[\frac{\rho_{22}-\rho_{33}}{\rho_{11}}\right] , \tag{3a}$$

where

$$\gamma_2 = \Gamma_{21}(1+2n_{21})/2 + \Gamma_{31}n_{31}/2 + \Gamma_{32}n_{32}/2 ,$$
$$\gamma_3 = \Gamma_{21}(1+n_{21})/2 + \Gamma_{31}(1+n_{31})/2 + \Gamma_{32}(1+2n_{32})/2 ,$$
$$n_p = \langle \hat{a}^\dagger\hat{a} \rangle . \tag{3b}$$

The complete expression including the dependence on Γ_{21}, Γ_{31}, and Γ_{32} is tedious to evaluate but presents no difficulties. Notice that in the absence of a coupling field Ω_c the spontaneous photon Boltzman factor is exactly equal to the atomic Boltzman factor for state $|3\rangle$ (ρ_{33}/ρ_{11}).

Figure 2 shows our result evaluated in the limit that $\Gamma_{21}, n_{21} \to 0$. This should be equivalent to the result that we obtained in Imamoğlu, Field and Harris [8] for the laser without inversion case. Notice that as $\Gamma_{32} \to \Gamma_{31}$ the steady-state photon number diverges. This is the lasing threshold condition as stated in Ref. [8] in the absence of collisional dephasing. As Γ_{32} exceeds Γ_{31} the steady-state photon temperature becomes negative, implying laser action. This system lases in steady state and does not satisfy the normal inversion condition for lasing that $\rho_{22}+\rho_{33} > \rho_{11}$. There is no inversion in any atomic basis set. It is not a normal Raman laser.

Figure 3 considers our result in the blackbody limit. In this case, we introduce a pumping rate from $|1\rangle$ (ground state) into $|2\rangle$ (i.e., $\Gamma_{21}, n_{21} > 0$). For now, we specifically neglect the decay Γ_{32} of state $|3\rangle$ to $|2\rangle$. Based on the results of Fig. 2, this decay will increase the emitted brightness, but conceptually it obscures our result. Provided $\Gamma_{31} \gg \Gamma_{21}$ and the radiation temperature of the $|1\rangle$-$|3\rangle$ reservoir is kept low, the spontaneous emission has a Boltzmann factor nearly equal to that of the $|1\rangle$-$|2\rangle$ reservoir and much greater than that of the atoms that are emitting the radiation. This is the primary result of this paper. Figure 3 shows the thermal occupation number of the spontaneous radiation as a function of $\Delta\omega_p$. The peak occurs at $\Delta\omega_c = \Delta\omega_p = 0$, the frequency of highest transparency. At this frequency, n_p is nearly as large as n_{21}, the thermal occupation number of the $|1\rangle$-$|2\rangle$ reservoir.

Under the conditions of Fig. 3, the ratio of the total excited-state population to ground-state population $[(\hat{\rho}_{22}+\hat{\rho}_{33})/\hat{\rho}_{11}]$ is less than 2×10^{-4}. Since the total population of the excited states is independent of a basis rotation, no rotation of the atomic basis can create an oc-

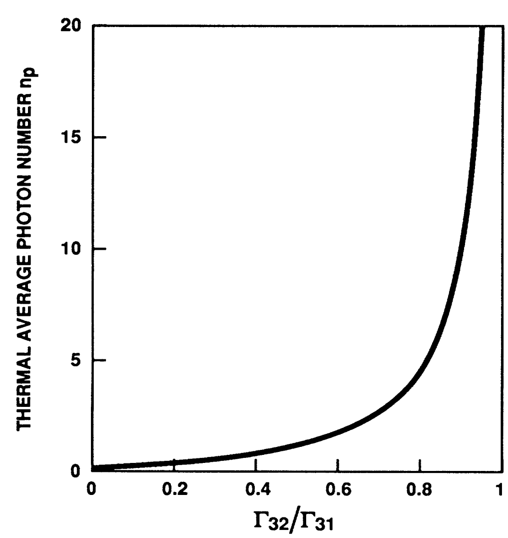

FIG. 2. Spontaneous photon number vs Γ_{32}/Γ_{31}. For this case, $\Gamma_{31}=1$, $n_{31}=0.1$, $\Gamma_{21}=n_{21}=n_{32}=\Delta\omega_p=0$, $\Delta\omega_c=0$, and Rabi frequency $\Omega_c=3$ in arbitrary units. The lasing threshold is reached as $\Gamma_{32}\to\Gamma_{31}$.

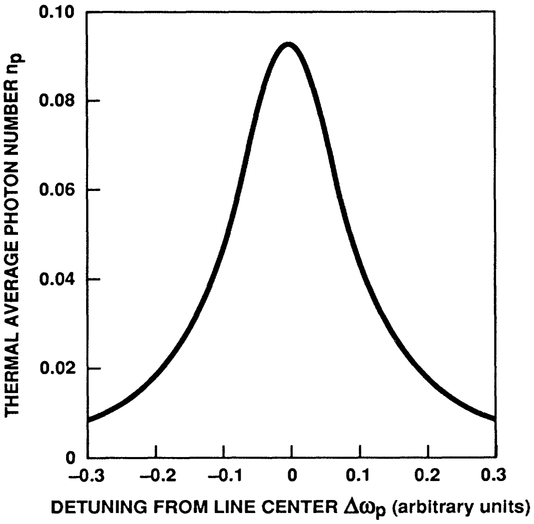

FIG. 3. Spontaneous photon number vs detuning $\Delta\omega_p$ from the transparency center ($\Delta\omega_c=0$). For this case, $\Gamma_{21}=0.001$, $n_{21}=0.1$, $\Gamma_{31}=1$, $n_{31}=\Gamma_{32}=n_{32}=0$, and $\Omega_c=3$. The peak value occurs at the center of the transparency and its value is close to the occupation number of the $|1\rangle$-$|2\rangle$ reservoir, $n_{21}=0.1$.

cupancy in an excited state larger than 2×10^{-4}. Since the process of emission of a spontaneous photon requires the stimulated absorption of a coupling laser photon in the field Ω_c, one may think of the spontaneous field as due to a stimulated Raman-type effect. It is important to note that a normal Raman-type effect would be unable to produce a photon occupation number greater than approximately 2×10^{-4} in this case. The coupling laser Ω_c creates a coherence between the $|1\rangle$-$|2\rangle$ reservoir and the spontaneous mode. This coherence allows the occupation number of the spontaneously emitted radiation to grow as large as n_{21}, the occupation number of the $|1\rangle$-$|2\rangle$ reservoir. The transparency allows the atoms to remain at a low temperature.

It is an apparent contradiction that the steady-state condition for the spontaneous mode is at a higher temperature than that of the $|1\rangle$-$|3\rangle$ reservoir. The resolution of this dilemma, of course, is that the atoms are not in thermal equilibrium to begin with and therefore the emission temperature in one mode need not be equal to the temperature of the environment in another mode. Nevertheless, this places a requirement on any practical way to realize the system. We need to impose a relatively cold temperature on the $|1\rangle$-$|3\rangle$ reservoir, and yet allow the spontaneous emission to build up at the transparency point.

To achieve the conditions called for in our calculation we require a cell that is long and narrow. The large aspect ratio allows most spontaneous photons to escape and thereby imposes the temperature of the surrounding environment on the atoms. Along the length of the atoms, the media must be optically thick in order to achieve the full photon occupation number predicted above. The spontaneous photons can be detected by placing a detector at one end of the long column of atoms. In general, practical constraints will dictate that the width of the atomic region be sufficient to capture some of the spontaneous photons. This will tend to increase the temperature of the $|1\rangle$-$|3\rangle$ reservoir and destroy the effect. In order to estimate this effect, we introduce an operator equation for the evolution of the $|1\rangle$-$|3\rangle$ thermal reservoir. This can be traced over the reservoir states just as the density equations are and then it becomes a c-number differential equation coupled to the density equations. As a crude approximation to this effect, we may use γ as the escape rate for photons and n_{ext} as the average photon number outside the atomic region. The equation for n_{31} is then

$$\frac{d\langle n_{31}\rangle}{dt}=\gamma(\langle n_{\text{ext}}\rangle-\langle n_{31}\rangle)+[\Gamma_{31}(1+\langle n_{31}\rangle)\rho_{33}-\Gamma_{31}\langle n_{31}\rangle\rho_{11}]/N\ , \quad (4)$$

where N is the number of modes being emitted into. This allows for a finite escape rate of photons from the optically dense region. The derivative may be set to zero and a steady-state solution may be found, which is consistent with the atomic populations and the thermal reservoir temperatures. The escape rate of photons for a particular case may be evaluated using the formulas for the optical absorption of a transparency given in Harris, Field, and Imamoğlu [1].

Practical considerations like collisional dephasing of the atoms and finite Doppler widths can be accounted for with the addition of the conventional terms in the density equations and averaging over atomic velocity distribution functions. These effects degrade the performance of the system, but do not render the effect unobservable.

We have analyzed the spontaneous emission into an electromagnetically induced transparency. The emitted brightness can be much greater than that which would be predicted based on the atomic populations and the Planck blackbody law. To achieve this large brightness, the thermal fields surrounding the atoms must be controlled using a geometry that allows photons of all wavelengths to escape.

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