Multiphoton ejection of strongly bound relativistic electrons in very intense laser fields

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We investigate multiphoton ejection probability of strongly bound electrons in relativistically intense laser fields. A solvable model of a Klein-Gordon electron bound in a finite-range separable potential and interacting with a circularly polarized plane-wave field is used for the analysis. For binding energies of the order of several keV the rates of electron ejection for ω=100 eV are found to be significant at relativistic intensities but are extremely small for ω=10 eV. For lower binding energies spectra are obtained for the available CO_2 laser frequency (ω=0.117 eV) and Nd laser frequency (ω=1.169 eV) Numerical results show the stabilization effect for both relativistic and nonrelativistic intensities and subthreshold frequencies.

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I. INTRODUCTION

Due to rapid developments [1] of laser intensity it is expected to be possible to experimentally investigate the interaction of deeply bound electrons (like those bound in highly charged ions) with very intense lasers. In view of the formidable mathematical difficulty of the necessary nonperturbative analysis involving real systems, it is of much interest to obtain a qualitative understanding of such problems from solvable models. An exactly solvable model for investigations of nonperturbative behavior of a bound electron interacting with a strong circularly polarized electromagnetic field is the well-known δ-potential model, which has been introduced by Berson [2] and Manakov and Rapoport [3]. This model, however, may not be used for intensities that are so high that the energy of oscillation of the electron in the field (quiver energy) becomes comparable to the rest mass energy, mc^2 of the electron. In other words, in the intensity domain

$$\frac{E_{\text{quiver}}}{mc^2} \approx 1, \tag{1}$$

it becomes necessary to analyze the problem both relativistically *and* nonperturbatively. For very strongly bound objects we need relativistic formulas also for the binding force itself.

Recently, we have introduced [4] a relativistic model of a bound Klein-Gordon (KG) electron interacting with a circularly polarized electromagnetic field (with full multipolar interaction) and obtained the exact solution of the corresponding KG equation. Since the KG equation involves the squaring of the potential, the zero-range δ potential, used in the nonrelativistic model [2, 3], cannot be used here. We have, therefore, chosen a separable finite-range potential that supports a discrete bound state (and the full continuum) as does the δ potential in the nonrelativistic model. Note that, originally, separable potentials have been introduced to study nuclear reactions [5–7] and more recently they have been used for the laser-atom interaction problem, in the nonrelativistic domain [8–12].

In this paper we investigate the total rate of ejection of deeply bound electrons, as a function of the binding energy and the field intensity, within the framework of the KG equation.

II. THE BOUND-STATE MODEL POTENTIAL

The KG equation of the model system is

$$[(i\partial_t - V_0|\tilde{\phi}\rangle\langle\tilde{\phi}|)^2 - \hat{\mathbf{p}}^2 - m^2]|\Psi(t)\rangle = 0 , \tag{2}$$

where the separable potential is defined in the form of a projection operator

$$\hat{V} = V_0|\tilde{\phi}\rangle\langle\tilde{\phi}| , \tag{3}$$

and the potential functions $\tilde{\phi}$ are taken in this work in the Gaussian form

$$\tilde{\phi}(\mathbf{x}) = N_0 e^{-\lambda^2 x^2} , \tag{4}$$

where $N_0 = (\frac{2\lambda^2}{\pi})^{3/4}$ is chosen such that

$$\langle\tilde{\phi}|\tilde{\phi}\rangle = 1 . \tag{5}$$

Note that the parameters λ and V_0 are arbitrary and can be chosen to imitate the bound state of interest.

The eigenstate $\Psi(\mathbf{x},t) = e^{-iEt}\Psi_E(\mathbf{x})$ of Eq. (2) can be readily seen to satisfy the integral equation

$$\Psi_E(\mathbf{x}) = \int d^3y\, G_E(\mathbf{x}-\mathbf{y})\tilde{\phi}(\mathbf{y})V_0(2E-V_0) \times \int d^3z\,\tilde{\phi}(\mathbf{z})\Psi_E(\mathbf{z}) , \tag{6}$$

and $G_E(\mathbf{x}-\mathbf{y})$ is the stationary Green's function of the free Klein-Gordon equation

$$(E^2 + \triangle_x - m^2)G_E(\mathbf{x}-\mathbf{y}) = \delta^{(3)}(\mathbf{x}-\mathbf{y}) . \tag{7}$$

If we now project both sides of Eq. (6) onto $\tilde{\phi}(\mathbf{x})$ and perform the integrals, we get the following equation for the eigenenergy E_0 of the bound state:

$$\frac{V_0(2E_0-V_0)}{2\lambda^3}\sqrt{m^2-E_0^2}\int_0^\infty dt \frac{e^{-t/2}}{\left(t+\frac{m^2-E_0^2}{\lambda^2}\right)^{3/2}} = -1 . \tag{8}$$

We can now fix one of the arbitrary parameters defining the potential by taking $\lambda = \sqrt{m^2-E_0^2}$, where E_0 is the solution of (8). In this way our energy eigenvalue equation becomes

$$\frac{V_0(2E_0-V_0)\xi}{2(m^2-E_0^2)} = -1, \tag{9}$$

and hence

$$E_0 = \sqrt{m^2 + \frac{V_0^2\xi^2}{4} - \frac{V_0^2\xi}{2}} + \frac{V_0\xi}{2} , \tag{10}$$

where $\xi = 2 - \sqrt{2\pi e}[1-\mathrm{erf}(\frac{\sqrt{2}}{2})] \approx 0.688\,641$ and erf is the error function [13]. If V_0 is negative this potential, according to (10), can support one positive-energy bound state [14].

III. LEVEL WIDTH IN A CIRCULARLY POLARIZED LASER FIELD

Let us now couple the system with the external laser field characterized by the electromagnetic potential A^μ. The KG equation satisfied by the wave function Ψ then reads

$$[(i\partial^\mu - eA^\mu)(i\partial_\mu - eA_\mu) - m^2]\Psi(x) = V_0(2i\partial_0 - V_0)\tilde{\phi}(\mathbf{x})\int d^3z\,\tilde{\phi}(\mathbf{z})\Psi(x_0,\mathbf{z}) , \tag{11}$$

in units where $c = \hbar = 1$. The potential A^μ has only spatial components

$$\mathbf{A}(x) = A_0[\mathbf{e}_1\cos(k\cdot x+\delta) - \mathbf{e}_2\sin(k\cdot x+\delta)] , \tag{12}$$

and $\mathbf{e}_1, \mathbf{e}_2$ are unit vectors in the directions of axes x and y. The four-vector k^μ in the frame we use has the form

$$k = (k_0, 0, 0, k_3) , \tag{13}$$

where $k^2 = 0$ and δ is a certain arbitrary initial phase of the field. We use the following notation for the products of four-vectors : $k\cdot x = k^\mu x_\mu = k_0x_0 - \mathbf{k}\cdot\mathbf{x}$.

We will now try to solve Eq. (11) similarly to the way we have done it, without laser field, in Sec. II, with the difference that the "eigenenergy" is now expected to be complex since we deal with an unstable situation. We will follow a path somewhat similar to that of Berson [2] and Manakov and Rapoport [3], the differences originating from the relativistic nature of the KG "electron" and non-δ character of the binding potential. The solution of (11) can formally be written as

$$\Psi(x) = \int d^4x' G(x,x')\tilde{\phi}(\mathbf{x}')V_0(2i\partial_0' - V_0) \times \int d^3y\,\tilde{\phi}(\mathbf{y})\Psi(x_0',\mathbf{y}) , \tag{14}$$

where $\tilde{\phi}$ is given by (4) and $G(x,x')$ is now the Green's function inside the field satisfying

$$[(i\partial^\mu - eA^\mu)(i\partial_\mu - eA_\mu) - m^2]G(x,x') = \delta^{(4)}(x-x'), \tag{15}$$

and containing for large r only outgoing waves. This Green's function can be found by using, for instance, the Fock-Schwinger proper time method [15, 16]. The result is the following:

$$G(x,x') = -\frac{1}{(4\pi)^2}\int_{-\infty}^0 \frac{d\tau}{\tau^2}\exp\left\{i\left[\frac{(x-x')^2}{4\tau} + \tau\left(2e^2A_0^2\frac{\cos k\cdot(x-x')-1}{[k\cdot(x-x')]^2} + e^2A_0^2 + m^2\right) + eA_0\left((x_1-x_1')\frac{\sin k\cdot x - \sin k\cdot x'}{k\cdot(x-x')} + (x_2-x_2')\frac{\cos k\cdot x - \cos k\cdot x'}{k\cdot(x-x')}\right)\right]\right\}. \tag{16}$$

If we put this into Eq. (14) and project both sides onto $\tilde{\phi}(\mathbf{x})$, we come to the following relation:

$$\chi(t) = \int d^3x \int d^3x' \int dt'\tilde{\phi}(\mathbf{x})G(\mathbf{x},t;\mathbf{x}',t')\tilde{\phi}(\mathbf{x}')V_0(2i\partial_{t'} - V_0)\chi(t') , \tag{17}$$

where $\chi(t) = \int d^3x\,\tilde{\phi}(\mathbf{x})\Psi(\mathbf{x},t)$.

We do not filter out, in this place, the time dependence of the kind $e^{in\omega t}$ (n photons absorbed or emitted) as has been done previously in nonrelativistic works [2, 3] to simplify the calculations, since the integration of the Green's function with spherically symmetric $\tilde{\phi}$ in (17) simplifies it automatically (more exactly, it is accomplished by the change of integration variables, as shown below). This observation is confirmed by the fact that the object $\int d^3x \int d^3x'\tilde{\phi}(\mathbf{x})G(\mathbf{x},t;\mathbf{x}',t')\tilde{\phi}(\mathbf{x}')$ depends only on $t-t'$. This is a great advantage of using circularly polarized light. To show this we rotate, around the third axis, the frames in which the integrals over x and x' are performed by the angle $-k\cdot(x+x')$ and make use of the fact that the form factor $\tilde{\phi}$ is an s-type function:

$$\int d^3x\int d^3x'\tilde{\phi}(\mathbf{x})G(\mathbf{x},t;\mathbf{x}',t')\tilde{\phi}(\mathbf{x}')$$
$$=-\frac{N_0^2}{(4\pi)^2}\int_{-\infty}^{0}\frac{d\tau}{\tau^2}\int d^3x\int d^3x' e^{-\lambda^2x^2}e^{-\lambda^2x'^2}$$
$$\times\exp\left\{i\left[\frac{(t-t')^2-(x_2-x_2')^2-(x_3-x_3')^2}{4\tau}+\tau(m^2+A_0^2)\right.\right.$$
$$\left.\left.-\frac{1}{4\tau}\left(x_1-x_1'-4eA_0\tau\frac{\sin\frac{1}{2}[k_0(t-t')-\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')]}{k_0(t-t')-\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}\right)^2\right]\right\}, \qquad (18)$$

where we use the old symbols x and x' for the new integration variables. In Eqs. (16) and (18), x_i's are ordinary spatial three-vector components. If we now take the Fourier transform of Eq. (17) we obtain the relation, which plays the role of the energetic eigenequation,

$$1=V_0(2E-V_0)<\tilde{\phi}|G_E|\tilde{\phi}>, \qquad (19)$$

where G_E is simply $\int_{-\infty}^{\infty}dTe^{iET}G(T)$ and $T=t-t'$. We shall use the shorthand notation $\langle\tilde{\phi}|G_E|\tilde{\phi}\rangle$ for $\int d^3x\int d^3x'\tilde{\phi}(\mathbf{x})G_E(\mathbf{x},\mathbf{x}')\tilde{\phi}(\mathbf{x}')$ below. The solution of Eq. (19) in general exists for a complex energy: $E_{\rm real}-i\frac{\gamma}{2}$ and γ is just the total ejection rate we are looking for.

In the momentum representation $\langle\tilde{\phi}|G_E|\tilde{\phi}\rangle$ is given by

$$\langle\tilde{\phi}|G_E|\tilde{\phi}\rangle=-\frac{iN_0^2}{8\lambda^6}\int_{-\infty}^{0}d\tau\int d^3p\,e^{-\frac{p^2}{2\lambda^2}}\exp\left[i\tau(p^2+m^2+e^2A_0^2-E^2)-ip_1 2eA_0\frac{\sin[\omega\tau(E-p_3)]}{\omega(E-p_3)}\right]. \qquad (20)$$

We note here that there are two sources of relativity in the laser-atom interaction problem. One of them corresponds to very highly energetic photons with energies comparable to the electron rest mass energy. This is the situation for x-ray or γ-ray interactions, in weak fields, as in elementary-particle physics. The other source is the very high laser field strength and nonrelativistic photon frequencies, and this is the case we are concerned with in this work. (The relativistic corrections can be introduced by the binding force too.) Consequently we neglect, in the above formula, p_3's in the last term in the exponent. This corresponds to the assumption $\omega\ll\sqrt{m^2-E_0^2}$, which can be seen already from (18) by rescaling $x_3\to x_3/\omega$, and similarly x_3', and observing that $\exp[-\lambda^2(x^2+x'^2)]$ becomes a strongly dumping factor if the above condition for ω is fulfilled [17]. This condition is well satisfied by the photon energy of presently available intense lasers. On the contrary, from the point of view of the field strength, current laser intensities are already on the border of the relativistic region ($10^{17}-10^{20}$ W/cm^2 depending on frequency). For example, for ω=1 eV and $I=10^{18}$W/cm^2, the electron quiver energy is comparable to its mass.

If we now perform the above-mentioned approximation together with the d^3p integral, we come to

$$\langle\tilde{\phi}|G_E|\tilde{\phi}\rangle=-\,e^{-i\frac{\pi}{4}}\frac{N_0^2\pi^{3/2}(\omega E)^{1/2}}{8\lambda^6}\int_0^{\infty}\frac{d\tau}{(\tau-i\frac{\omega E}{2\lambda^2})^{3/2}}$$
$$\times\exp\left[-i\tau\frac{m^2+e^2A_0^2-E^2}{\omega E}+i\frac{e^2A_0^2\sin^2\tau}{\omega E(\tau-i\frac{\omega E}{2\lambda^2})}\right], \qquad (21)$$

where τ has been rescaled: $\tau\to\frac{1}{\omega E}\tau$. For $\frac{m^2+e^2A_0^2-E^2}{\omega E}\gg1$, we can find the τ integral using the saddle-point method. The approximate position of the saddle point is

$$\tau_0\simeq-i\left[\frac{\sqrt{m^2-E^2}}{eA_0}\left(1-\frac{1}{9}\frac{m^2-E^2}{e^2A_0^2}\right)+\frac{\omega^2E^2e^3A_0^3}{8\lambda^4(m^2-E^2)^{3/2}}\right], \qquad (22)$$

under the conditions [18]

$$\beta_1\equiv\frac{e^2\mathcal{E}_0^2}{2m\omega^2\varepsilon_0}\gg1$$
$$\beta_2\equiv\frac{e\mathcal{E}_0}{2^{5/2}m^{1/2}\varepsilon_0^{3/2}}\ll1, \qquad (23)$$
$$\beta_3\equiv\frac{1}{2}\beta_1\beta_2^2=\frac{e^4\mathcal{E}_0^4}{128\omega^2m^2\varepsilon_0^4}\ll1,$$

where we have introduced the external electric field $\mathcal{E}_0=\omega A_0$ and binding energy $\varepsilon_0=m-E_0$. The conditions for β_1 and β_2 are equivalent to those considered in the nonrelativistic model [2] ($V\gg w$, $V\ll1$) and in [3] ($\gamma\gg1$, $F\ll F_0$). The condition for β_3 is new and arises due to the finite-range potential used here. Actually it is more severe than that for β_2 and, therefore, it suffices to consider only β_1 and β_3 [19].

We now choose the steepest path for the integration in (21), which goes down along the imaginary axis up to τ_0 and then along a distorted curve to complex infinity. Inserting the result of the integration in (19), we obtain

$$1=-\frac{V_0(2E-V_0)\sqrt{m^2-E^2}}{2^{3/2}\lambda^3}\int_0^\infty dz\frac{e^{-z}}{(z+\frac{m^2-E^2}{2\lambda^2})^{3/2}}+i\sqrt{\pi}\frac{V_0(2E-V_0)eA_0\omega E}{2^{9/2}\lambda^3(m^2-E^2)}$$
$$\times\exp\left[-\frac{2}{3}\frac{(m^2-E^2)^{3/2}}{eA_0\omega E}\left(1-\frac{1}{15}\frac{m^2-E^2}{e^2A_0^2}\right)-\frac{e^2A_0^2}{2\lambda^2}\left(1+\frac{\omega EeA_0}{4\lambda^2\sqrt{m^2-E^2}}\right)\right]$$
$$+(\text{real nonleading terms})\ . \tag{24}$$

In the above formula "real" means "real for $\gamma=0$." The solution of this implicit transcendental equation in the first approximation $\mathrm{Re}E\approx E_0$ leads to the formula

$$\gamma=-\sqrt{\frac{\pi}{2}}\frac{e\mathcal{E}_0V_0(2E_0-V_0)}{8\lambda}\left[m^2-E_0^2+\lambda^2+V_0(2E_0-V_0)/4\right]^{-1}$$
$$\times\left[1-\frac{2\lambda^2(m^2-E_0^2)}{E_0[m^2-E_0^2+\lambda^2+V_0(2E_0-V_0)/4](2E_0-V_0)}\right]^{-1}$$
$$\times\exp\left[-\frac{2}{3}\frac{(m^2-E_0^2)^{3/2}}{e\mathcal{E}_0E_0}\left(1-\frac{\omega^2}{15}\frac{m^2-E_0^2}{e^2\mathcal{E}_0^2}\right)-\frac{e^2\mathcal{E}_0^2}{2\omega^2\lambda^2}\left(1+\frac{e\mathcal{E}_0E_0}{4\lambda^2\sqrt{m^2-E_0^2}}\right)\right]\ . \tag{25}$$

Putting $\lambda=\sqrt{m^2-E_0^2}$ and expressing V_0 through E_0 [cf. Eq. (10)], we find

$$\gamma=\frac{1}{8}\sqrt{\frac{\pi}{2}}\frac{e\mathcal{E}_0}{\sqrt{m^2-E_0^2}}\left[\xi-\frac{1}{4}-\frac{\xi(m^2-E_0^2)}{E_0\left[E_0+\sqrt{E_0^2+\frac{2}{\xi}(m^2-E_0^2)}\right]}\right]^{-1}$$
$$\times\exp\left[-\frac{2}{3}\frac{(m^2-E_0^2)^{3/2}}{e\mathcal{E}_0E_0}\left(1-\frac{\omega^2}{15}\frac{m^2-E_0^2}{e^2\mathcal{E}_0^2}\right)-\frac{e^2\mathcal{E}_0^2}{2\omega^2(m^2-E_0^2)}\left(1+\frac{e\mathcal{E}_0E_0}{4(m^2-E_0^2)^{3/2}}\right)\right]\ . \tag{26}$$

Equation (26) is reminiscent of the nonrelativistic formulas obtained by Berson [2] and, independently, by Manakov and Rapoport [3]. It is effectively a generalization of the zero-range nonrelativistic case to the finite-range relativistic case of present interest.

IV. QUANTITATIVE RESULTS

Before proceeding with the numerical results let us look briefly into the regions of intensity where conditions for β_1 and β_3 for the applicability of Eq. (26) may be ensured. Thus, if we take, for example, ω=1 eV, the following restrictions for β's must hold:

$$\beta_1\approx 2.5\times10^{-13}\frac{I}{\varepsilon_0}\gg 1\ ,\qquad \beta_3\approx 2\times10^{-27}\frac{I^2}{\varepsilon_0^4}\ll 1\ , \tag{27}$$

where I should be expressed in W/cm^2 and ε_0 in electronvolts. For strongly bound electrons; for example, in highly ionized atoms with atomic number $Z\sim 10$ or, in

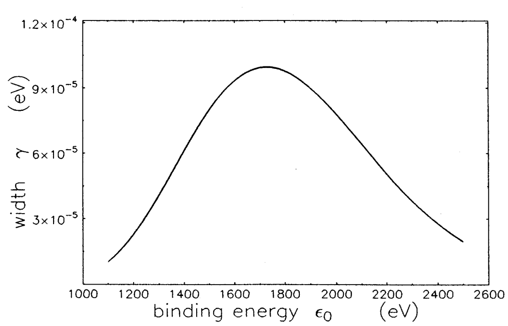

FIG. 1. Width of the bound state as the function of binding energy for $I=10^{21}$ W/cm^2 and $\omega=100$ eV.

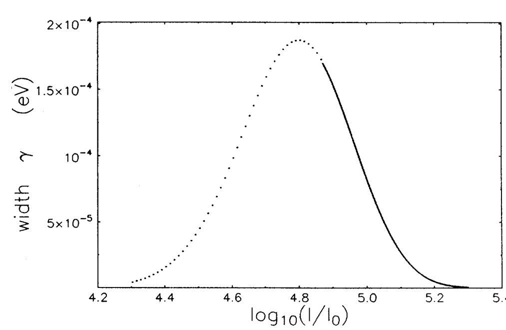

FIG. 2. Width of the bound state as the function of laser intensity for $\omega=100$ eV and atomic number $Z=12$ which corresponds to $\varepsilon_0=1963$ eV. I_0 corresponds to the value 10^{16} W/cm^2. Here and also in Figs. 3, 5, and 6, the dotted line constitutes the continuation of the curve to the region where the conditions for the derivation of Eq. (26) are not well satisfied.

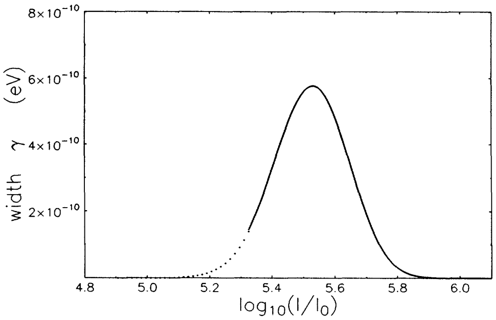

FIG. 3. Width of the bound state as the function of laser intensity for $\omega = 100$ eV and atomic number $Z = 20$ ($\varepsilon_0 = 5472$ eV). $I_0 = 10^{16}$ W/cm^2.

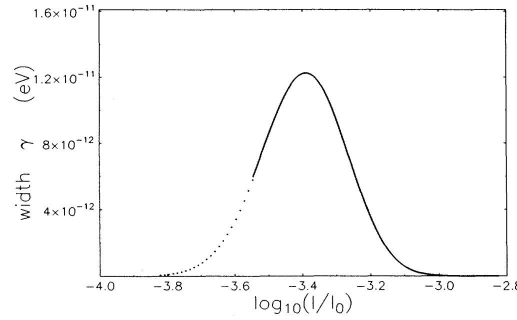

FIG. 5. Dependence of γ on the laser intensity for the Li atom. $\varepsilon_0 = 5.390$ eV and $\omega = 0.117$ eV (CO_2 laser).

other words, $\varepsilon_0 = m[1 - \sqrt{1-(Z\alpha)^2}] \approx 1$ keV, both the conditions for β's (23) are satisfied in the wide relativistic region of intensities $10^{17} - 10^{19}$ W/cm^2. For $Z > 20$ they are satisfied for I to $10^{18} - 10^{20}$ W/cm^2. For a higher frequency; for example, $\omega = 10$ eV, the relativistic region of intensities starts from $10^{18} - 10^{19}$ W/cm^2. The restrictions on the intensity and the binding energy now take the form

$$\beta_1 \approx 2.5 \times 10^{-15}\frac{I}{\varepsilon_0} \gg 1,$$
$$\beta_3 \approx 2 \times 10^{-29}\frac{I^2}{\varepsilon_0^4} \ll 1\,, \tag{28}$$

again with I in W/cm^2 and ε_0 in eV. These conditions are again satisfied for a range of values of Z and intensities ($10^{19} - 10^{21}$ W/cm^2) [20].

In Fig. 1 we present the total rate γ as a function of the binding energy ε_0, for the frequency $\omega = 100$ eV and intensity $I = 10^{21}$ W/cm^2. (In these conditions the quiver energy constitutes about 10% of the electron rest mass.) It is interesting to observe that the rates first increase with increasing binding energy, reach a maximum around 1.7 keV and finally give rise to a bell-shaped curve.

In Fig. 2 we show the intensity dependence of the rate for a fixed binding energy $\varepsilon_0 = 1963$ eV, which corresponds to an effective charge of 12, and for $\omega = 100$ eV. It is evident from the figure that the rate shows a maximum with increasing intensity and then decreases rapidly beyond $I = 6.3 \times 10^{20}$ W/cm^2. This behavior is similar to the well-known stability effect discussed extensively for the nonrelativistic systems recently [21–28]. Note that in the present case the stability effect occurs for a frequency much below the threshold of electron ejection. Similar behavior is found for effective $Z = 20$, but the absolute values are very much smaller in this case (Fig. 3). At the maximum the values of the parameters are $\beta_1 = (\frac{4\varepsilon_0}{3\omega})^{2/3}$ and $\beta_3 = \frac{1}{2}(\frac{\omega}{36\varepsilon_0})^{2/3}$. In the analytical formula (26) the presence of the second term in the exponent is responsible for the decrease of γ for very large I. It should be noted that in the nonrelativistic δ-potential model mentioned earlier only the first term in the exponent in (26) is present, causing a monotonical increase of γ with I without the stability behavior seen here.

Investigations of the rates of ejection of deeply bound electrons at lower frequencies show extremely low probabilities at relativistic intensities. As an illustration, in Fig. 4 we present the data for $\omega = 10$ eV and $I = 10^{19}$ W/cm^2, where the rates are shown as a function of the binding energy. We observe that the rates are distributed over a bell-shaped curve as before (cf. Fig. 1), but are extremely small (ten orders of magnitude smaller) in absolute values. We found also that the rates decrease even more drastically at this frequency with a further increase in intensity and are, therefore, not demonstrated here.

Equation (26) may be applied to the nonrelativistic region of intensities as well. The main features in this intensity domain are found to be the same as those shown

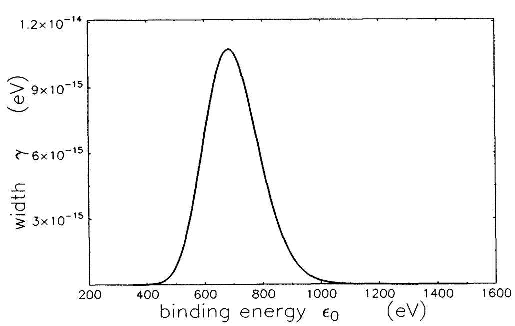

FIG. 4. Width of the bound state as the function of binding energy for $I = 10^{19}$ W/cm^2 and $\omega = 10$ eV.

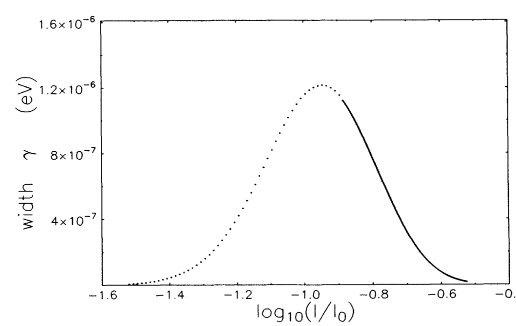

FIG. 6. Dependence of γ on the laser intensity for the He atom. $\varepsilon_0 = 24.581$ eV and $\omega = 1.169$ eV (Nd laser).

above. Below we present some plots of γ as a function of laser intensity for the binding energy corresponding to that of a neutral Li atom ($\varepsilon_0 = 5.390$ eV, $\omega = 0.117$ eV) (Fig. 5) and of a neutral He atom ($\varepsilon_0 = 24.581$ eV, $\omega = 1.169$ eV) (Fig. 6). They are, as dictated by Eq. (26), the bell-shaped curves showing the same stability effect.

In conclusion, we have shown, with the help of the solution of a relativistic model based on the Klein-Gordon equation of a deeply bound electron, that the probability of ejection of electrons for binding energies in the range of several keV can be significant for $\omega = 100$ eV, at relativistic intensities, but is extremely small for $\omega = 10$ eV. It is also shown that the so-called *stabilization* effect can occur for deeply bound electrons for subthreshold frequencies and relativistic intensities similarly to the way it occurs in nonrelativistic conditions.

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[19] The third condition constitutes a restriction on the upper limit for the intensities but, as seen in Sec. IV, for deeply bound electrons, where ε_0 is large compared to ω, the allowed intensities can be very high.
[20] The limitations can be understood since, if the notion of γ is to make sense, the external field cannot be too strong—this field can perturb the atom but cannot destroy it. If we take, however, a deeply bound particle, the binding is not so easily destroyed in these laser intensities.
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