Nonclassical vibrational states in a quantized trap

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The quantized center-of-mass (c.m.) motions of a single two-level atom or ion confined into a one-dimensional harmonic potential and interacting with a single-mode classical traveling-wave laser field are examined. We demonstrate that trap quantum states with remarkable nonclassical properties such as quadrature and amplitude-squared squeezing and sub-Poissonian statistics can be generated in this simple trap model when the c.m. motion is initially in certain coherent trap states. Our analyses also indicate that there exist some time regions where the production of nonclassical vibrational states is possible even if squeezing or sub-Poissonian statistics do not appear.

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Previously, laser radiation was used successfully to reduce the relative motion of atoms or ions. It was demonstrated that a single ion can be cooled down to its zero-point vibrational energy [1] in a Paul or Penning trap [2,3]. Two recent experiments [4,5] have observed the transitions of laser-cooled neutral atoms between vibrational levels in one-dimensional (1D) optical molasses [6], and proved that the trapped atoms could also be cooled down to their vibrational ground states. Therefore, the c.m. motion of ions or atoms in extremely low kinetic temperature should be treated quantum mechanically [7–9]. Ions in a Paul or Penning trap can be approximately regarded as being constrained to move in a harmonic potential [10], while on the other hand atoms in optical molasses are significantly trapped in optical potential wells [11] when their temperature is low enough [12]. This potential depends upon the atomic internal states and is periodic. Hence, it energy spectrum consists of bands [9,13]. Here, we concentrate on considering a relatively simple case. An atom in a certain m_F sublevel of the ground state in lin⊥lin 1D optical molasses, for example, an ^{85}Rb $5s_{1/2}(F=3)$ atom in the $m_F=3$ sublevel of the ground state, as has been investigated in Ref. [5], is supposed to be well localized in the bottom of an optical potential well. Then the c.m. motions of the atom behave approximately as a harmonic oscillator.

In what follows, we take into account that a two-level atom or ion in a harmonic potential, whose motions of the external degrees of freedom are quantized, interacts with a single-mode classical light field, which is tuned resonantly to the internal transitions between the atomic or ionic ground and excited state. As the classical light field exciting or deexciting the two-level atom or ion, the vibrational states of the c.m. motions change since atomic or ionic stimulated absorption or emission processes always accompany with momentum exchanging with the driven laser field. Theoretical calculations predicted that the atomic or ionic inversion in this simple model exhibited quantum collapses and revivals [10], which is the same as those in the ordinary Jaynes-Cummings model [14] except that the role of the quantized radiation field is replaced by the quantized c.m. motion. In this article, we restrict our attention to the quantum fluctuations of the position and momentum operators and show that squeezed vibrational states can be generated when the atomic or ionic c.m. motions are initially in some coherent states. It is found that higher-order squeezing such as amplitude-squared squeezing for the trap quanta can also exist in some time regions during the time evolution of the system. The trap quanta distribution is examined quantitatively. The parameter Q turns out to be negative in some ranges of time intervals, which characterizes sub-Poissonian statistics. Extending a criterion developed by Agarwal *et al.* [15] for characterizing the nonclassical properties of the field even if it does not exhibit squeezing or sub-Poissonian statistics, we test the nonclassical character of the quantized c.m. motions. It is interesting to note that there may exist some other nonclassical properties besides squeezing or sub-Poissonian statistics.

Since the trap potential is assumed to be harmonic, the position and momentum operators x and p can be written in terms of creation and annihilation operators for the trap quanta, $a^\dagger$, and a as

$$x=\left[\frac{\hbar}{2M\nu}\right]^{1/2}(a+a^\dagger),\quad p=i\left[\frac{\hbar\nu M}{2}\right]^{1/2}(a^\dagger-a)\,. \tag{1}$$

Then, the Hamiltonian of our simple trap model in the rotating-wave approximation (RWA) and in the rotating frame takes the form

$$H=\hbar\nu(a^\dagger a+\tfrac{1}{2})+\frac{\hbar\delta\sigma^z}{2}+\tfrac{1}{2}\hbar\Omega(F\sigma^+ +F^*\sigma^-)\,, \tag{2}$$

where σ^z, σ^+, and σ^- are the Pauli spin matrices of the atomic or ionic internal operators, ν is the trap frequency, Ω denotes Rabi frequency, and δ represents the detuning between the internal transition frequency ω_0 and the laser frequency ω_L, $\delta=\omega_0-\omega_L$. F and F^* are defined by: $F=(F^*)^*=\exp[i\epsilon(a+a^\dagger)]$ with the parameter ϵ being given by $\epsilon=\sqrt{E_r/E_\nu}$. Here, $E_r=\hbar^2k^2/2M$ and

$E_\nu=\hbar\nu$ are the classical recoil energy of the atom or ion and the energy of the trap quantum, respectively. In the Lamb-Dicke limit [16], ϵ is small, which correspond to the well-spaced trap states. In writing the above Hamiltonian, we have omitted the effects of spontaneous emissions. This is reasonable only for the situation $\Omega \gg \Gamma$ (Γ denotes the spontaneous decay rate of atomic or ionic internal excited state), i.e., only when the spontaneous decay rate is very small compared to the Rabi frequency of the classical driven laser field can we neglect the spontaneous decay terms. The first term in our Hamiltonian corresponds to c.m. motion in a 1D harmonic potential; the second one is the atomic or ionic internal energy; and the third one denotes the coupling between the driven field and the atom or ion. Note that for the c.m. motion in the 1D harmonic potential, the vibrational states are quantized with energy difference $\hbar\nu$ between each other. Suppose $|m\rangle$ is one of the quantized vibrational levels, $a^\dagger a|m\rangle=m|m\rangle$, and the vibrational energy (including kinetic and potential energy) is $(m+\frac{1}{2})\hbar\nu$. If the energy difference $\hbar\nu$ is larger than the natural width of the excited states and Rabi splitting, i.e., $\nu \gg \Omega$, $\nu \gg \Gamma$, the transitions between vibrational levels will dominantly affect the atomic or ionic c.m. motion. For simplicity, we focus on a heating transition involving the exchange of only one trap quantum. In such a case, the driven laser is tuned to the first vibrational sideband on the blue side, $\delta=-\nu$. Hence the internal transition from the ground state to the excited state heats atoms or ions and therefore the trap quantum number m increases by 1 ($\Delta m=1$), while deexciting decelerates the atomic or ionic c.m. motion and reduces m ($\Delta m=-1$). In the limit $\nu \gg \Omega$, all other transitions oscillate with sufficiently high frequencies. We ignore their effects in RWA. For $\epsilon \ll 1$ and not-too-large quantum number m ($m\epsilon^2 \ll 1$), the associated matrix elements of operators F and F^* approximate

$$\langle m+1|F|m\rangle=(\langle m|F^*|m+1\rangle)^* \cong i\epsilon\sqrt{m+1}\ . \tag{3}$$

Note that, in the last part of the above equations, $m\epsilon^2$ and higher-order terms are neglected. It should be borne in mind that such an approximation becomes unsatisfactory when m is large ($m\epsilon^2 \simeq 1$). Fortunately, for an initial coherent vibrational state with small average trap quantum $\bar{m}$, the above approximation holds. In the following calculations, we therefore concentrate on considering such a situation. After some short algebra, we derive the time evolution operators $U(t)=\exp(-iHt/\hbar)$. It can be expanded as

$$U(t)=\sum_{m=0}^{\infty} \exp[-i(m+1)\nu t][\cos(\lambda t)|e,m+1\rangle\langle e,m+1|+\sin(\lambda t)|e,m+1\rangle\langle g,m| +\cos(\lambda t)|g,m\rangle\langle g,m|-\sin(\lambda t)|g,m\rangle\langle e,m+1|], \quad \lambda=\tfrac{1}{2}\Omega\epsilon\sqrt{m+1}\ , \tag{4}$$

where $|g\rangle$ and $|e\rangle$ correspond to the internal ground and excited state of atom or ion, respectively. $|e,m\rangle$ and $|g,m\rangle$ are the entangle states of internal and external degrees of freedom, i.e., $|e,m\rangle=|e\rangle\otimes|m\rangle$, $|g,m\rangle=|g\rangle\otimes|m\rangle$. If the initial density matrix of the system is $\rho(0)$, the reduced density matrix $\rho(t)$ for external degrees of freedom at time t can be written as

$$\rho_{\rm red}(t)=\langle e|U(t)\rho(0)U^\dagger(t)|e\rangle +\langle g|U(t)\rho(0)U^\dagger(t)|g\rangle\ . \tag{5}$$

The quadrature and amplitude-squared operators for trap quanta are defined in similar forms as those for quantized radiation:

$$X_1=\frac{a+a^\dagger}{2},\quad X_2=\frac{a-a^\dagger}{2i}\ , \tag{6a}$$

$$Y_1=\frac{A^2+A^{\dagger 2}}{2},\quad Y_2=\frac{A^2-A^{\dagger 2}}{2i}\ , \tag{6b}$$

where $A=\exp(i\nu t)a$ and $A^\dagger=\exp(-i\nu t)a^\dagger$ are slowing varying operators of the trap quanta. For the harmonic potential, the trap operators a and $a^\dagger$ obey $[a,a^\dagger]=1$, which confirms the following commutation relations for X_1 and X_2, and for Y_1 and Y_2:

$$[X_1,X_2]=i/2,\quad [Y_1,Y_2]=i(2N+1)\ , \tag{7}$$

where $N=A^\dagger A$ is the number operator of the trap quanta. The uncertainty relations, then, are

$$\langle\Delta X_1\rangle\langle\Delta X_2\rangle \geq \tfrac{1}{4},\quad \langle\Delta Y_1\rangle\langle\Delta Y_2\rangle \geq \langle N+\tfrac{1}{2}\rangle\ . \tag{8}$$

The minimum uncertainty states with $\langle(\Delta X_1)^2\rangle=\langle(\Delta X_2)^2\rangle=\frac{1}{4}$ and $\langle(\Delta Y_1)^2\rangle=\langle(\Delta Y_2)^2\rangle=\langle N+\frac{1}{2}\rangle$ are the vibrational coherent states $|\alpha\rangle$, $a|\alpha\rangle=\alpha|\alpha\rangle$. Unambiguously, vibrational ground state $|0\rangle$ is a coherent state. The quadrature squeezing in X_i or amplitude-squared squeezing in Y_i for c.m. motion exists if

$$\langle(\Delta X_i)^2\rangle<\tfrac{1}{4} \text{ or } \langle(\Delta Y_i)^2\rangle<\langle N+\tfrac{1}{2}\rangle,\quad i=1,2\ . \tag{9}$$

The quantum fluctuation can be calculated from

$$\langle(\Delta K_i)^2\rangle=\mathrm{tr}(\rho(t)K_i^2)-[\mathrm{tr}(\rho(t)K_i)]^2,\quad K=X,Y,\quad i=1,2\ . \tag{10}$$

To measure the trap quantum number distributions, a parameter Q is defined as

$$Q=\frac{\langle a^{\dagger 2}a^2\rangle-\langle a^\dagger a\rangle^2}{\langle a^\dagger a\rangle}\ . \tag{11}$$

For sub-Poissonian statistics, Q is negative. We next introduce a quantity as a measure of the nonclassical

property for the quantized c.m. motion even if it does not exhibit squeezing and sub-Poissonian statistics, which takes an analogous form as that for quantized radiation in Agarwal *et al.* [15]:

$$A_3 = \det m^{(3)} / [\det \mu^{(3)} - \det m^{(3)}] , \tag{12}$$

where the matrix $m^{(3)}$ is defined by

$$m^{(3)} = \begin{bmatrix} 1 & m_1 & m_2 \\ m_1 & m_2 & m_3 \\ m_2 & m_3 & m_4 \end{bmatrix}, \quad m_n = \langle a^{\dagger n} a^n \rangle . \tag{13}$$

The matrix $\mu^{(3)}$ is obtained from $m^{(3)}$ by the replacement $m_n \rightarrow \mu_n = \langle (a^\dagger n)^n \rangle$. It was demonstrated that in the nonclassical region $-1 < A_3 < 0$.

Numerical simulations are performed in accordance with the above formulas. We assume that the atom or ion is initially in its ground state $|g\rangle$, while the c.m. motion in a coherent state $|\alpha\rangle$ with an average vibrational quantum number $\bar{m} = |\alpha|^2$, i.e., $\rho(0) = |g,\alpha\rangle\langle g,\alpha|$. In Fig. 1, $\langle (\Delta X_1)^2 \rangle$ or $\langle (\Delta X_2)^2 \rangle$ is plotted for a different initial vibrational state $|e^{i\phi}\rangle$ with the phase ϕ being given by 0 (curve *a*), $\pi/18$ (*b*), $\pi/4$ (*c*) for $\langle (\Delta X_1)^2 \rangle$; or $\pi/2$ (curve *a*), $4\pi/9$ (*b*), $\pi/4$ (*c*) for $\langle (\Delta X_2)^2 \rangle$. It is clear that, when ϕ varies between 0 and $\pi/18$ (or between $4\pi/9$ and $\pi/2$), there exists time regions in which $\langle (\Delta X_1)^2 \rangle$ [or $\langle (\Delta X_2)^2 \rangle$] turns out to be less than $\frac{1}{4}$. In this region squeezing in X_1 (or X_2) occurs. The maximum squeezing in X_1 (or X_2) is at the point $\phi = 0$ (or $\phi = \pi/2$). Figure 2 shows $F_i = \langle (\Delta Y_i)^2 \rangle - \langle N + \frac{1}{2} \rangle$ $(i = 1,2)$ as functions of the dimensionless time $\Omega t/2\pi$. Again, we find that amplitude-squared squeezed states in Y_1 (or Y_2) can be generated and last for a short while, when $0 \leq \phi \leq \pi/15$ (or $8\pi/45 \leq \phi \leq \pi/4$). The maximum squeezing in Y_1 (or Y_2) arrives when $\phi = 0$ ($\phi = \pi/4$). In Figs. 3 and 4, we illustrate the quantum fluctuations of X_1 for $\alpha = 4$ (curve *a*), 6 (*b*), 8 (*c*), 10 (*d*) and of Y_1 for $\alpha = 6$ (curve *a*), 8 (*b*), 10 (*c*), respectively. It is indicated that the initial coherent states $|\alpha\rangle$ evolve to be quadrature or amplitude-squared squeezed states in some ranges of time intervals. The meaning of trap quantum squeezing can be outlined qualitatively as follows: The squeezing in X_1 means that the position variations of the atomic or ionic c.m. motion in a harmonic potential is less than those of minimum uncertainty wave packet; states squeezed in X_2 can be interpreted as states with homogeneous c.m. velocity distribution, in fact, their quantum fluctuations are smaller than those of zero-point vibrational or ground trap states. Squeezing in Y_1 or Y_2 manifests the reduction of quantum fluctuations of the real or imaginary part of the slowly varying amplitude-squared operator.

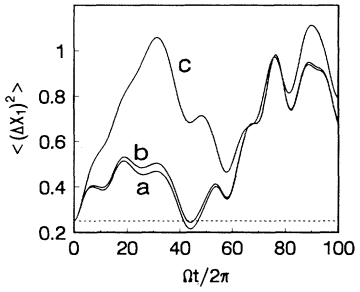

FIG. 1. The quantum fluctuation $\langle (\Delta X_i)^2 \rangle$ $(i = 1,2)$ as functions of dimensionless time $\tau = \Omega t/2\pi$ for $\epsilon = 0.05$, $\nu/\Omega = 50$, $\delta/\Omega = -50$. Initially the two-level atom or ion is assumed to be in its ground state $|g\rangle$, while the center-of-mass motion is in a coherent state $|e^{i\phi}\rangle$. The curves *a*, *b*, and *c* correspond to $\phi = 0$, $\phi = \pi/18$, and $\phi = \pi/4$, for $\langle (\Delta X_1)^2 \rangle$; or (curve *a*) $\phi = \pi/2$, (*b*) $\phi = 4\pi/9$, and (*c*) $\phi = \pi/4$, for $\langle (\Delta X_2)^2 \rangle$.

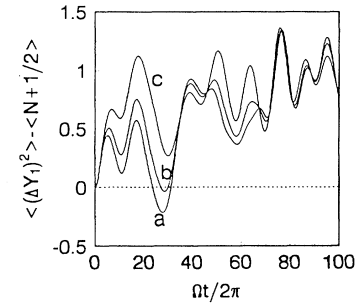

FIG. 2. The quantum fluctuation $F_i = \langle (\Delta Y_i)^2 \rangle - \langle N + \frac{1}{2} \rangle$ $(i = 1,2)$. Y_1 and Y_2 are the real and the imaginary part of the amplitude-squared operator. The initial density matrix is supposed to be $\rho(0) = |g,\alpha\rangle\langle g,\alpha|$, $\alpha = e^{i\phi}$, and ϕ is chosen as (curve *a*) $\phi = 0$, (*b*) $\phi = \pi/15$, and (*c*) $\phi = \pi/8$, for F_1; or (curve *a*) $\phi = \pi/4$, (*b*) $\phi = 11\pi/60$, and (*c*) $\phi = \pi/8$, for F_2. Other parameters are the same as those in Fig. 1.

The parameter Q is evaluated for initial density matrix $\rho(0) = |g,\alpha\rangle\langle g,\alpha|$, $\alpha = 1,2$. The results are depicted in Fig. 5, curves *a* and *b*. The diagrams clearly show that Q oscillates and reaches a negative value many times in the time range $\Omega t/2\pi = 0$–100. In Fig. 6, curves *a* and *b*, we give the plots of the time variation of the parameter A_3. Comparing Figs. 5 and 6, we immediately see that there are some ranges of time intervals where Q is positive while A_3 is negative. This ensures the existence of a non-

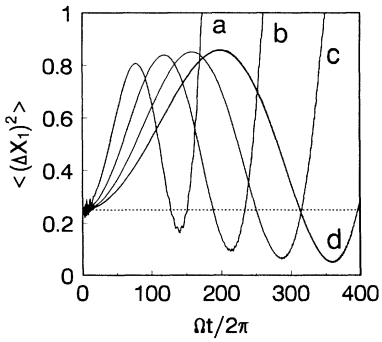

FIG. 3. The time evolution of quantum fluctuation of operator X_1. The initial density matrix is $\rho(0) = |g,\alpha\rangle\langle g,\alpha|$, $\alpha = 4$ (curve *a*), 6 (*b*), 8 (*c*), and 10 (*d*). ϵ, ν, Ω, and δ are the same as those in Fig. 1.

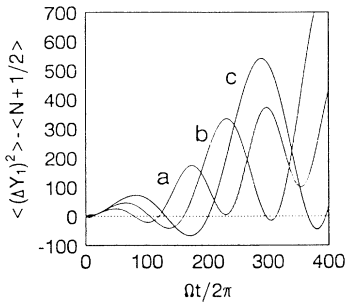

FIG. 4. The time variation of quantum fluctuation of $F_1=\langle(\Delta Y_1)^2\rangle-\langle N+\frac{1}{2}\rangle$. The curves *a*, *b*, *c* correspond to the case of initial density matrix $\rho(0)=|g,\alpha\rangle\langle g,\alpha|$, $\alpha=6$, 8, 10. Other parameters are the same as in Fig. 3.

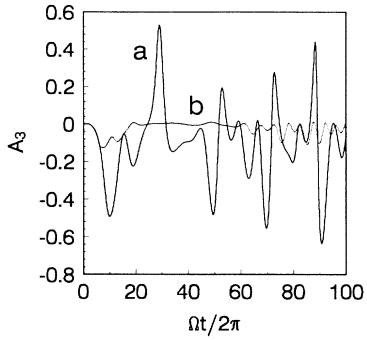

FIG. 6. Parameters A_3 as a function of dimensionless time $\tau=\Omega t/2\pi$. The initial density matrix is $\rho(0)=|g,\alpha\rangle\langle g,\alpha|$, (curve *a*) $\alpha=1$, (*b*) $\alpha=2$. Other parameters are the same as in Fig. 3.

classical property for the quantized c.m. motion in regions where it does not exhibit sub-Poissonian statistics.

Let us discuss briefly the physical mechanism underlying the generation of the above-mentioned nonclassical features. Approximation adopted in the above considerations implies that the involved transitions in our configuration are between $|g,m\rangle$ and $|e,m+1\rangle$, with $m=0, 1, 2, \ldots$, which leads to a simplification of the laser-atom coupling terms in the Hamiltonian H:

$$H_{\text{coupl}}=\frac{i}{2}\epsilon\hbar\Omega(a^{\dagger}\sigma^{+}-a\sigma^{-}) . \tag{14}$$

The first term and the second term in the right side of the above equation are associated with heating and cooling of the particle, respectively. These two processes arise coherent redistribution of the trap quanta with the particle being in internal upper or lower state and establish coherences between trap quanta creation an annihilation. These two-photon coherences are in turn responsible for the appearance of such nonclassical features as squeezing and sub-Poissonian trap quanta statistics.

As examples, we next estimate briefly the experimental possibilities of our analyses. For ions in a Paul or Penning trap, we consider a typical situation used in a Diedrich [1] sideband cooling scheme: $^{89}Hg^{+}$ ions interact with classical laser field which is tuned resonantly

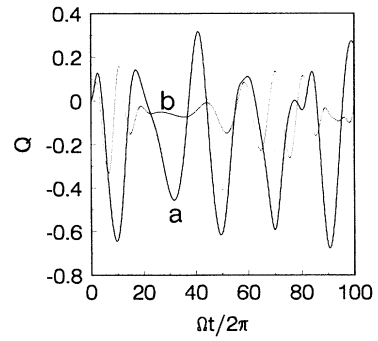

FIG. 5. Parameters Q as a function of dimensionless time $\tau=\Omega t/2\pi$. Initially the system is assumed to be described by $\rho(0)=|g,\alpha\rangle\langle g,\alpha|$, (curve *a*) $\alpha=1$, (*b*) $\alpha=2$. Other parameters are the same as in Fig. 3.

to the narrow $^2S_{1/2}$-$^2D_{5/2}$ electric quadrupole transitions with wavelengths $\lambda=281.5$ nm. The experimental relevant parameters are as follows: the trap frequency is $\nu=2.96$ MHz, the decay rate is $\Gamma=11$ Hz [17]. These give $\epsilon=0.07$. Allow for the trap quantum number distribution of a coherent vibrational state with small average trap quanta $\bar{m}$, the large m terms can be neglected. Hence, $m\epsilon^2\ll 1$ holds. Clearly there exists an extensive domain for Ω to be chosen to fulfill both the conditions $\nu\gg\Omega$ and $\Omega\gg\Gamma$ simultaneously. While for atoms in 1D optical molasses, an important experimental consideration is the requirement of negligible population transferring out of the involved levels. Take, for example, the experiment of Jessen *et al.* [5] into account. Two counterpropagating laser beams (lin⊥lin) were tuned red of the $5S_{1/2}(F=3)$-$5P_{3/2}(F'=4)$ transition and formed the 1D optical molasses. It was shown that optical pumping out of the $m_F=3$ sublevel of the $F=3$ hyperfine level was strongly suppressed and that the atomic motion in the bottom of the optical well approximated an undamped harmonic oscillator. Because the ratio of the squared Clebsch-Gordan coefficients for the transitions $m_F=3$-$m_{F'}=4$ vs $m_F=3$-$m_{F'}=2$ is 28 to 1, the latter transitions can be omitted and the atomic internal degrees of freedom can be described sufficiently as a two-level system ($m_F=3$ and $m_{F'}=4$). An additional single-mode σ^{+} polarized laser light, which is tuned to the blue side of the $m_F=3$-$m_{F'}=4$ transition, provides coupling between the two internal levels. Carefully set the Rabi frequency and the detuning of the two counterpropagating laser so as the trap frequency ν satisfies $\nu\gg\Gamma$. A choice of the Rabi frequency Ω of the classical driving laser field can then be made to satisfy $\nu\gg\Omega$ and $\Omega\gg\Gamma$.

In conclusion, previous discussions demonstrate that both quadrature and amplitude-squared squeezed vibrational states can be generated for the quantized center-of-mass motion of two-level atoms or ions constrained to move in a one-dimensional harmonic potential. We point out that some initial coherent states of the quantized c.m. motions will evolve to be states exhibiting sub-Poissonian statistics in some ranges of time intervals. It was proven by Agarwal and others [18–21] that nonclassical states could also be produced in a harmonic oscillator with a

time-dependent frequency, which described the quantum motion of particles in a Paul trap. Our results here predict the existence nonclassical properties, such as squeezing, higher-order squeezing, and sub-Poissonian statistics, for the quantized c.m. motions of ions in a Paul or Penning trap and atoms in optical molasses. We remark that nonclassical vibrational states can be obtained even if the parameter Q is positive.

Note added. A very similar investigation has been recently published by Cirac, Parkins, Blatt, and Zoller [22]. Therein, they proposed to exert multichromatic excitation of a trapped ion by standing- and traveling-wave light fields for preparing coherent squeezed states of motion in an ion trap [22]. Particularly, they offered a mechanism to detect actually the nonclassical character of the vibrational states of a trapped ion. Therefore, it is reasonable to believe that the results predicted here are experimentally observable. This appears to be quite promising to the study of the nonclassical properties of the center-of-mass motions in a quantized trap. On the other hand, along with the early predicted quantum collapses and revivals in a quantized trap, the time-dependent dynamics of the c.m. motions described here confirm again its quantum nature.

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