Amplitude squeezing from spectral-hole burning: A semiclassical theory

J. Arnaud
Equipe de Microoptoélectronique de Montpellier, Université de Montpellier II, Sciences et Techniques du Languedoc, Place E. Bataillon, 34095 Montpellier Cédex 2, France
(Received 31 August 1992; revised manuscript received 8 January 1993)

Spectral-hole burning (SHB) profoundly affects the modulation and noise properties of laser oscillators and amplifiers at high optical power. The present semiclassical theory of SHB shows that the optical gain should be considered a function of the carrier number and photon rate (rather than photon number) plus a fluctuation at the shot-noise level (for full population inversion). Constant-voltage-driven laser diodes generate amplitude-squeezed light, a result not predicted by previous theories that treat gain compression in a formal way. Amplitude and phase noise of oscillators and amplifiers are considered.

PACS number(s): 42.50.Lc, 42.55.Px, 42.50.Ar

I. INTRODUCTION

Amplitude and phase noise of laser oscillators and amplifiers at high output power are analyzed in this paper on the basis of a revised semiclassical theory. In the context of the present paper "semiclassical" means that only classical (commuting) functions of time are employed. Single-mode laser diodes are particularly considered, in which the polarization follows adiabatically the optical field. The photon lifetime is usually small compared with the carrier lifetime. For the sake of clarity, spontaneous carrier recombination (radiative and nonradiative) is neglected. This is a reasonable approximation for modern laser diodes that operate at, say, 20 times the threshold current. Furthermore, only the zero-frequency limit is considered in detail. To obtain the general solution it suffices to restore the dN/dt and dm/dt electron and photon storage rates. Nonessential effects such as the $1/f$ noise are ignored.

An oscillator of any kind essentially consists of an emitting and an absorbing element. The resonator that defines the oscillation frequency is unimportant as long as only slow amplitude fluctuations are considered. Stable oscillation requires that the rate at which photons are generated by the emitting element be a sublinear function of the photon number if the absorbing element is linear. The key question is how the primary noise sources are affected by the nonlinearities, which are essential for oscillators as we just discussed, and important for amplifiers as well.

Nonlinearities first arise from the dependence of the optical gain on carrier number, or on the number of atoms in the excited state in gas lasers. This kind of nonlinearity is well understood (see, e.g., [1]). But many observations made on laser diodes (e.g., the strong damping of relaxation oscillations at high power) are best explained by assuming that the gain decreases as the optical field intensity increases, the carrier number being kept constant. This effect, called "gain compression" or "nonlinear gain," attributed to various mechanisms such as spectral-hole burning (SHB), is not well understood. In order to clarify how the primary noise sources are affected by gain compression we have considered two specific models. In one [2], a linear negative resistance is followed by a lossless Kerr's medium. In the other, treated in the present paper, the nonlinearity is caused by SHB. Both models lead to identical conclusions, namely, that under idealized conditions the optical gain should be considered a function of carrier number and electron-photon conversion *rate* (not photon number), plus a fluctuation at the shot-noise level irrespectively of the nonlinearity. In an equivalent electrical circuit, the emitting element is represented by a resistance whose value depends on the nonlinearity, but the series-noise-voltage spectral density (representing the intrinsic noise source) remains at the shot-noise level [3].

Let us briefly discuss the nature of SHB. Interacting carriers (i.e., carriers separated in energy by approximately $h\nu$, where h denotes Planck's constant and ν the optical frequency) recombine quickly at high power levels, electrons and holes are not in thermal equilibrium within their respective bands, and a hole builds up in the gain-versus-frequency curve while injected carriers accumulate at higher energy (hot carriers). The rate at which the spectral hole in the conduction band is replenished by carrier interaction is approximately proportional to the departure of the mean state occupation from its thermodynamic equilibrium value. Similar considerations apply to the valence band. Because we are interested in the consequences of SHB rather than with the detailed mechanism itself, this simplified picture (see the Appendix) should suffice. Detailed theories of SHB can be found in [4–6].

Semiclassical concepts have been much used during the last decade in the optical engineering literature. These papers deal with rather complicated structures (strained quantum wells, optical or electrical feedbacks, internal gratings, etc.) but the basic principles employed may not be entirely consistent. It is therefore of interest to critically review three semiclassical concepts that may be equivalent in special cases but should, in general, be distinguished. McCumber's quantum theory [7] ends up with a semiclassical prescription that emphasizes the corpuscular aspects of light. Henry's theory (Chapter 2 of

[8]) views noise as the interference between a strong oscillating field and a spontaneously emitted field. From our viewpoint, Lax's semiclassical theory [9] is easiest to generalize being based on the fluctuation-dissipation theorem.

Following McCumber [7], consider any number of boxes containing either electrons (number N) or photons (number m). Electrons from one box may be converted into photons of another box, or the converse. Time rates of change of N or m are the sums of known functions of the populations plus random functions of time called Langevin's forces. The spectral density of the Langevin force relating to a box is the sum of the absolute values of the corpuscular rates entering or leaving that box (in appropriate units). The cross-spectral density between Langevin's forces relating to any two boxes is opposite to the corpuscular rate from one box to the other. McCumber's theory prescription applies only to conversion of electron-hole pairs and photons: For example, the electronic flow from a battery to the laser active material through some resistor, far from being at the shot-noise level, does not fluctuate at all, if $kT \ll h\nu$ [10]. One must also be careful to distinguish the fluctuations of the detected outgoing photonic flow from the fluctuations of the number of photons in the laser cavity.

According to a picturesque model often presented for explaining the nature of laser noise, spontaneous emission contributes a small randomly phased field component to the steady-state oscillation field, the added photon rate being equal to the reciprocal of the photon lifetime in the optical cavity (for complete population inversion). The important term in this picture is the cross-product of the steady-state field and the spontaneously emitted field, hence the occurrence of a factor of 2. This concept forms the basis of Henry's semiclassical theory. The merit of such "standard rate equations" is to describe phase as well as amplitude fluctuations. The cross-spectral densities differ essentially by a factor of 2 from those appearing in McCumber's theory (see [1]). Yet, Henry's and McCumber's theories may give the same expressions for measurable quantities provided adequate interpretations are made and in special situations. In particular, a shot-noise term (attributed to the detector photocurrent) must be added in Henry's theory, which assumes that the electrons are injected independently. The theory can be modified to account for below-shot-noise injected-current fluctuations (modified rate equations (MRE), see Chapter 2 of [8]). But because intensity-noise spectral densities may be negative the theory becomes conceptually unclear.

According to the semiclassical theory presented by Lax [9], two independent noise sources drive the oscillator, one relating to the emitting element ("dipole noise"), and one to the absorbing element ("vacuum fluctuation"). For (positive or negative) *linear* resistances, the spectral densities of the associated noise voltages are equal to the absolute values of the resistances (fluctuation-dissipation theorem). The essential zero-point-fluctuation term being omitted in Nyquist's paper, it is more accurate from an historical standpoint to refer here to Planck himself rather than to Nyquist. The above prescription needs not apply to nonlinear elements. This is easily seen by modeling the laser oscillator by a parallel rather than series circuit, with a current-noise-source spectral density equal to the conductance. If parallel and series circuits are treated analogously, different expressions for amplitude noise are obtained. This inconsistency shows that the primary (voltage or current) noise sources in general depend on the nonlinearity. The noise source is independent of the nonlinearity, however, in the representation in Eq. (2) below.

To pursue Lax's model, one may first consider that the emitting resistance is linear but that its value depends on the time-varying carrier number N. The dipole noise remains stationary because the relative variations of N, and therefore of the resistance, turn out to be small. Of course, one must account for the variation of the resistance due to changes of N, the latter being given by a rate equation into which *the dipole noise enters again.* These considerations lead to a semiclassical theory [1] that predicts spectral densities in agreement with the quantum theory [10] for second-order correlations and large particle numbers. Note incidentally that other parameters besides N, e.g., temperature or strain, could be treated analogously provided exact conservation laws are available.

It is tempting to account for gain compression in a formal way writing the photonic rate R (number of photons emitted per unit time) as

$$R=\mathcal{G}^0(N,m)m+\alpha \;, \tag{1}$$

where $\mathcal{G}^0$ is the gain and m the photon number, and to *assume* that the spectral density of the intrinsic noise source $\alpha(t)$ remains at the shot-noise level for full population inversion (note that the approximation $m+1\approx m$ has been made on the grounds that m is a large number, and thus spontaneous emission in the mode is omitted). This assumption made in a previous paper by this author [1] and indirectly in [11] unfortunately appears to be valid for weak nonlinearities only. The correct formulation for one-photon processes, justified later in this paper, is that $\mathcal{G}^0$ should be considered a function of R rather than a function of m, if one insists that the noise source be at the shot-noise level. The detailed expressions are then also simpler. We therefore assert that

$$R=\mathcal{G}^0(N,R)m+\rho \;, \quad \mathcal{S}_{\rho}=R \;, \tag{2a}$$

where $\mathcal{S}$ denotes double-sided spectral densities. Let us emphasize that the same rate R appears on both sides of Eq. (2a). All the results in this paper are essentially based on Eq. (2a). In the special case of constant-voltage drive N is constrained to be a constant N_0 and $\mathcal{G}^0$ is a function of R only.

It may be helpful to exemplify Eqs. (1) and (2a) by considering N_0 three-level atoms, n of them being in the upper state. Stimulated emission causes these n atoms to move to the lower state at rate R. There is instantaneous decay to the ground state. The pump moves the electrons back up regularly to the upper state at a rate J proportional to the number N_0-n of atoms in the ground state, i.e., at time intervals $1/(N_0-n)$. At low frequency the storage rate dn/dt can be neglected, and thus, omitting a constant, $J=R=N_0-n$, or $n=N_0-R$. The stimulated emission rate R being proportional to the number n of atoms in the upper state and to the number m of photons in the cavity, plus some intrinsic noise source $\mathscr{r}$, we can write a relation of the form in Eq. (2a) (with $N=N_0$),

$$R=(N_0-R)m/m_s+\mathscr{r} \;, \tag{2b}$$

where m_s is a constant. If this relation is written in the form of Eq. (1),

$$R=\left[\frac{N_0/m_s}{1+m/m_s}\right]m+\mathscr{u} \;, \quad \mathscr{u}=\frac{\mathscr{r}}{1+m/m_s} \;, \tag{1'}$$

the factor that multiplies m exhibits the usual saturation form. The point is that, while the spectral density of $\mathscr{r}$ in Eq. (2b) is likely to be proportional to n, this is not so for $\mathscr{u}$ in Eq. (1′). Furthermore, if $\mathscr{r}$ is at the shot-noise level as we prove in Sec. II, the spectral density of $\mathscr{u}$ obviously depends on the nonlinearity.

We will show that Eq. (2) predicts sub-Poissonian photon statistics from *constant-voltage-driven* laser diodes as a result of SHB, while Eq. (1) would predict that this cannot be achieved if $\mathscr{u}$ is at the shot-noise level. Modified rate equations are even farther off, predicting large intensity noise in that situation [1].

The proposed formalism can be written in various forms that are more or less convenient depending on the laser model. If V and I are proportional to voltages and currents at optical frequency ν, evolution equations may be written either for the admittance I/V, for V itself, or for rates V^*I (the asterisk indicates complex conjugation). The first form was employed in most previous papers by this author [12]. The second one is closest to the quantum theory in [10] that provides the evolution of the internal-field operator A which, semiclassically, corresponds to our V. The latter formulation is employed in [1] and here. For spatially extended configurations a wave formalism is to be preferred.

The spectral properties of the noise source $\mathscr{r}$ are given in Sec. II. Equation (2) enables us to relate δR, δm, and $\delta\mathscr{G}^0$, where δ refers to first-order deviations from steady-state values. It is shown in the Appendix that $\mathscr{G}^0$ may be written as a function of N, minus a term proportional to R. This enables us to relate $\delta\mathscr{G}^0$ to δN and δR. The carrier rate equation in Sec. III provides the carrier-number variation δN in terms of δR. Considering all these results together, a relation between δR and δm applicable to both emitters and absorbers follows. This completes the theory as far as intensity noise is concerned.

Section IV provides an expression for amplitude noise for a simple oscillator model when SHB is significant, considering particularly constant-voltage drives. In Sec. V the output noise of laser amplifiers with SHB is given. This author has shown previously that electrical feedback from a linear amplifier of power gain $\mathscr{G}$ is capable of reducing amplitude fluctuations by a factor $(2\mathscr{G}-1)$ below the initial shot-noise level, thereby preserving minimum uncertainty. This result is shown in Sec. VI to apply to nonlinear amplifiers as well, and applications are suggested. Note that $\mathscr{G}$ represents the output-to-input power ratio, while $\mathscr{G}^0$ in Eqs. (1) and (2) describes photon-number time rates of change.

The general formulas pertaining to phase fluctuations are given in Sec. VII. It is shown in Sec. VIII that the material phase-amplitude coupling factor α should be divided by a factor significantly larger than unity as a result of SHB and low-impedance drive combined effects. Thus SHB effectively *reduces* the linewidth, except at very high power where the material α factor increases, a feature that may contribute to linewidth rebroadening. A simple relationship between the three measurable cross-spectral densities (electrical voltage, amplitude, and phase fluctuations) is proven to hold in general in Sec. IX. The conclusion is in Sec. X.

II. BASIC NOISE SOURCE

To ensure that the same formulation applies to absorbers and emitters R is defined as the *absorbed* photon rate and is therefore negative for emitters. This seemingly unnatural sign convention proves convenient to treat sequences of absorbing and emitting elements. We first consider two-level atoms and then show how the results can be applied to semiconductors.

The rate R at which photons are absorbed by n two-level atoms in the ground state through k-photon processes (e.g., two-photon absorption, $k=2$) is

$$R=GP^k+\mathscr{r}=kJ \;, \quad G\equiv An \;, \tag{3}$$

where A is a constant, P denotes the square of the voltage V at optical frequency ν (proportional to the optical field acting on the atoms), and J the electronic rate induced by the transitions from the lower to the upper state. For two-photon absorption, for example, two photons are required to generate an electron: $R=2J$. For detectors, the upper state is the continuum and the emitted electrons constitute the measurable external current. $\mathscr{r}$ represents some intrinsic noise source whose properties will be discussed below. The relation in Eq. (3) reduces to the first relation in Eq. (2a) for $k=1$ if we introduce the capacitance C of a tuned circuit and the photon number $m=CP$ [see Fig. 1(b)]. The gain $\mathscr{G}^0=G/C$ for a conductance $G=An$. Thus $\mathscr{G}^0m=GP$. In semiconductors, n represents the number of states interacting with the optical field. The dependence of n on the total carrier number N and recombination rate R will be discussed later.

The spectral density of the $\mathscr{r}(t)$ process may be derived from a simple intuitive argument: Consider the special case in which electron and photon numbers: n and m (or P) are somehow constrained to remain constant (number states). Stimulated absorption events should be independent because the atoms cannot "communicate," so to speak, with one another through induced field fluctuations. Accordingly, the fluctuations of J are at the shot-noise level: $\mathscr{S}_{\delta J}=J$. In that situation $\mathscr{r}=k\delta J$ according to Eq. (3), and thus the spectral density of $\mathscr{r}$ is equal to $k^2J=kR$. The photon-number variance calculated from this expression can be shown to agree exactly with Agarwal's master equation for k-photon processes [13] in the large m limit. It follows that for some $R(P)$ law the spectral density of $\mathscr{r}$ is equal to PdR/dP.

Laser diodes are usually driven through high electrical resistances. The fluctuations of the injected rate J are then specified from the outside, e.g., by raising the series-resistance temperature, and some fluctuation in photon-number results. The opposite situation of constant photon number results in fluctuations of J at the shot-noise level. The above considerations also apply to stimulated emission, all the atoms being in that case maintained in the upper state. Generalization to incomplete population inversion with n_1 atoms in the lower state and n_2 atoms in the upper state is straightforward.

For one-photon processes ($k=1$) the result $\mathcal{S}_{\not{r}}=R$ in Eq. (3) follows also from the fluctuation-dissipation theorem [9]. According to Eq. (3) with $k=1$, the first-order variations of P, R, and G are related by

$$\frac{\delta P}{P}=\frac{\delta R}{R}-\frac{\delta G}{G}-\frac{\not{r}}{R} , \tag{4a}$$

$$S_r=\eta R , \quad \eta\equiv 1-2n_p\equiv\frac{n_1+n_2}{n_1-n_2} . \tag{4b}$$

Upper bars indicating average values are omitted since no confusion with instantaneous values may arise. Up to Sec. VII all the quantities are real. The conductances are assumed to be frequency-independent, and therefore frequency fluctuations do not react back on amplitude fluctuations. For emitters $n_2>n_1$, and η, G, R, and J are negative. P and $\mathcal{S}_{\not{r}}$ of course are always positive.

In semiconductors it may happen that the upper and lower states of equal electronic momenta are both filled with electrons, or that neither state is occupied, in which case interaction with the optical field cannot occur. Therefore the number n_1 (respectively, n_2) of state pairs with one electron (respectively, none) in the upper state and none (respectively, one) in the lower state can be written as

$$n_1=Ff_v(1-f_c) , \quad n_2=Ff_c(1-f_v) , \tag{5a}$$

where f_c and f_v denote the probability that the conduction- and valence-band states, respectively, are occupied. The number F of interacting states is approximately

$$F\approx\rho h/(2\pi\tau_{\text{in}}) , \tag{5b}$$

where ρ denotes the density of states (number of states per unit energy interval) and $\tau_{\text{in}}\approx 0.1$ ps the dephasing time. The precise expression of F is not important for our discussion. We consider here only states whose energy spacing $\approx h\nu$ is appropriate for interaction with the optical field, the other states being relevant to spontaneous carrier recombination (neglected in this paper) and scattering.

According to Eq. (5), G may be written

$$G=A[f_v(1-f_c)-f_c(1-f_v)]=A(f_v-f_c) . \tag{6}$$

The positive constant A depends on the semiconductor considered and operating temperature. In the case of weak transverse guidance, the confinement factor and thus A would decrease as the total carrier number N increases because the plasma effect reduces the refractive index and the confinement. But from now on, strong index confinement is assumed.

According to the discussion in the Appendix, G has the approximate linear form

$$G(N,R)=G_0(N)-BR , \tag{7}$$

the dependence of B on N being neglected for simplicity, although it may significantly enhance the differential gain.

If we use the relation $R=GP$ applicable to average values, Eq. (7) reads to first order

$$\frac{\delta G}{G}=\frac{g\delta N}{N}-\frac{\beta\delta R}{R} , \tag{8}$$

where we have introduced the differential gain parameter $g\approx 2$, and the SHB parameter $\beta\approx 0.1$, defined according to

$$g\equiv(N/G)G_{0N} , \quad \beta\equiv BP , \tag{9}$$

where subscripts denote partial derivatives. G_{0N} is negative, and therefore g is negative for an absorber ($G>0$) and positive for an emitter ($G<0$). It is shown in the Appendix that in the limit of large carrier densities

$$\beta=(1-f_c+f_v)/(f_c-f_v) . \tag{10a}$$

For symmetrical conduction and valence bands, we have $f_v=1-f_c$, and Eq. (10a) may be solved for f_c according to

$$f_c=\frac{1+\beta/2}{1+\beta} . \tag{10b}$$

If we eliminate δG from Eqs. (4) and (8), we obtain

$$\frac{\delta P}{P}=\frac{(1+\beta)\delta R}{R}-\frac{g\delta N}{N}-\frac{\not{r}}{R} . \tag{11}$$

When the emitting element is constant-voltage driven, $\delta N=0$, and the carrier equation derived in the next section is not needed. In that special case, intensity noise follows in a straightforward manner from Eq. (11).

For completeness, we give the expression of the population inversion factor n_p defined in Eq. (4),

$$n_p\equiv\frac{n_2}{n_2-n_1}=\frac{f_c(1-f_v)}{f_c-f_v}=\frac{f_c^2}{2f_c-1} . \tag{12a}$$

If we use the expression in Eq. (10b) applicable to symmetrical bands, n_p may be written in terms of β

$$n_p=\frac{(1+\beta/2)^2}{1+\beta} . \tag{12b}$$

This relation rests on the assumption that population inversion is complete at low power levels, i.e., $n_p=1$ if $\beta=0$.

III. CARRIER RATE EQUATION

The purpose of this section is to derive a relation between δN and δR, using the law of conservation of electron number and Ohm's law applied to the electrical circuit connected to the element. The expressions apply equally to absorbing and emitting elements, but absorbing elements (e.g., saturable absorbers or photovoltaic cells under intense illumination) are first considered.

The carrier rate equation

$$R = J + \frac{dN}{dt} + S + \mathscr{S} \tag{13}$$

expresses the fact that the absorbed photon rate R provides an output electronic rate J and spontaneous recombination rate S, the remaining carriers being stored at rate dN/dt. The spectral density of the noise term $\mathscr{S}$ is equal to the average rate S in the case of *radiative* spontaneous recombinations. For simplicity, as indicated earlier, we neglect spontaneous recombination, and consider only the zero-frequency limit, in which case Eq. (13) reduces to $R = J$.

Consider next the electrical load of the absorbing element. It consists in a battery of voltage U_s defined as positive for a forward bias, and a series resistance R_s (including the device series resistance). Let U denote the voltage across the intrinsic diode, a known function of the carrier number N. According to Ohm's law

$$J = [U(N) - U_s]/eR_s \ , \tag{14}$$

where e is the absolute value of the electron charge. U_s is less than $h\nu$ for an absorber (and is usually negative to shorten the detector response time) and U_s exceeds $h\nu$ for an emitter.

To first order, we obtain by combining the relation $R = J$ and Eq. (14)

$$\frac{-g_s \delta N}{N} = \frac{\delta R}{R} \ , \quad -g_s \equiv \frac{NU_N}{eRR_s} \ , \tag{15}$$

the minus sign being introduced so that g_s is positive for emitters.

At room temperature, $NU_N \approx 50$ mV, and the current eR may be of the order of 0.1 A. For an emitting element submitted to a constant-voltage power supply, R_s reduces to the diode internal series resistance (unless some active electronics is used), of the order of 1 Ω. With these values, $g_s \approx 0.5$. Nyquist's noise associated with the series resistance is neglected.

When Eq. (15) is introduced in Eq. (11), we finally obtain

$$\frac{\delta P}{P} = \frac{(1+\kappa)\delta R}{R} - \frac{\mathscr{R}}{R} \ , \quad \kappa \equiv \frac{g}{g_s} + \beta \ . \tag{16}$$

Let us clarify the significance of this result, considering first a linear absorbing element, e.g., an ideal detector. P is proportional to the square of the optical field to which the element is submitted, while R denotes the rate at which photons are absorbed. $\delta P/P$ is then equal to $\delta R/R$ ($\kappa = 0$), plus a noise term whose spectral density is the photon rate reciprocal.

Consider next an emitting element. The proportionality factor is $1 + g/g_s + \beta$. The term g/g_s is proportional to the driver series resistance R_s. If R_s is infinite, it follows from Eq. (16) that $\delta R = 0$, irrespectively of SHB. This is the driving condition employed by Yamamoto, Machida, and Nilsson [10] to generate amplitude-squeezed light. If, in contradistinction $R_s = 0$, the term g/g_s vanishes, but the spectral-hole-burning term β remains. For an oscillator with linear absorbing element, stable operation is then possible only as a result of SHB: $\beta > 0$. If $R_s = 1$ Ω for example, we obtain $g/g_s \approx 4$. As shown in the appendix, β is proportional to the output power P_{out}, and is of the order of 0.1 for a typical vertical-cavity diode delivering 6 mW. The term g/g_s is also proportional to the output power according to the definition of g_s in Eq. (15) since $R = J$ is proportional to P_{out}. Thus the nonlinearity factor κ is of the form $(AR_s + B\tau_{\text{in}}^2)P_{\text{out}}$ where A and B are constants. In that respect, SHB simply augments the external series resistance. But the effect of SHB on phase fluctuations is distinctly different from that of external resistances.

IV. OSCILLATOR NOISE

Figure 1(a) represents a laser oscillator emitting light fully converted into an electrical current by an ideal detector. The laser oscillator, driven by a battery of voltage U_s in series with a resistance R_s, consists of a direct-band-gap semiconductor enclosed in a Fabry-Pérot cavity. For near-unity mirror reflectivities a simple lumped circuit representation is appropriate. The laser oscillator is then represented in the schematic of Fig. 1(b) by an emitting element in parallel with a resonating circuit tuned at the oscillating frequency ν_0 and an absorbing element.

The steady-state admittance of the emitting element is a negative conductance $G \equiv I_e/V$. The detector, or absorbing load, is a linear positive conductance $G_a \equiv I_a/V$, and we have $G + G_a = 0$ in the steady state. As long as the load is perfectly matched optically, the physical sepa-

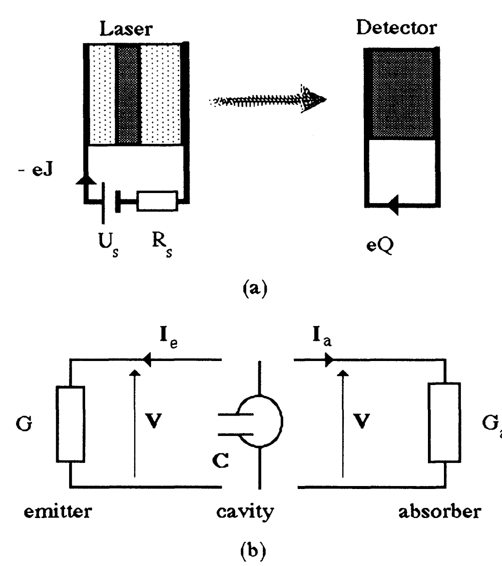

FIG. 1. (a) A "vertical-cavity" laser driven by voltage U_s and series resistance R_s emits light, fully collected by an ideal detector. (b) Schematic representation, consisting of an emitting element, an optical cavity, and an absorbing element, shown separately. V and I are voltage and current at optical frequency ν, divided by $\sqrt{2h\nu}$. I_e relates to the emitting element, I_a to the absorbing element.

ration between the laser and the load is immaterial. For the absorbing element the photonic rate is denoted by Q instead of R, and the noise source is denoted by $\mathscr{q}$ instead of $\mathscr{r}$. Since, according to the schematic in Fig. 1(b), the emitting and absorbing elements are connected together, the P parameter (equal to the square of the voltage V across the circuit at frequency ν_0) is the same for both elements. We have at low base band frequencies

$$\frac{\delta P}{P}=\frac{(1+\kappa)\delta R}{R}-\frac{\mathscr{r}}{R}\ , \quad \mathcal{S}_{\mathscr{r}}=\eta R\ , \tag{17}$$

$$\frac{\delta R}{R}=\frac{\delta Q}{Q}\ , \tag{18}$$

$$\frac{\delta P}{P}=\frac{\delta Q}{Q}-\frac{\mathscr{q}}{Q}\ , \quad \mathcal{S}_{\mathscr{q}}=Q\ , \tag{19}$$

where Eq. (17) coincides with Eq. (16), while Eq. (19), applicable to the absorbing element, is obtained by setting $\kappa=0$. Let us recall that the η factor defined in Eq. (4) is equal to -1 in the case of complete population inversion. The cavity Eq. (18) is the same as Eq. (3) of [1] for $\Omega=0$, and we need not distinguish $-R$ and Q.

Subtracting Eq. (17) from Eq. (19) we obtain

$$-\kappa\delta Q=\mathscr{r}+\mathscr{q}\ . \tag{20}$$

δQ represents the fluctuation of the detected current if the absorber is identified with a detector. Its spectral density follows from Eq. (20) if we use the expressions for the spectral densities of $\mathscr{r}$ and $\mathscr{q}$ in Eqs. (17) and (19). It is now convenient to introduce the population inversion factor n_p in place of $\eta\equiv 1-2n_p$.

$$\mathcal{S}_{\delta Q}=2Qn_p/\kappa^2\ . \tag{21}$$

For constant-voltage drive, $R_s=0$ and $\kappa=\beta$ according to Eqs. (15) and (16). Using the expressions given for n_p in Eq. (12b) applicable to symmetrical bands, Eq. (21) can be written

$$Q^{-1}\mathcal{S}_{\delta Q}=2(1+\beta/2)^2(1+\beta)^{-1}\beta^{-2}\ . \tag{22}$$

This expression shows that sub-Poissonian statistics is achieved ($\mathcal{S}_{\delta Q}<Q$) if the SHB parameter $\beta>1.63$. According to Eq. (22), the fluctuations of the detected current would vanish in the limit of infinite β values. This is, however, a rather extreme situation since the optical gain tends to zero in that limit, and the population inversion factor n_p goes to infinity. Let us recall that the result in Eq. (22) that predicts the possibility of amplitude squeezing for a constant-voltage-driven laser diode rests entirely on the formulation in Eq. (2) and Ohm's law.

For the sake of comparison, consider the previously reported expression for photonic noise based on the formulation in Eq. (1). The result [Eqs. (27) and (20e) of [1]] reads

$$Q^{-1}\mathcal{S}_{\delta Q}=1+2(n_p-\gamma)/\gamma^2\ . \tag{23}$$

Comparison of Eq. (16) and Eq. (28) of [1], in the zero-impedance drive limit ($\kappa=\beta$, $\chi\to\infty$), reveals that the $1-\gamma$ nonlinearity factor of [1] is the reciprocal of $1+\beta$. The limit $\beta\to\infty$ thus corresponds to $\gamma\to 1$, and the result in [1] (which coincides with the result in [11]) would predict that amplitude squeezing cannot be achieved with zero-impedance drive, since $n_p>1$. Using for n_p the expression in Eq. (12b) and setting $1/\gamma=1+1/\beta$, Eq. (23) can also be written explicitly in terms of β.

Figure 2 shows the variation of $Q^{-1}\mathcal{S}_{\delta Q}$ (relative photonic or amplitude noise) as a function of β, as given by Eq. (22) (plain line), and as given by Eq. (23) (dotted line) for constant-voltage drive. The exact result for a 1-Ω series resistance is also shown.

As is well known, SHB is detrimental to modulation bandwidth in the usual high-impedance drive conditions. However, SHB *enhances* the modulation bandwidth in the case of low-impedance drive. For a constant-voltage drive, the -3-dB modulation bandwidth is $\beta/(1+\beta)$ times the cold cavity linewidth. Thus the modulation bandwidth may approach the cavity linewidth at high power if low-impedance drive proves practical.

V. AMPLIFIER AMPLITUDE NOISE

The amplifier model consists of an emitting element, i.e., a negative conductance, the separation between incident and reflected waves being effected with the help of an optical circulator [14].

According to the wave formalism familiar to microwave and optical engineers input and output waves a and b are defined by

$$2a=V+I\ , \quad 2b=V-I \tag{24}$$

assuming without loss of generality that the characteristic impedance is unity. V and I are voltages and currents divided by $\sqrt{2h\nu}$. From the steady-state values we define the field gain

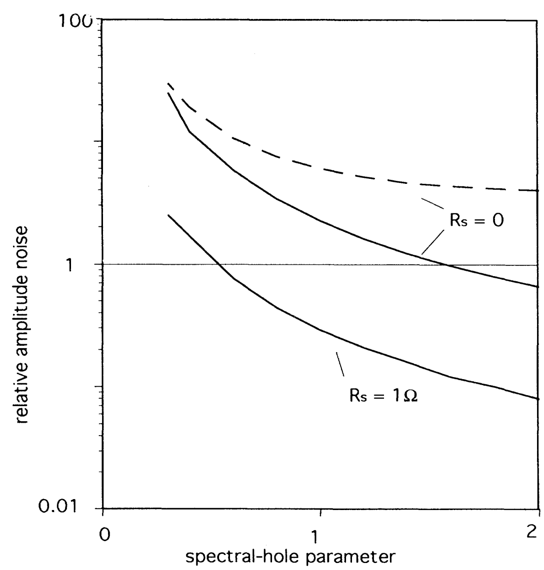

FIG. 2. Variation of the relative photonic (or amplitude) noise $Q^{-1}\mathcal{S}_{\delta Q}$ (unity at the shot-noise level) as a function of the spectral-hole burning parameter β, proportional to optical power. R_s denotes the series resistance. The zero value corresponds to constant-voltage drive. Plain lines are from the exact results [Eq. (22), and Eqs. (21), (16)] essentially based on Eq. (2). Dotted line: result [1,11], essentially based on Eq. (1).

$$\mathscr{g}\equiv\frac{b}{a}=\frac{1+G}{1-G}\ ,\quad G\equiv\frac{I}{V}\ . \tag{25}$$

For an emitter, $\mathcal{G}\equiv\mathscr{g}^2>1$ is the power gain, while for an absorber $\mathcal{G}<1$.

The photonic rate R and the square P of the voltage V are, using Eq. (24),

$$R=VI=a^2-b^2\ ,\quad P=V^2=(a+b)^2\ . \tag{26}$$

The first-order variations of R and P are

$$\delta R=2a\,\delta a-2b\,\delta b\ ,\quad \delta P=2(a+b)(\delta a+\delta b)\ . \tag{27}$$

Using Eq. (27), Eq. (16) is readily converted into

$$(a+\kappa b)2\delta b=(b+\kappa a)2\delta a-\varkappa\ , \tag{28a}$$

$$\mathcal{S}_{\varkappa}=\eta R=\eta(a^2-b^2)\ , \tag{28b}$$

where $\eta=1$ for an absorber and $\eta=-1$ for an emitter with complete population inversion. The spectral density of $2\delta a$ (or $2\delta b$) is unity when light is in the coherent state and it vanishes when light is in the number state. The small-signal power gain $(2b\delta b/2a\delta a)$ follows from Eq. (28a) without the noise term. It is equal to $\sqrt{\mathcal{G}}$ if $\kappa=1$, for example.

The spectral density of the output wave fluctuations can be obtained in a straightforward manner from Eq. (28). Assuming that the *input* wave fluctuation δa is independent of the internal noise source $\varkappa$ we have

$$(1+\kappa\mathscr{g})^2\mathcal{S}_{2\delta b}=(\mathscr{g}+\kappa)^2\mathcal{S}_{2\delta a}+\eta(1-\mathcal{G})\ . \tag{29}$$

For *linear* amplifiers or absorbers $\kappa=0$ and Eq. (29) reduces to known results (but here the derivation is semiclassical). For example, if the input wave is in the coherent state ($\mathcal{S}_{2\delta a}=1$), the attenuated output wave is in the coherent state as well ($\mathcal{S}_{2\delta b}=1$) at zero temperature ($\eta=1$): Random decimation leaves the Poissonian statistics unchanged except for the average rate. More generally ($T>0$), Eq. (29) coincides with the master equation result [15] when the photon number is much larger than $N_2/(N_1-N_2)$.

For an ideal amplifier with a coherent state at the input, $\mathcal{S}_{2\delta b}=2\mathcal{G}-1$ can be interpreted as the sum of the signal shot noise plus the beat between the amplified wave and the field spontaneously generated in the mode (see, e.g., [16]). The so-called spontaneous-spontaneous beat noise not accounted for here can be neglected when the input photon rate is much larger than the optical amplifier bandwidth [17].

Let us now turn our attention to nonlinear amplifiers. For input light in the coherent state ($\mathcal{S}_{2\delta a}=1$) and any κ value we obtain in the large gain limit from Eq. (29), remembering that $\eta=1-2n_p$,

$$\mathcal{S}_{2\delta b}=2n_p/\kappa^2\ . \tag{30}$$

As we have seen, nonzero κ values can be obtained with constant-voltage drive because of SHB. The output is amplitude squeezed if $\kappa^2>2n_p$.

For finite gain values, the simplest situation is when a constant current is injected, in which case $\delta R=2a\,\delta a-2b\,\delta b=0$ (implying $\kappa\to\infty$). Then

$$\mathcal{S}_{2\delta b}=\mathcal{S}_{2\delta a}/\mathcal{G}\ . \tag{31}$$

Amplitude squeezing is perfect in the limit of infinite gain.

VI. AMPLIFIERS WITH ELECTRICAL FEEDBACK

The result in Eq. (31) is not the best that can be achieved for a given gain because the fluctuation of the electrical voltage across the diode is information not employed so far. This author has shown [18] that when the voltage across the series resistance R_s driving an ideal linear amplifier is fed forward to an amplitude modulator, one can achieve in place of Eq. (31) the better result (with $\mathcal{S}_{2\delta a}=1$)

$$\mathcal{S}_{2\delta b}=1/(2\mathcal{G}-1)\ . \tag{32}$$

This result is easily established from Eq. (28a) for *any κ value* by minimizing the spectral density of

$$2\delta b-F\delta R=2\delta b+F(2b\,\delta b-2a\,\delta a)\ , \tag{33}$$

with respect to the feedback factor F.

The other quadrature, multiplied by $2\mathcal{G}-1$ by the linear amplifier, is not otherwise affected. Coherent states at the input also remain minimum-uncertainty states for multiphoton processes.

Quite generally, schemes employing atoms in the upper state (optical amplifiers) are potentially more useful than conventional detectors because they are sensitive to vacuum fluctuations [19]. One must ensure, however, that spontaneous carrier recombination is negligible compared with stimulated recombination, a condition best fullfilled with microcavities. The "squeezing amplifier" just described can be inserted from place to place in a phase-modulated optical link to prevent conversion of amplitude fluctuations into phase fluctuations through the fiber Kerr's effect. The alternative use of parametric amplifiers [20] is at the moment difficult to implement.

At the output end of a lossy fiber light is essentially in the coherent state. High-gain squeezing amplifiers inserted before phase-shift-keyed balanced homodyne receivers may enhance somewhat the signal-to-noise ratio. Note, however, that the neglect of the spontaneous-spontaneous beat noise made in the above theory is permissible only when the signal-to-noise ratio is already rather high. Because the phase fluctuations are amplified and the statistics may not be Gaussian [21], a detailed system analysis is required to establish whether improvement in bit-error rate can be obtained.

VII. PHASE FLUCTUATIONS

According to simple theories neglecting gain compression, a laser diode cannot be frequency modulated by varying slowly the injected current, aside from thermal effects. Indeed, the frequency deviation is proportional to αf, where α denotes the phase-amplitude coupling factor (ratio of changes of the imaginary and real parts of the optical conductivity for a small change of carrier density) and f the base band frequency. Experimentally, a plateau of the order of 0.1 GHz/mA is, however, measured in the low-frequency range (10–100 MHz) for index-guided laser diodes. This is a further indication that a mechanism such as SHB unclamps the carrier number. Spectral holes being symmetrical, they are not expected to affect much the refractive index at the operating frequency according to the Kramers-Kronig relationship. The actual gain is clamped at the loss value, however, and the carrier number N increases as the injected current increases, the spectral hole getting more pronounced. This leads to a decrease of the refractive index and an increase of the oscillation frequency because of the plasma effect and gain curve shift to higher optical frequencies (band filling and hot-carrier injection).

To discuss phase fluctuations, let us introduce a complex rate $R \equiv V^*I = YP + \mathscr{r}$, where the notation $P \equiv |V|^2$ is used, and the conductance G is now considered more generally an admittance Y whose steady-state value remains real. The deviation of R from its (real) steady-state value is denoted $\delta R \equiv \delta R' + i\delta R''$. Thus the quantity previously denoted δR is now denoted $\delta R'$. Similarly, the noise source $\mathscr{r}$ is now a complex quantity $\mathscr{r} \equiv \mathscr{r}' + i\mathscr{r}''$, where $\mathscr{r}'$ and $\mathscr{r}''$ are independent and have the same spectral density as given earlier for $\mathscr{r}$. We obtain

$$\delta R \equiv \delta R' + i\delta R'' = \delta(V^*I) = \delta(YP) + \mathscr{r}' + i\mathscr{r}'' \ , \tag{34a}$$

$$\delta Y = \delta G + i\alpha\delta G_0 \ . \tag{34b}$$

We have introduced in Eq. (34b) the phase-amplitude coupling factor α, whose value is of the order of -5 at a wavelength of 1.55 μm. For the reason mentioned earlier (symmetrical hole), the imaginary part of Y is considered proportional to the thermal equilibrium conductance deviation δG_0 rather than to the actual conductance deviation δG. We do not consider here the unsymmetrical effect of hot carriers.

From Eq. (34), the imaginary part $\delta R''$ of δR is given by

$$\frac{\delta R''}{R} = \frac{\alpha g \delta N}{N} + \frac{\mathscr{r}''}{R} = \frac{-\alpha(g/g_s)\delta R'}{R} + \frac{\mathscr{r}''}{R} \ , \tag{35}$$

where g and g_s were introduced according to their definition in Eq. (9), and Eq. (15) was used.

VIII. OSCILLATOR PHASE NOISE

We consider the same laser diode model as in Sec. IV, again in the limit of small frequencies and negligible spontaneous recombination. According to Eq. (35) applied to the emitting element and to the linear absorbing element (with R changed to Q), the optical frequency deviation $\delta\nu$ is

$$4\pi\tau_p \delta\nu = \frac{\delta R''}{R} - \frac{\delta Q''}{Q} = \frac{\mathscr{r}''}{R} - \frac{\mathscr{q}''}{Q} - \frac{\alpha(g/g_s)\delta R'}{R}$$
$$= \frac{-(\mathscr{r}'' + \mathscr{q}'')}{Q} + \frac{(\alpha/c)(\mathscr{r}' + \mathscr{q}')}{Q} \ , \tag{36a}$$

$$c \equiv 1 + \beta g_s / g \ . \tag{36b}$$

Let us recall that β is the SHB parameter, proportional to the output power $P_{\text{out}} = h\nu Q$, g is a differential gain parameter, and g_s is inversely proportional to the driver resistance R_s. Equation (16) for κ has been used. $\tau_p = C/G_a$ is the photon lifetime, C denoting the capacitance of the tuned circuit shown in Fig. 1. $-\delta R' = \delta Q'$ is given in Eq. (20). In the steady state, $R + Q = 0$.

The (full width at half power) linewidth $\Delta\nu$ is equal to 2π times the spectral density of $\delta\nu$. Because the noise sources in Eq. (36) are independent, and the spectral densities of $\mathscr{q}'$ and $\mathscr{q}''$ are both equal to Q, while the spectral densities of $\mathscr{r}'$ and $\mathscr{r}''$ are both equal to $\eta R = (2n_p - 1)Q$, we obtain from Eq. (36)

$$2\pi Q \Delta\nu = \frac{1}{2}\left[\frac{n_p}{\tau_p^2}\right]\left[1 + \left[\frac{\alpha}{c}\right]^2\right] . \tag{37}$$

Equation (37) shows that Lax's expression [9] for the linewidth of a laser diode is preserved. But the material α factor is divided by a factor c larger than unity as a result of SHB and low electrical drive impedance (or, almost equivalently, high threshold current, if spontaneous carrier recombination is considered). Since usually $\alpha^2 \gg 1$, SHB may importantly reduce the laser linewidth [22,23]. If $R_s = 1\ \Omega$, $NU_N = 50$ mV, $g = 2$, $eQ = 5$ mA, $\beta = 0.1$, the c parameter in Eq. (36b) is equal to 1.5 and the laser linewidth is reduced by SHB by a factor ≈ 2.25 at low and moderate powers. At high optical powers, carrier-number enhancement entails an increase of the intrinsic α factor that may contribute to linewidth rebroadening. Other causes have been invoked (mode instability, side modes, TM modes, $1/f$ noise, induced gratings, etc.) to explain this effect.

Let us now consider the wave formalism. From the definition of $R \equiv V^*I$, $V = a + b$, $I = a - b$, and remembering that the steady-state values of a and b are real, we obtain

$$\delta R'' = \text{Im}\{\delta[(a+b)^*(a-b)]\} = b2\delta a'' - a2\delta b'' \ , \tag{38}$$

$$\delta\phi_{\text{out}} - \delta\phi_{\text{in}} = \frac{\delta b''}{b} - \frac{\delta a''}{a} = \frac{-\delta R''}{2ab} \ . \tag{39}$$

For vanishing α, the linewidth $\Delta\nu$ of a unidirectional ring-type laser with concatenated (emitting or absorbing) elements is, from Eq. (39) and (35), a simple sum

$$2\pi\Delta\nu\tau^2 = \tfrac{1}{4}\sum_k \left[\eta\left[\frac{1}{b^2} - \frac{1}{a^2}\right]\right]_k \ , \tag{40}$$

where τ is the round-trip time. Note that all the terms in this sum are positive because η is positive when $a > b$ (loss), and negative when $b > a$ (gain).

Equation (40) can be shown to coincide with Eq. (47) of [24] when $\alpha = 0$. For nonzero α factors one must first solve for $\delta R'$. The linewidth expression then involves in addition to the term shown in Eq. (40) a sum over products of the form $\alpha_i \alpha_j$. The case of shot-noise injection, however, is remarkably simple because only squares (α_i^2) enter. The results in [24] are most easily obtained by considering concatenated absorbing and emitting elements as we do here.

The Green's-function method used by many authors

[25–27] may not be quite accurate above threshold. Indeed, unless the mirror reflectivities are near unity, it is not permissible to assume that the optical field factorizes into a modal function and a function of time: the independent noise sources distort the mode profile. This conclusion has also been reached by Nilsson and others (Chapter 3 of [8]), and Prasad from a different viewpoint [28].

IX. CORRELATIONS

The important relation between normalized cross-spectral densities, denoted by C, established in [29] without SHB,

$$C_{\delta Q'\delta\nu}=C_{\delta Q'\delta U}C_{\delta U\delta\nu} \tag{41}$$

or equivalently

$$\mathcal{S}_{\delta Q'/Q\delta\nu}\mathcal{S}_{\delta U\delta U}=\mathcal{S}_{\delta Q'/Q\delta U}\mathcal{S}_{\delta U\delta\nu}\ , \tag{42}$$

where $\delta Q'$, $\delta\nu$, and δU represent, respectively, photon rate, frequency, and electrical voltage fluctuations, follows quite generally from the fact that $\delta\nu$ is the sum of a term proportional to δN (or δU) and a noise term $(\not{p}''-q'')$ uncorrelated with both δN and $\delta Q'$. Since the spectral densities in Eq. (42) are independent of the attenuation [1], Q can be replaced by the detected rate D.

X. CONCLUSION

We have presented a theory of amplitude and phase noise of high-power index-guided laser diodes that accounts for spectral-hole burning. The gain must be considered a function of carrier number and photon rate (rather than photon number) plus a fluctuation that *does not depend on the nonlinearity*. Examples of application have been worked out relating to both oscillators and amplifiers. It is shown in particular that amplitude-squeezed light can be generated by constant-voltage laser diode, a result not predicted by previous theories that treat gain compression in a formal manner (i.e., without investigating specific mechanisms). Quantum results for atoms (see, e.g., [30]) can alternatively be obtained from the semiclassical theory.

In conventional edge-emitting lasers, side modes are separated from the main mode by approximately 100 GHz, and symmetrically located. The frequency separation being small compared with the scattering-time reciprocal, SHB is essential to mode competition [31]. This important subject has not been treated in the present paper. Statistical fluctuations of the optical gain should also be considered in an improved laser-diode noise theory.

ACKNOWLEDGMENT

The Equipe de Microoptoélectronique de Montpellier is "Unité de Recherches Associée au CNRS No. 392."

APPENDIX: SPECTRAL-HOLE BURNING

Spectral-hole burning is described for an absorbing element but the expressions also apply to emitting elements. At small and moderate optical power electrons (holes) are in thermal equilibrium within the conduction (valence) band and obey Fermi's statistics. The conductance G then depends only on the total carrier number N, the optical frequency being considered fixed. But at high optical power, G is a function of both N and the absorbed photon rate R.

Within the relaxation-time approximation [4], the rates at which spectral holes in the conduction and in the valence bands are replenished are proportional to the respective deviations of mean state occupancy from the thermal equilibrium values. If we neglect the storage rate for *interacting* carriers, spectral-hole-filling rates must be equal to the interband generation rate R from electron-rate conservation. Thus

$$R=\frac{F}{\tau_e}(f_c-f_{c0})=\frac{F}{\tau_h}(f_{v0}-f_v)\ , \tag{A1}$$

where the 0 subscripts refer to thermal equilibrium values, and f_c and f_v denote the mean occupation of the conduction and valence bands, respectively. For absorbers $f_c>f_{c0}$, but for emitters the converse holds. An approximate expression for the number F of interacting carriers was given in Eq. (5b) of the main text. The electron and hole relaxation times τ_e and τ_h are of the order of 0.1 ps. Measured electron and hole mobilities give only lower bounds to τ_e and τ_h since near-elastic scattering events reduce mobilities without much affecting τ_e and τ_h that depend mostly on carrier-carrier scattering [32]. In the case of rare-earth ions in a perturbating glass matrix, inhomogeneous broadening may be accounted for by summing up the various gain contributions, leading to a $(1+m/m_s)^{-1/2}$ saturation law, where m is the photon number. This seems to be the approach in [33]. However, when the strong coupling between interacting states is considered a behavior closer to the usual form in Eq. (1′) obtains [6].

If the expressions in Eq. (A1) are substituted into Eq. (6), G assumes the form

$$G=A(f_v-f_c)=G_0(N)-B(N)R\ , \tag{A2}$$

$$G_0\equiv A(f_{v0}-f_{c0})\ , \tag{A3a}$$

$$B\equiv\left[\frac{A}{F}\right](\tau_e+\tau_h)\approx 2\pi\left[\frac{A}{\rho h}\right]\tau_c^2\ , \tag{A3b}$$

$$\tau_c^2\equiv\tau_{\rm in}(\tau_e+\tau_h)\ . \tag{A3c}$$

As the carrier density increases, carrier-carrier scattering events become more frequent and the SHB parameter B, proportional to τ_c^2, decreases, i.e., $dB/dN<0$. The dependence of B on N, which is usually ignored, may influence significantly the laser diode properties. However, for simplicity, this dependence is neglected in the main text.

Using the expression for R in Eq. (A1), G in Eq. (A2), and the relation $R=GP$ applicable to average values, the dimensionless parameter $\beta\equiv BP$ can be written

$$\beta=(f_{c0}-f_c+f_v-f_{v0})/(f_c-f_v)\ . \tag{A4}$$

If the population inversion is complete at low fields, we have $f_{c0}=1$, $f_{v0}=0$.

A practical expression for β is

$$\beta \approx \frac{0.15 P_{\text{out}} (\lambda_0^2 / \mathcal{A}) \tau_c^2}{\ln(1/\mathcal{R})} , \tag{A5}$$

where the total output optical power P_{out} is expressed in mW, and the scattering time τ_c in ps. λ_0 denotes the free-space wavelength, $\mathcal{A}$ the active emitting area, and $\mathcal{R}$ the mirror reflectivities. The numerical factor has been calculated for GaAs at room temperature but is approximately applicable to other III-V compounds as well. The details relating to this estimate of β are too lengthy to be given here.

For a vertical-cavity laser diode of 6.4 μm^2 area, $\mathcal{R}=0.99$ and $\tau_c=0.1$ ps, we calculate $\beta=0.1$ at a total generated power of 6 mW. β is expected to increase at lower temperatures because of lower carrier densities and reduced optical-phonon population. There is much uncertainty on the scattering times. However, scattering times much larger than those quoted above would imply deep spectral holes in the spontaneous emission spectrum, a feature rarely observed.

Spatial diffusion in many respects mimics SHB. In particular, carriers in the high-band-gap layers separating quantum wells diffuse in and out, with characteristic times in the order of tens of picoseconds. The comparatively slow spatial carrier diffusion may explain the mediocre modulation bandwidth of quantum-well lasers at high power. The introduction of a single phenomenological β parameter perhaps suffices to account for the observed dynamics of single-mode index-guided laser diodes.

Let us make a few observations relating to spontaneous carrier recombination and intraband scattering fluctuations. Consider first a simple example: Light-emitting diodes (LED) generate light by the process of radiative spontaneous recombination, each electron being converted into one photon. For a constant-voltage drive, the electrical current fluctuations are thus at the shot-noise level. But consider q light-emitting diodes in series submitted to a constant voltage. Elementary circuit theory shows that the fluctuations of the current flowing through this chain of LED's are now at a much lower value: $1/q$ times the shot-noise level.

This observation suggests that, generally, the spectral density of current fluctuation relative to shot noise is of the form $s=(n/S)(dS/dn)$, where $S(n)$ represents the recombination rate and n the number of carriers that can contribute to spontaneous recombination (to be distinguished from the total carrier number N). For example, $s=2$ for Augers's effect, and $s=\frac{1}{2}$ for recombination with impurity levels half-way between the valence and the conduction bands (deep-level traps) [34]. It is usually considered that the rate at which carriers recombine spontaneously radiatively is proportional to the square of N, while Auger's recombinations would exhibit a third-power law. These conclusions, however, hold only at high temperature. As far as noise is concerned, the important fact is that radiative recombination is a one-photon process, and Auger's effect is a two-electron process.

For Ohmic current leaks, the above expression gives $s \approx 2kT/h\nu$ in agreement with Nyquist's formula if we use the generally accepted value $NdU/dN \approx 2kT$, where U denotes the voltage across the diode, and $n \approx N$. Nyquist's formula is indeed preserved to first order when a direct current flows through a resistor [35,36]. Thus Ohmic leaks or spatial diffusion do not introduce new significant noise terms. Similarly, carrier-carrier scattering within a band does not introduce intrinsic noise, while intraband scattering by optical phonons introduces noise only at the low $2kT/h\nu$ level. The above considerations appear to be consistent with observations made by various authors.

[1] J. Arnaud, Phys. Rev. A **45**, 1775 (1992).
[2] J. Arnaud, Opt. Quantum Electron. (to be published).
[3] Y. Vidal, S. Gaillard, and J. Arnaud (unpublished). The equivalent circuit given in this paper shows that amplitude squeezing can be achieved with pump fluctuations at the shot-noise level at nonzero frequencies.
[4] H. Haug, Z. Phys. **200**, 57 (1967).
[5] M. T. Tavis, IEEE J. Quantum Electron. **19**, 1302 (1983).
[6] A. Tomita and S. Suzuki, IEEE J. Quantum Electron. **27**, 1630 (1991), see Fig. 5.
[7] D. E. McCumber, Phys. Rev. **141**, 306 (1966).
[8] *Coherence, Amplification and Qauntum Effects in Semiconductor Lasers,* edited by Y. Yamamoto (Wiley, New York, 1991).
[9] M. Lax, Phys. Rev. **160**, 290 (1967).
[10] Y. Yamamoto, S. Machida, and O. Nilsson, Phys. Rev. A **34**, 4025 (1986).
[11] A. Karlsson, Phys. Rev. A **43**, 3741 (1991).
[12] J. Arnaud, Electron. Lett. **23**, 450 (1987).
[13] G. S. Agarwal, Phys. Rev. A **1**, 1445 (1970).
[14] Y. Fujii, J. Lightwave Technol. **9**, 1238 (1991).
[15] R. Loudon, *The Quantum Theory of Light* (Clarendon, Oxford, 1986).
[16] J. Arnaud, IEEE J. Quantum Electron. **QE4**, 893 (1968).
[17] Y. Yamamoto, IEEE J. Quantum Electron. **QE16**, 242 (1980); J. C. Simon, J. Lightwave Technol. **LT5**, 260 (1987); D. W. Smith, *ibid.* **5**, 1466 (1987).
[18] J. Arnaud, Electron. Lett. **25**, 1 (1989).
[19] M. Ueda and M. Kitagawa, Phys. Rev. Lett. **68**, 3424 (1992).
[20] Y. Mu and C. M. Savage, J. Opt. Soc. Am. B **9**, 65 (1992).
[21] J. H. Shapiro, IEEE Phot. Technol. Lett. **4**, 647 (1992); P. A. Humbler, *ibid.* **4**, 650 (1992).
[22] G. Morthier, P. Vanwikelberge, F. Buytaert, and R. Baets, Proc. IEE J **137**, 30 (1990).
[23] J. Arnaud, Electron. Lett. **27**, 2354 (1991).
[24] J. Arnaud, IEEE J. Quantum Electron. **QE25**, 668 (1989).
[25] B. Tromborg, H. Olesen, X. Pan, IEEE J. Quantum Electron. **27**, 178 (1991). An improved formulation is X. Pan, B. Tronborg, H. Olesen, and H. E. Lassen, IEEE Phot. Technol. Lett. **4**, 1213 (1992).
[26] G. H. Duan, P. Gallion, J. Holtz, and J. C. Bouley, J.

Phys. (France) III **2**, 1651 (1992).
[27] M. Claassen and W. Harth, Proc. IEE J **139**, 50 (1992).
[28] S. Prasad, Opt. Commun. **85**, 227 (1991).
[29] M. Estéban, Ph.D. thesis, University of Montpellier II, France, 1991.
[30] M. I. Kolobov, L. Davidovich, E. Giacobino, and C. Fabre, Phys. Rev. A **47**, 1431 (1993).
[31] G. R. Gray and G. P. Agrawal, IEEE Phot. Technol. Lett. **3**, 204 (1991).
[32] R. Binder, D. Scott, A. E. Paul, M. Lindberg, K. Hennenberger, and S. W. Koch, Phys. Rev. B **45**, 1107 (1992); C. B. Kim, En. T. Peng, C. B. Su, W. Rideout, and G. H. Cha, IEEE Phot. Technol. Lett. **4**, 969 (1992).
[33] G. P. Agrawal, Appl. Phys. Lett. **57**, 1 (1990).
[34] M. J. Buckingham, *Noise in Electronics Devices and Systems* (Wiley, New York, 1983).
[35] A. J. Dekker, H. Hickman, and T. M. Chen, Am. J. Phys. **59**, 609 (1991).
[36] Y. Yamamoto and H. A. Haus, Phys. Rev. A **45**, 6596 (1992).