Reply to "Comment on 'Further investigations of the operationally defined quantum phase' "

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It is pointed out in response to Hradil and Bajer [preceding Comment, Phys. Rev. A **48**, 1717 (1993)] that $\hat{C}_M$ and $\hat{S}_M$ cannot both be measured together by scheme 1, whereas they can by scheme 2. There is no phase angle whose cosine and sine both vanish, and putting $C+iS$ equal to zero for certain measurement outcomes breaks the connection with phase.

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In their Comment on our paper, Hradil and Bajer [1] argue that we have given the "wrong description" of our measurement scheme 1, in that the corresponding cosine $\hat{C}$ and sine $\hat{S}$ operators should commute [2,3]. They then go on to represent the input field in an enlarged four-mode Hilbert space. We do not agree with this treatment. The position $\hat{q}$ and momentum $\hat{p}$ of a particle can also be measured successively and independently if the same quantum state is prepared before each measurement, yet $\hat{q}$ and $\hat{p}$ do not commute because they cannot be measured together. For the same reason, the operators $\hat{C}_M$ and $\hat{S}_M$ for scheme 1, which cannot be measured simultaneously, do not commute either.

Also, it is claimed by Hradil and Bajer [1] that measurement by our scheme 1 is "fully equivalent" to measurement by scheme 2. That may be so in their interpretation of the measurement, but it is definitely not true in our approach, which is based on the analogy with classical optics. To see this, we need only point out that the combination of outcomes $m_3=1$, $m_4=0$, $m_5=0$, $m_6=0$ leads to perfectly meangingful and compatible values of the cosine and sine via scheme 2, but not via scheme 1. As an explicit counterexample of the claim by Hradil and Bajer [1], we consider the two-mode coherent state $|v_1\rangle|v_2\rangle$ with $|v_1|, |v_2| \ll 1$. If we choose the constant K for scheme 1 so that $\langle \hat{C}_M^2+\hat{S}_M^2\rangle=1$, then a scheme-1 measurement yields the result

$$\langle \hat{C}_M\rangle \approx \frac{\sqrt{2}|v_1|\,|v_2|\cos(\arg v_2-\arg v_1)}{(|v_1|^2+|v_2|^2+2|v_1|\,|v_2|)^{1/2}} , \quad (1)$$

whereas scheme 2 gives

$$\langle \hat{C}_M\rangle \approx \frac{\cos(\arg v_2-\arg v_1)}{|v_1|/|v_2|+|v_2|/|v_1|} . \quad (2)$$

As $|v_1|, |v_2|\to 0$, $\langle \hat{C}_M\rangle$ tends to zero in the first case, but not in the second.

Second, we do not agree with the procedure of interpreting both the values of C and S as zero when the experimental outcomes are $m_4=m_3$ and $m_6=m_5$, so as to avoid the renormalization that we have adopted [2,3]. Such outcomes are not "noisy data," as Hradil and Bajer describe them, and we discount them only because they do not lead to meaningful values of the cosine and sine. There is no phase angle whose cosine and sine are both zero, and making both $C=0$ and $S=0$ breaks all connection with what one generally understands by phase, and leads to meaningless values at times [4].

[1] Z. Hradil and J. Bajer, preceding Phys. Rev. A **48**, 1717 (1993).
[2] J. W. Noh, A. Fougères, and L. Mandel, Phys. Rev. A **45**, 424 (1992).
[3] J. W. Noh, A. Fougères, and L. Mandel, Phys. Rev. A **46**, 2840 (1992).
[4] Z. Hradil, Phys. Rev. A **47**, 4532 (1993).

