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Laser-noise suppression in the dressed-atom approach. II. Minimization principle for conventionally pumped lasers

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The intrinsic property of laser systems with many internal degrees of freedom to generate a squeezed light is studied. The dressed-atom approach to fluctuations, described in the preceding paper [Khazanov, Koganov, and Shuker, Phys. Rev. A **48**, 1661 (1993)], is employed. A minimization principle which governs noise quenching in laser systems is described comprehensively. This principle allows one to avoid complicated quantum-mechanical calculations to assess the squeezing capacity of the system. It is shown that noise in a laser system can be decomposed into noise states. These states interact coherently. For instance, each nonactive level in a multilevel lasing scheme may represent separate noise states under certain conditions while both lasing levels make only one noise state. The squeezing capacity of the system is determined by a quantity called the *noise dimension.* The theory is extended to laser schemes with many photons (two-photon generation, more than one lasing transition, etc.). The validity of the minimization principle is established for this type of system. Some consequences from the theory, which are relevant to the experiment, are discussed.

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I. INTRODUCTION

Squeezed-light generation can be achieved by two different schemes. In the preceding paper, paper I (Ref. [1]), we discussed an introduction of the regularity via pumping with suppressed fluctuations [2–7]. Another method of noise suppression is based upon the intrinsic feature of multilevel systems to generate squeezed light [8–16]. Regular pumping schemes have an evident disadvantage: to have reached a squeezed output in a laser one should have had a squeezed input (i.e., squeezed pumping). This seems to discredit the idea of a laser itself as an active device. Indeed, how can a more coherent light be obtained than that given by a laser? Moreover, the crucial difference between the active device—the laser—and the passive one is that the former is able to generate a coherent light from an incoherent input. In this sense the regularly pumped laser is rather a passive device. It just transforms a coherent input into a coherent output. In light of this reasoning, the second method of laser noise suppression seems to be a more appealing one. In contrast to the regular pumping it can be regarded as an active device.

The present paper is designed for a comprehensive description of a principle of noise suppression in active devices, i.e., lasers. This principle governs the noise suppression in a number of systems in which the pump can be assumed to be stationary (see the specification in Ref. [1]). Qualitatively, the mechanism of noise suppression can be formulated as follows. In a multilevel laser system with internal degrees of freedom each level (internal degree) is a source of noise. Since the noise of each level has the same physical origin, i.e., spontaneous emission noise, the levels are "correlated" and able to interact coherently. Incoherent processes (i.e., pumping and decay) impose a specific restriction on this interaction. For instance, a group of levels between which there is an intensive incoherent exchange (as compared to other levels) is statistically identical and makes only one source of noise. The output noise is determined by the coherent interaction of such groups. As will be shown, the squeezing capacity of the system grows with the number of such groups. It is remarkable that such a coherent addition of noises is intrinsic for any active system. In the conventionally pumped two-level laser the coherent addition of noises of different atoms gives rise to the shot-noise limit. While in the case of the conventional two-level laser a coherent summation of noises from different atoms results in a Poissonian distribution of amplitude, in a multilevel system the addition of noises from the different levels gives rise to a sub-Poissonian distribution. Our treatment is based upon the dressed-atom approach described in Ref. [1]. Several important examples of multilevel systems are discussed in recent publications [8–16]. The semiconductor laser that demonstrated a very high squeezing degree [17] can be viewed as such a multilevel system [15,16]. Indeed, it has been recently reported that such a laser cannot be regarded as a two-level system with implication on the laser photon statistics [18]. In the present work we briefly address this noise reduction degree of semiconductor laser.

The setup of the paper is as follows. In Sec. II we apply the dressed-atom approach to the conventionally pumped multilevel laser and discuss the intrinsic noise quenching property of the system with internal degrees of freedom. A minimization principle that governs the noise reduction is formulated. In Sec. III it is shown that the minimization principle holds true for many-photon and multiphoton systems. The hierarchy of photon statistics for such systems is discussed.

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II. CONVENTIONAL MULTILEVEL PUMPING

A. Stationary fluctuations in laser systems with internal degrees of freedom

In this section we consider the influence of internal degrees of freedom on squeezing phenomenon. Here we deal with the simplest case of three-level generation [8]. Consider the three-level laser (Fig. 1) with atomic relaxation defined in the usual manner [see Eq. (5) of I, abbreviated Eq. (1-5), at $n=2$]. Dressed-atom formulation of the problem (1-11)-(1-17) results in the following set of equations:

$$\frac{1}{\kappa}\frac{\partial}{\partial t}\rho_f = 2\frac{\partial}{\partial \eta}\eta\left[1-\frac{N}{\kappa\gamma_\perp}R\right]\rho_f \ , \tag{1}$$

$$\frac{1}{\gamma_\perp}\frac{\partial}{\partial t}R + R - \rho_1 + \rho_0 = \theta_R \ , \tag{2}$$

$$\frac{\partial}{\partial t}\rho_0 + (\gamma_{01}+\gamma_{02})\rho_0 - \gamma_{01}\rho_1 - \gamma_{20}\rho_2 - \frac{\gamma_\parallel}{2}xR = \theta_0 \ , \tag{3}$$

$$\frac{\partial}{\partial t}\rho_1 + (\gamma_{10}+\gamma_{12})\rho_1 - \gamma_{21}\rho_2 - \gamma_{01}\rho_0 + \frac{\gamma_\parallel}{2}xR = \theta_1 \ , \tag{4}$$

$$\frac{\partial}{\partial t}\rho_2 + (\gamma_{21}+\gamma_{20})\rho_2 - \gamma_{12}\rho_1 - \gamma_{02}\rho_0 = \theta_2 \ , \tag{5}$$

with N being the number of atoms (all the atoms are identical and the number of atoms is fixed); κ is the field damping constant; $\gamma_\perp$ is the dipole moment relaxation constant. Notations ρ_0, ρ_1, and ρ_2 stand for diagonal matrix elements of the dressed-atom matrix r_1 [see Eq. (1-14)]

$$\rho_0 \equiv (r_1)_{00} \ , \quad \rho_1 \equiv (r_1)_{11} \ , \quad \rho_2 \equiv (r_1)_{22} \ . \tag{6}$$

Other quantities are defined as follows:

$$z \equiv \sqrt{\eta}e^{i\varphi} \ , \quad x \equiv \frac{\eta}{\tilde{\eta}}, \quad \tilde{\eta} \equiv \frac{\gamma_\perp\gamma_\parallel}{4} \ ,$$
$$\gamma_\parallel = \frac{2(\gamma_{01}+\gamma_{10})(\gamma_{21}+\gamma_{20})}{\gamma_{12}+\gamma_{02}+2(\gamma_{20}+\gamma_{21})} + \frac{2[\gamma_{02}(\gamma_{12}+\gamma_{10}+\gamma_{21})+\gamma_{12}(\gamma_{20}+\gamma_{10})]}{\gamma_{12}+\gamma_{02}+2(\gamma_{20}+\gamma_{21})} \ . \tag{7}$$

The quantity R stands for the real dipole moment of the dressed atom according to the definition

$$(r_1)_{01} = \frac{izR}{\gamma_\perp} \ . \tag{8}$$

Fluctuation terms θ_i stem from the right-hand side (rhs) of the master equation (1-14) and have the following explicit form:

$$\theta_R = -\frac{\partial}{\partial \eta}\left[\rho_1 - \frac{x}{2m}R^2\right]\ln\rho_f \ , \tag{9}$$

$$\theta_0 = \frac{\partial}{\partial \eta}\left\{\frac{\gamma_\parallel}{2}xR(\rho_0-1)\right\}\ln\rho_f \ , \tag{10}$$

$$\theta_i = \frac{\partial}{\partial \eta}\left\{\frac{\gamma_\parallel}{2}xR\rho_i\right\}\ln\rho_f \quad (i=1,2) \ , \tag{11}$$

$m \equiv (\gamma_\perp/\gamma_\parallel)$. For the sake of simplicity we assume here only three nonzero rate constants γ_{21}, γ_{12}, and γ_{02} so that

$$\gamma_{10} = \gamma_{01} = \gamma_{20} = 0 \ . \tag{12}$$

Standard perturbation theory described in Ref. [1] results in the following expression for the Q parameter:

$$Q = -\frac{2\gamma_{21}\gamma_{02}}{(\gamma_{02}+\gamma_{12}+2\gamma_{21})^2} \ , \tag{13}$$

provided the field is strong ($x \gg 1$). The result (13) one can find from our work [8]. Minimization of the Q parameter upon pumping rate γ_{21} results in

$$Q_{\min} = -\frac{\gamma_{02}}{4(\gamma_{02}+\gamma_{12})} \tag{14}$$

at

$$2\gamma_{21} = \gamma_{02} + \gamma_{12} \ . \tag{15}$$

It turned out that the condition of minimization of the Q parameter coincided with another condition. Let us consider the populations of the levels. "Classical" (i.e., $\theta_R = \theta_i = 0$, $i=0,1,2$) solution of Eqs. (2)–(5) is (provided the field is strong)

$$\rho_0 = \rho_1 = \frac{\gamma_{21}}{\gamma_{02}+\gamma_{12}+2\gamma_{21}} \ , \tag{16}$$

$$\rho_2 = \frac{\gamma_{02}+\gamma_{12}}{\gamma_{02}+\gamma_{12}+2\gamma_{21}} \ . \tag{17}$$

One may notice that the minimizing condition (15) coincides with the following condition:

$$\rho_2 = \rho_0 + \rho_1 \ . \tag{18}$$

In what follows we shall see that it is not by chance that Eq. (18) is identical to minimization condition (15). We shall see that the two lasing levels constitute one "noise state" and Eq. (18) corresponds to some minimization principle according to which all the noise states (nonactive level 2 makes another noise state in our case) have to be equally populated.

The minimal value $Q=-\frac{1}{4}$ (25% squeezing) occurs

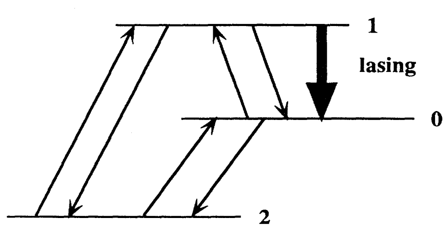

FIG. 1. General scheme of three-level lasing.

under the limit $\gamma_{02} \gg \gamma_{12}$. In this limit one has a unidirectional rate process and condition (15) changes into

$$2\gamma_{21} = \gamma_{02} \; . \tag{19}$$

Condition (19) has been generalized for multilevel systems by Ralph and Savage [9,13,14] and Ritsch *et al.* [10,12]. They have identified the mechanism of noise quenching for the unidirectional pumping process and have established that the multiple recycling of the active electrons results in the deterministic pumping process and, thus, suppresses a stochasticity in a laser system. The results obtained for the three-level generation case have prompted the more general consequences of coherent interaction of noise states (noises of different levels or groups of levels). This concept will be formulated in the follow section.

B. Noise states and minimization principle

Here we discuss a common principle of laser light coherence and introduce all notions associated with it. We emphasize that the systems we discuss in this paper possess an intrinsic property to convert an incoherent energy of pumping to coherent one of noise-reduced laser light. It means that we do not consider here the systems with any external coherences such as regularly or coherently pumped lasers [2–7] to which our principle is not relevant. Before we deal with a formal definition and quantitative assertions we discuss qualitatively the phenomenon of coherent addition of noises from different sources in laser.

Coherent addition of noises from different atoms in a conventional two-level laser

The principle difference between the laser light and classical light is the coherence of the former. From a statistical point of view the classical light consists of incoherent wave packets and the total noise is a simple sum of different atom noises. There is, apparently, no correlation between the noises of different atoms. The laser light is coherent and the noise properties of different radiating dipoles are correlated. This statistical coupling of the different radiating dipoles is so strong that the laser noise corresponds to the noise of only one radiating dipole. The drastic reduction of noise to the noise of a single atom is clearly seen in the transition from the below-threshold photon statistics to the above-threshold statistics. Indeed, the incoherent light of a laser below threshold is described by the Plank distribution with the photon number variance in the form [19,20]

$$\sigma^2 = \langle n^2 \rangle - \langle n \rangle^2 = \langle n \rangle (\langle n \rangle + 1) \; , \tag{20}$$

where $\langle n \rangle$ is the mean photon number. Provided $\langle n \rangle \gg 1$ the variance normalized to mean photon number is proportional to $\langle n \rangle$ or to the number N of radiative dipoles, i.e.,

$$\frac{\sigma^2}{\langle n \rangle} \cong \langle n \rangle \sim N \; . \tag{21}$$

Above threshold the photon statistics is given by a Poisson distribution with normalized variance,

$$\frac{\sigma^2}{\langle n \rangle} \equiv 1 + Q \sim 1 \; , \tag{22}$$

with Q being Mandel parameter (in this case $Q \to 0$). It is distinctly seen from formulas (21) and (22) that the transition from incoherent light to coherent light corresponds to the transition from the incoherent sum of noises of different radiating dipoles [Eq. (21)] to the noise of a single atom (22). Thus we emphasize that above threshold the amplitude noise features a strong collective behavior. In fact, the state of a field with N atoms in a cavity of quality q remarkably coincides with one with a single atom in the cavity but with a quality qN (see Fig. 2). It has been established [19,21–23] that the transition from Eq. (21) to Eq. (22) features a first-order phase-transition behavior with the pump parameter as the control parameter. Thus, this phase transition is associated with strong coherent coupling of different atoms, when the system as a whole exhibits a noise like a single atom.

It has been shown [8–14] that additional internal degrees of freedom in radiating dipoles may give rise to further reduction of amplitude noise below the Poissonian one. For instance, the squeezing capacity of a three-level generation scheme is 25% [8]. Certainly, the internal degrees of freedom can be quite diverse and complicated. These can be a multilevel system, a semiconductor, a system with several active levels, etc.

We shall see in what follows that under certain conditions a coherent addition of noises from different internal degrees of freedom is also possible, which results in further noise reduction. This noise reduction is caused by a certain different order imposed on the system and features a second-order phase-transition behavior.

We start with the following definition: The number n is noise dimension (ND) of the system if the two following conditions are satisfied.

(a) The states of the system are decomposed into n groups so that transition rates between states belonging to one group are much larger than transition rates between the states belonging to different groups. Each such group is called the noise state of the system.

(b) The population of every noise state has the same order of magnitude.

First comments. Fast transitions inside the noise state

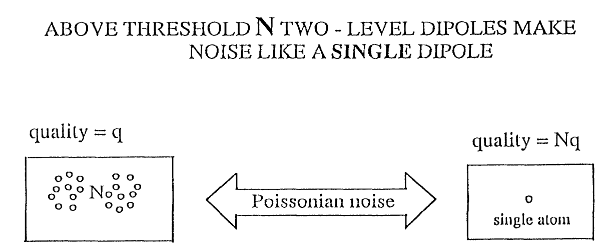

FIG. 2. Coherent addition of noises for two-level atom lasing. Correlated agents are radiated dipoles. The limit of classical correlations is a Poissonian noise. To overcome this limit the additional (quantum) structure is required.

physically mean that states belonging to the same noise state are statistically degenerate. In other words, they "bear" the same noise and statistically manifest themselves like one state. For instance, two lasing levels, at strong saturating field, make only one noise state. Thus the noise dimension of the system is the full number of noise states. Let us consider some examples of systems with different noise dimensions.

(1) Four-level system with lasing transition between levels 0 and 1 (Fig. 3). At strong field the levels 0 and 1 make only one noise state. If we adopt the same order of magnitude for all relaxation constants, i.e., $\gamma_{21} \sim \gamma_{32} \sim \gamma_{03}$, then the noise dimension is equal to 3. If, for instance, $\gamma_{21} \sim \gamma_{32} \ll \gamma_{03}$ then the levels 1, 0, and 3 make one noise state and the noise dimension of the system becomes equal to 2. When otherwise $\gamma_{21} \ll \gamma_{32} \sim \gamma_{03}$ all levels 0, 1 , 2, and 3 degenerate into one noise state and the noise dimension becomes equal to 1. Then, in fact, the noise properties of such a system would be identical to those of a two-level system. The last case results in a Poissonian noise.

(2) Semiconductor laser. From a conceptual point of view all degrees of freedom in a semiconductor are "internal" and this example is particularly important for our concept. If intraband transitions are much faster than those between the different bands we get a system with the noise dimension equal to 1. This fact distinctly shows that the noise dimension of the system has nothing to do with the number of energy levels (the latter is very large for the semiconductor case). If, to the contrary, all the transition rates are of the same order the noise dimension grows to infinity and one can expect the remarkable noise reduction observed in experiments.

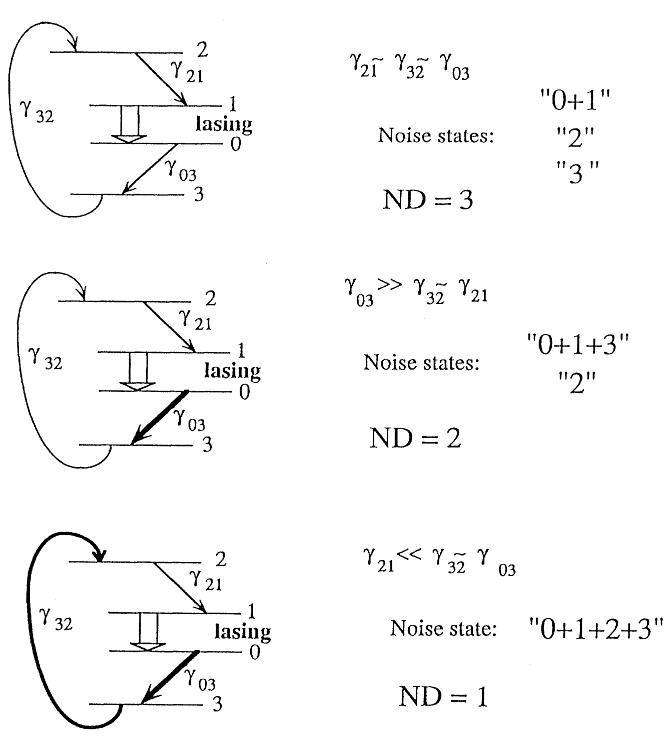

FIG. 3. Illustration of a noise dimension (ND) idea for an example of the four-level scheme.

As mentioned above, it was reported that semiconductor lasers cannot be described by a two-level model but rather it requires at least four levels and that this has important consequences on the laser photon statistics [18]. Thus the regular pumping description of a semiconductor laser is not appropriate as it is based on a two-level model. In terms of the present work, the semiconductor laser is a multilevel system, the noise dimension of which should be calculated in order to determine its noise reduction capacity. It is suggested here that such a description can account for the high squeezing degree reported for the semiconductor lasers [17].

(3) Four-level lasing with two lasing transitions (Fig. 4). We assume that the field saturates both of the lasing transitions. In this case, each pair of the lasing levels constitutes one noise state. So, the maximal noise dimension of such a system is equal to 2.

It should be noted that there is some analogy between the notion of the noise dimension and the concept of adiabatic elimination of the fast variables widely used in the analysis of nonlinear, dynamical systems [24]. Indeed, there is a hierarchy of times in the problem. The shortest time scale refers to the transition times within one noise state. Populations of the noise states are slow variables with respect to the population of levels within these noise states. Therefore, the reduction to the noise states relates to the adiabatic elimination of the fast variables of the system and the noise dimension corresponds to the remaining number of slow atomic variables (i.e., noise states) which "slave" the fast variables.

However, the noise dimension is a more general notion than the number of the slow variables. Indeed, as we will see later (see Sec. III), fluctuation properties of the systems with more than one lasing transition are really determined by the effective noise dimension which is not, necessarily, equal to the number of the remaining slow variables. This inequality is a consequence of the symmetry of such systems. An effective noise dimension may become, for example, a fractional quantity (as seen in Sec. III E).

The principle we put forward establishes a linkage between fluctuations and noise dimension of the system. It reads as follows: In a system with the noise dimension equal to n the minimal noise is reached if and only if all the noise states are equally populated. The larger the noise dimension of the system is, the larger is the possible noise reduction.

Thus, this principle requires equalizing the population

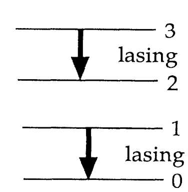

FIG. 4. Four-level lasing scheme with two coherent transitions.

of different noise states.

A numerical example depicted in Fig. 5 illustrates the above formal assertion. The system shown consists of a total of nine levels and lasing takes place among levels 1 and 2. It is assumed that levels 5, 4, and 3 have fast non-radiative transitions between themselves. The same is true for levels 6, 7, 8, and 9. Division into noise states is evident. The lasing levels 1 and 2 make noise state a. Two other noise states are made by the group with levels 3, 4 and 5 (group b) and the group with levels 6, 7, 8, and 9 (group c). The noise dimension of the system is equal to 3. So, according to the above principle, each group has to have the same population, which is equal to $\frac{1}{3}$. Fast transitions inside each group equalize the populations of levels belonging to the same group. In other words, inside each group one has a statistical equilibrium. Therefore, the distribution of populations of the levels is that represented by the last column in Fig. 5.

The formulated principle selects the most correlated state of the system in which one can find alternative symmetry of the noise. Like the coherent addition of noises from different dipoles at the first laser phase transition, the noises from different internal degrees of freedom are added coherently, establishing a different symmetry. Thus, this is cooperative behavior of the noise states via specially tailored incoherent pumping which gives rise to the suppression of spontaneous emission noise. We emphasize that the quantum correlations dealt with here are the correlations between different parts of one and the same system rather than correlations between different systems [25].

In this section we discuss the most simple examples of systems for which the minimization principle is valid.

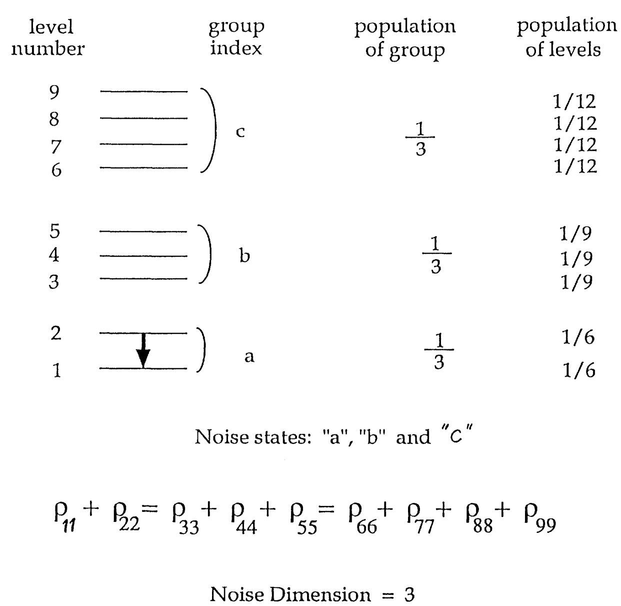

FIG. 5. This figure illustrates the minimization principle for an example of the nine-level system with lasing transition between levels 2 and 1.

Then we shall consider more sophisticated examples.

(a) A two-level system in the case of strong saturating field consists only of one noise state. The minimization principle is satisfied trivially.

(b) Consider three-level lasing (Fig. 1). The noise dimension of the system is equal to 2 (both lasing levels make one state). As has been shown, the noise is minimal when pumping rate γ_{21} satisfies condition (15) in which case relation (18) takes place and, thus, the minimization principle is satisfied. It may be noted that the global minimum of the Q parameter [see Eq. (14)] corresponds to the unidirectional process (no decay of upper lasing level, i.e., $\gamma_{12}=0$). In this particular case the minimization principle is also satisfied.

In the following sections we consider in detail more sophisticated examples of laser systems for which the minimization principle holds true.

C. Multilevel generation

In this section we discuss a multilevel generation and provide simultaneously an example which may be used for illustration of our common principle. Consider a multilevel laser system and assume, for simplicity, an initially unidirectional process of pumping (see Fig. 6). The general master equations (1-14)–(1-17) result in the following set of equations written down for fluctuating populations of dressed-atom levels ρ_i and the real dipole moment R

$$\frac{1}{\kappa}\frac{\partial}{\partial t}\rho_f=2\frac{\partial}{\partial\eta}\eta\left[1-\frac{N}{\kappa\gamma_\perp}R\right]\rho_f\ , \qquad (23)$$

$$\frac{1}{\gamma_\perp}\frac{\partial}{\partial t}R+R-S=\theta_R\ ,\quad S\equiv\rho_1-\rho_0\ , \qquad (24)$$

$$\frac{\partial}{\partial t}\rho_0+\gamma_{0n}\rho_0-\frac{\gamma_\parallel}{2}xR=\theta_0\ , \qquad (25)$$

$$\frac{\partial}{\partial t}\rho_1-\gamma_{21}\rho_2+\frac{\gamma_\parallel}{2}xR=\theta_1\ , \qquad (26)$$

$$\frac{\partial}{\partial t}\rho_i+\gamma_{i,i-1}\rho_i-\gamma_{i+1,i}\rho_{i+1}=\theta_i\ ,$$

$$i=2,3,\ldots,n-1\ , \qquad (27)$$

$$\frac{\partial}{\partial t}\rho_n+\gamma_{n,n-1}\rho_n-\gamma_{0n}\rho_0=\theta_n\ , \qquad (28)$$

with fluctuation θ terms of the form

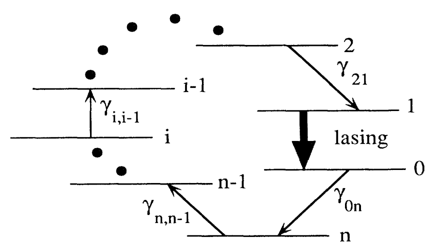

FIG. 6. Multilevel generation scheme. Maximal squeezing is reached under the condition $\rho_0+\rho_1=\rho_2=\cdots=\rho_n=1/n$.

$$\theta_R = \frac{\partial}{\partial\eta}\left[-\rho_1 + \frac{x}{2m}R^2\right]\ln\rho_f , \tag{29}$$

$$\theta_0 = \frac{\partial}{\partial\eta}\left[\frac{\gamma_\parallel x R}{2}(-1+\rho_0)\right]\ln\rho_f , \tag{30}$$

$$\theta_i = \frac{\partial}{\partial\eta}\left[\frac{\gamma_\parallel x R}{2}\rho_i\right]\ln\rho_f , \quad i=1,2,\ldots,n . \tag{31}$$

Successive adiabatic elimination of the atomic variables results in the following value of the mandel parameter:

$$Q = -\frac{1}{2} + \beta - \frac{\beta^2}{2}\sum_{i=2}^{n}\alpha_i(2-\alpha_i) + O(f) , \tag{32}$$

where

$$\alpha_i \equiv \frac{\gamma_{0n}}{\gamma_{i,i-1}} , \quad f \equiv (1+x)^{-1} ,$$

$$\beta = (2+\alpha_2+\cdots+\alpha_n)^{-1} , \quad \gamma_\parallel = 2\gamma_{0n}\beta .$$

The minimal value of Q is determined by the condition

$$\frac{\partial Q}{\partial \alpha_i} = 0 , \quad i=2,\ldots,n , \tag{33}$$

from which we readily find

$$\alpha_i = 2 , \quad i=2,\ldots,n , \tag{34}$$

or

$$\gamma_{21} = \gamma_{32} = \cdots = \gamma_{n,n-1} = \tfrac{1}{2}\gamma_{0n} . \tag{35}$$

Now we shall verify whether condition (35) is identical to that required by the minimization principle or not. The noise dimension is easily determined. Adopting for all the transition rates γ_{ij} to be of the same order of magnitude we obtain that each nonlasing level represents one separate noise state, and the two lasing levels make one noise state. Thus, the noise dimension is equal to n. Note that if one of the constants γ_{ij} becomes much larger than the others the noise dimension decreases by 1. This is explicitly seen in Eq. (32) in which the number of addendums is reduced by 1. The relevant populations in the strong-field limit ($x \gg 1$ so that $f \to 0$) are

$$\rho_{ii} = \alpha_i\beta , \quad i=2,\ldots,n , \tag{36}$$

$$\rho_{00} + \rho_{11} = 2\beta . \tag{37}$$

Upon equating the rhs of Eq. (36) to that of Eq. (37) we arrive exactly at Eq. (34), which completes the proof. Thus, we have found that our minimization principle minimizes the noise of the system. Under condition (35) we get for the Mandel parameter

$$Q = -\frac{n-1}{2n} , \tag{38}$$

with n being the noise dimension of the system. It yields 50% squeezing at the limit of $n \to \infty$. Equation (38) is in agreement with the result of Ritsch *et al.* [10,12] and Ralph and Savage [9], in which case Eq. (38) was obtained as a result of heuristic use of Eq. (35). Equation (35) in turn generalizes Eq. (19). Figure 7 displays the dependence of the Q parameter upon noise dimension n and the ratio of the rate constants α_i for the case of equal constants $\alpha_i = \alpha = \text{const}$, where $i=2,\ldots,n$. Formula (38) corresponds to curve 1 for which the minimization principle is satisfied. The figure illustrates the noise dimension idea as well. At $\alpha \gg 1$, i.e., at $\gamma_{0n} \gg \gamma_{n,n-1}$, the levels 0, 1, and n are statistically degenerated into one noise state and the noise dimension is lowered by 1.

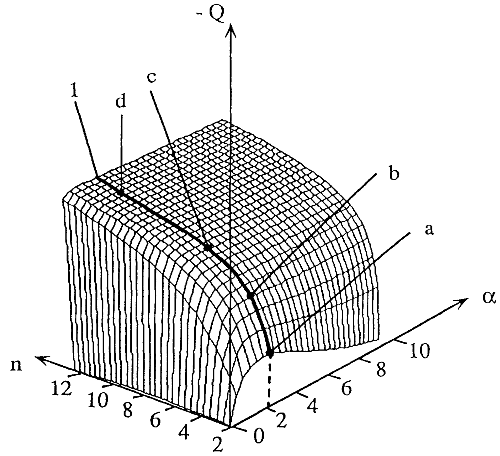

FIG. 7. The Q parameter as a function of the ratio of the pump rates and noise dimension (ND). Curve 1 corresponds to the maximum squeezing, when the requirements of the minimization principle are satisfied. The dots on this line represent the relevant squeezing levels: a corresponds to a ND of 2, $Q=-\frac{1}{4}$, and squeezing of 25%; similarly, b, c, and d have ND's of 3, 5, and 12, $Q=-\frac{1}{3}$, $-\frac{2}{5}$, and $-\frac{11}{24}$, and squeezing of 33%, 40%, and 46%, respectively. Note that a ND of 12 already results in a squeezing level very close to the ultimate value (when ND approaches infinity).

Consider an opposite case where $\alpha \ll 1/(n-1)$. In accordance with the noise dimension definition, the levels $0,1,2,\ldots,n$ make one noise state and the noise dimension is equal to 1. The noise becomes Poissonian. This result agrees well with the results of Refs. [9–14]. It is clear that the presence of the slow transition in the system drastically increases the noise. This evident consequences has a far-reaching experimental significance since it indicates the major cause of the fluctuation growth.

As argued above, the noise dimension of the semiconductor laser can be quite high, which may account for the success of noise reduction experiments in the semiconductor laser [17].

III. MANY-PHOTON AND MULTIPHOTON SYSTEMS

In this section we extend our theory to include the very important case of interaction. Hitherto we discussed the laser system with only one coherent transition. However, real examples of interaction in quantum optics have sometimes more than one coherent transition. These are, for instance, four-wave mixing (when coherent energy of the signal mode is transferred to coherent energy of the idler mode), lasing with many laser transitions (this takes place for vibrational molecules laser media), lasing in a semiconductor, etc. Recently, a two-photon lasing has been observed [26]. We shall show in this section that the two-photon process may be statistically identical to another process with two "one-photon" transitions. So the theory we present in this section is established to be relevant to this case as well.

Here we investigate a system with two and several lasing transitions and establish the validity of the minimization principle for this kind of the laser systems. In the case of more than one photon, namely, a few lasing transitions, the competition among these transitions on the cooperative behavior of all other internal noise states may hinder the noise reduction in the system. Thus the noise dimension which plays an important part in the squeezing phenomenon effectively decreases. The fractional noise dimension is introduced to account for this effect. This may result in a decrease in noise reduction capability.

Moreover, we shall see that systems with more than one lasing transition exhibit a remarkable feature. These can generate different photon statistics which continuously transform from super-Poissonian to sub-Poissonian just by varying the incoherent rate constants. This result seems to reveal a different statistical nature of two (multi)photon process. Here we restrict our consideration by interaction only with one and the same field mode. Nevertheless, our results are applicable to interaction with two and several modes.

A. Minimization principle for a system with more than one lasing transition

Consider the scheme which comprises five levels with two lasing transitions [Fig. 8(a)]. First of all, it should be noted that the presence of the two lasing transitions is expected to worsen noise properties, since lasing photons from transition 4→3 and from transition 1→2 are anticorrelated. Indeed, if radiation of a lasing photon occurs in one transition the radiation from the other transition becomes impossible simultaneously. In what follows, this elementary reasoning is confirmed by exact calculations. The one-particle interaction Hamiltonian from this scheme is

$$H_{\text{int}}=g(\sigma_{34}+\sigma_{01})b^{\dagger}+\text{H.c.} , \quad (39)$$

where $b^{\dagger}$ and b are cavity-mode operators, $\sigma_{ij}=|i\rangle\langle j|$ are atomic transitions operators, and g is the coupling constant. Let us determine the noise dimension of the system. Provided the field is strong, levels 3 and 4 make one noise state, likewise do levels 0 and 1. Provided all quantities γ_{21}, γ_{32}, and γ_{04} are of the same order of magnitude, the maximal noise dimensions of the system is equal to 3. According to the minimization principle the fluctuations are minimized under the condition

$$\rho_4+\rho_3=\rho_2=\rho_1+\rho_0=\tfrac{1}{3} , \quad (40)$$

where ρ_i are normalized populations, i.e., $\rho_0+\rho_1+\rho_2+\rho_3+\rho_4=1$. Now examine a validity of the minimization principle. The general master equation with interaction Hamiltonian (39) results in the following

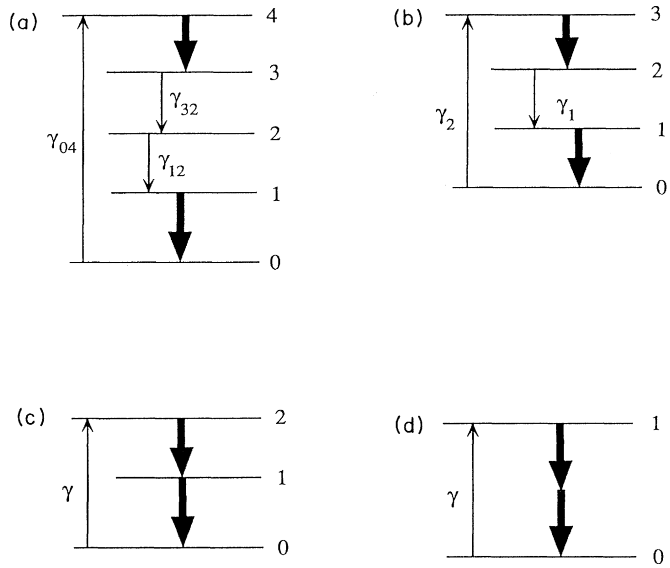

FIG. 8. This figure illustrates the continuous transition from the scheme with two "one-photon" lasing transitions (a) to one "two-photon" transition (d).

set of equations:

$$\frac{1}{\kappa}\frac{\partial}{\partial t}\rho_f=2\frac{\partial}{\partial\eta}\eta\left[1-\frac{N}{\kappa\gamma_\perp}(R_1+R_2)\right]\rho_f , \quad (41)$$

$$\frac{\partial}{\partial t}\rho_0+\gamma_{04}\rho_0-\frac{\gamma_\parallel}{2}xR_1=\theta_0 , \quad (42)$$

$$\frac{\partial}{\partial t}\rho_1-\gamma_{21}\rho_2+\frac{\gamma_\parallel}{2}xR_1=\theta_1 , \quad (43)$$

$$\frac{\partial}{\partial t}\rho_2+\gamma_{21}\rho_2-\gamma_{32}\rho_3=\theta_2 , \quad (44)$$

$$\frac{\partial}{\partial t}\rho_3+\gamma_{32}\rho_3-\frac{\gamma_\parallel}{2}xR_2=\theta_3 , \quad (45)$$

$$\frac{\partial}{\partial t}\rho_4-\gamma_{04}\rho_0+\frac{\gamma_\parallel}{2}xR_2=\theta_4 , \quad (46)$$

$$\frac{1}{\gamma_\perp}\frac{\partial}{\partial t}R_{1,2}+R_{1,2}-S_{1,2}=\theta_{R_{1,2}} , \quad (47)$$

$$S_1\equiv\rho_1-\rho_0 , \quad S_2\equiv\rho_4-\rho_3 , \quad (48)$$

with small fluctuations terms in the form

$$\theta_0=\frac{\partial}{\partial\eta}\left\{\frac{\gamma_\parallel x}{2}[(-1+\rho_0)R_1+R_2\rho_0]\right\}\ln\rho_f , \quad (49)$$

$$\theta_1=\frac{\partial}{\partial\eta}\left\{\frac{\gamma_\parallel x}{2}(R_1+R_2)\rho_1\right\}\ln\rho_f , \quad (50)$$

$$\theta_2=\frac{\partial}{\partial\eta}\left\{\frac{\gamma_\parallel x}{2}(R_1+R_2)\rho_2\right\}\ln\rho_f , \quad (51)$$

$$\theta_3=\frac{\partial}{\partial\eta}\left\{\frac{\gamma_\parallel}{2}[(-1+\rho_3)R_2+R_1\rho_3]\right\}\ln\rho_f , \quad (52)$$

$$\theta_4=\frac{\partial}{\partial\eta}\left\{\frac{\gamma_{\parallel}x}{2}(R_1+R_2)\rho_4\right\}\ln\rho_f\ , \tag{53}$$

$$\theta_{R_{1,2}}=-\frac{\partial}{\partial\eta}\left\{\rho_{1,4}-\frac{x}{2m}R_{1,2}^2\right\}\ln\rho_f\ . \tag{54}$$

The notations are standard ones. Real dipole moments of atoms are introduced as follows:

$$(r_1)_{01}=\frac{izR_1}{\gamma_\perp}\ ,\quad (r_1)_{34}=\frac{izR_2}{\gamma_\perp}\ . \tag{55}$$

The straightforward usage of the standard perturbation theory [1] yields the classical (without fluctuations) solution at the strong-field limit:

$$\rho_0^{(0)}=\rho_1^{(0)}=\frac{\alpha_2}{d}\ ,\quad \rho_3^{(0)}=\rho_4^{(0)}=\frac{1}{d}\ ,\quad \rho_2^{(0)}=\frac{\alpha_1}{d}\ , \tag{56}$$

$$\alpha_1\equiv\frac{\gamma_{32}}{\gamma_{21}}\ ,\quad \alpha_2\equiv\frac{\gamma_{32}}{\gamma_{04}}\ ,\quad d\equiv 2+\alpha_1+2\alpha_2\ . \tag{57}$$

Evaluation of the Mandel parameter yields the expression

$$Q=\frac{1}{2}-\frac{4(\alpha_1+2\alpha_2+\alpha_1\alpha_2)}{d^2}\ . \tag{58}$$

The minimum $Q=-\frac{1}{6}$ is reached at $\alpha_1=2$, $\alpha_2=1$, which results in the following values for the level populations [see formulas (56)]:

$$\rho_0^{(0)}=\rho_1^{(0)}=\tfrac{1}{6}\ ,\quad \rho_3^{(0)}=\rho_4^{(0)}=\tfrac{1}{6}\ ,\quad \rho_2^{(0)}=\tfrac{1}{3}\ . \tag{59}$$

We see that condition (40) is satisfied, which completes the proof of the minimization principle validity for this particular case of the laser system.

Then we skip details and present just the result of calculation of fluctuations for the case of lasing as it is shown in Fig. 9:

$$Q=\frac{1+\alpha_2}{d}-\frac{(2\alpha_2+\alpha_3)(2+\alpha_1+\alpha_3)}{d^2}-\frac{2(2+\alpha_1)(\alpha_1+\alpha_2)}{d^2}+\frac{2(\alpha_1^2+\alpha_3^2)}{d^2}+\frac{\alpha_1-\alpha_3}{2d}\ , \tag{60}$$

with

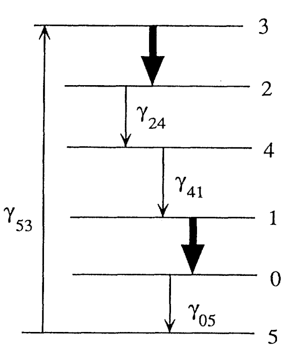

FIG. 9. Six-level scheme with two lasing transitions.

$$\alpha_1\equiv\frac{\gamma_{24}}{\gamma_{41}}\ ,\quad \alpha_2\equiv\frac{\gamma_{24}}{\gamma_{05}}\ ,\quad \alpha_3\equiv\frac{\gamma_{24}}{\gamma_{53}}\ ,\quad d\equiv 2+\alpha_1+2\alpha_2+\alpha_3\ . \tag{61}$$

Result (60) will be useful for discussion of the general structure of noise in laser systems which follows this section. One can readily check that the minimal noise case, which occurs at $\alpha_1=\alpha_3=2$, $\alpha_2=1$, exactly corresponds to the conditions of the minimization principle, namely, noise states (the noise dimension is equal to 4 in this case) occur to be equally populated, i.e.,

$$\rho_0+\rho_1=\rho_2+\rho_3=\rho_4=\rho_5\ , \tag{62}$$

B. Continuous transition from one-photon to two-photon statistics in a laser

Now we shall follow the photon statistics transformation while parameters α_1,α_2 in Eq. (58) are changed.

(i) $\alpha_1\ll 1$. In this case the levels 0, 1, and 2 make one noise state and the noise dimension is equal to 2. The minimum $Q=0$ is reached at $\alpha_2=1$. Thus, we have arrived at Poissonian photon statistics. The last limit is identified with the scheme displayed in Fig. 8(b) at $\gamma_1=\gamma_2$. For the scheme with one lasing transition and at the same noise dimension equal to 2, it follows from Eq. (38) that $Q=-\frac{1}{4}$.

(ii) $\alpha_1\sim\alpha_2\ll 1$. This limiting case corresponds to Fig. 8(c). In this case, all levels make the single noise state, so the noise dimension is equal to 1. The quantity Q is equal to $\frac{1}{2}$, which remarkably coincides with the two-photon laser statistics obtained by Cheng and Haken [27]. So that with respect to photon statistics, the cascade scheme with one-photon resonance [Fig. 8(c)] is identical to the scheme with two-photon resonance [Fig. 8(d)] discussed in Ref. [27], in which case the interaction Hamiltonian was supposed to be a quadratic one with respect to the field operators, i.e.,

$$H_{\rm int}=[g\sigma(b^\dagger)^2+{\rm H.c.}]\ . \tag{63}$$

The above examples exhibit the relation between the one-photon process statistics and the two-photon process statistics, which are just the different limits of one and the same general case. Once again we arrive at the conclusion that the noise properties of the two-lasing transition scheme are worse than those of the one-lasing transition scheme for which, at the same noise dimension equal to 1, Eq. (38) results in $Q=0$.

(iii) Now consider the continuous transition from case (i) [Fig. 8(b)] to case (ii) [Figs. 8(c) or 8(d)]. Such a transition one can follow while varying the parameter $\epsilon=\gamma_1/\gamma_2$ from 0 to infinity (see Fig. 10). State F_0 with $\gamma_1=\gamma_2$ has the noise dimension equal to 2 and satisfies the minimization principle requirements, i.e., both noise states are equally populated. Moving along the curve $F_0F_1(F_0F_2)$ corresponds to disturbing the symmetry characterizing the state F_0 with the noise dimension equal to 2. The limit of such a breakdown of symmetry is the final state $F_1(F_2)$. These states are once again

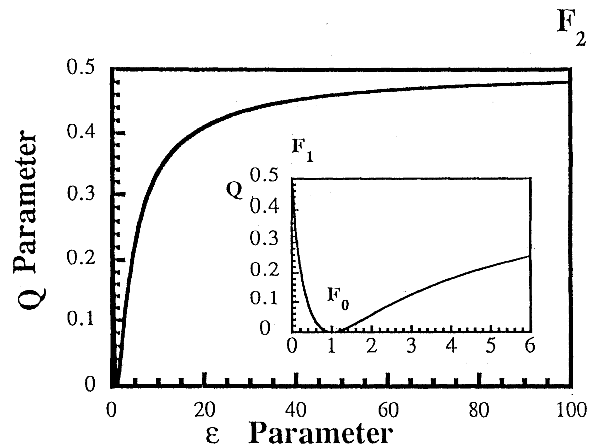

FIG. 10. Q parameter for the scheme is represented by Fig. 8(b) as a function of $\epsilon=\gamma_1/\gamma_2$. States F_0, F_1, and F_2 satisfy the minimization principle requirements. The crucial difference between the state F_0 and the states F_1, F_2 is the difference in their ND. In state F_0 the ND is 2, while in states F_1 and F_2 the ND is 1. In intermediate regions between state F_0 and F_1, F_2 the minimization principle requirements are not satisfied.

characterized by the symmetry required by the minimization principle but with the noise dimension lowered by 1. Thus the transition $F_0F_1(F_0F_2)$ from the one-photon process to the two-photon process is accompanied by lowering of the noise dimension and consequently by worsening of noise properties. The worsening of the noise property of the two-photon process can also be readily understood from the simple fact that in the latter case the photons are created by pairs which hinders the stabilization of the field intensity. Thus, a hierarchy of different photon statistics is established. The part of the control parameter at this photon statistics transformation is played by the noise dimension of the lasing scheme. Our consideration reveals a different statistical nature of the two-photon process. It appears to be a result of a breakdown of the symmetry characterizing a system with the noise dimension equal to 2 and a transformation to a different symmetry characterizing a system with the noise dimension equal to 1. Thus, it is shown that the squeezing capacity of the systems with two lasing transitions is worse than that of the schemes with one lasing transition, provided the noise dimension is the same.

C. How can one interpret the sub-Poissonian statistics in the two-level case?

It follows from the standard Scully-Lamb-Haken theory [20] as well as from our consideration that two-level atoms generate a Poissonian light well above threshold (the dressed-atom effects are insignificant in this case). Nevertheless, it has been known for quite a long time [19,28,29] that some small squeezing may be achieved in principle at intermediate field when the Rabi frequency is of the same order as the atomic decay rate. Interpretation of this peculiarity has been done by Kazantsev and Surdutovich [19]. It is easily done if we recall that in the case of unsaturating field the spontaneous emission probability becomes minimal under some intermediate field. The phenomenon may be understood in a framework of the minimization principle as well. Indeed, in the case of unsaturating field the lasing levels no longer make a single noise state. As a matter of fact, their noise state splits into two and a noise dimension increases up to 2. However, this growth of squeezing capacity is hindered by the fact that the minimization principle cannot be satisfied (the populations cannot be equalized before a saturation is achieved) for a two-level system. This is the reason that the squeezing is achieved but at a very modest level (about 5%).

D. Minimization principle and a reduction of description

In this section we shall see how the minimization principle allows one to reduce a description of a complicated laser system. This reduction is based upon an alternative symmetry that one can find in a laser system which satisfies the conditions required by the principle. We start with one preliminary remark. What sort of reduction in the description do we mean? Let us recall a coherent interaction of the two-level atoms in conventional laser. In this case, we had a reduction of description since the collective properties resulted in that we dealt with the density matrix of the single particle (dressed atom) instead of multiparticle correlation forms. In other words, we reduced our description up to the "single-particle" one. Physically this reduction is invoked by the strong laser coherence far above threshold. So a different symmetry in a laser system is accompanied with the reduction of description. To some extent, the coherent interaction of different noise states looks like the discussed interaction of the different two-level atoms. When the minimization principle is satisfied the system may be characterized by some different symmetry and a reduction in description is possible as well. Physically it means that some noise state may become indistinguishable and one operates with only one noise state instead of many. We demonstrate this reduction for a particular example and then generalize the results.

Consider again the laser scheme depicted in Fig. 8(b). We have already established that the minimization principle requires that the constants γ_1 and γ_2 be equal (in order to equally populate the noise states). In this case the two lasing transitions (noise states) becomes indistinguishable and we can operate just with the quantities of only one transition. This symmetrization results in the following relations:

$$R_1=R_2 \quad \text{(for dipole moments)}\,, \tag{64}$$

$$\rho_0=\rho_2\,, \quad \rho_1=\rho_3 \quad \text{(for populations)}\,. \tag{65}$$

Equations (64) and (65) reduce the description of the system from six quantities (ρ_0, ρ_1, ρ_2, ρ_3, R_1, and R_2) down to three. We skip the straightforward calculations which repeat mostly the derivation of Eq. (58) or (60). The amplitude fluctuation result perfectly agrees with that obtained from Eq. (58), under conditions $\alpha_1=0$ and $\alpha_2=1$, or that obtained from Eq. (60), under conditions $\alpha_1=0$, $\alpha_2=1$, and $\alpha_3=0$. It reads as

$$Q=0\ . \tag{66}$$

This result indicates that this scheme is statistically identical to the conventional two-level (one-photon) result where the noise is known to be a Poissonian one as well [19,20].

Now consider again the scheme represented by Fig. 9. Without any quantum-mechanical consideration (using the classical rate equations only) one can easily check that the symmetry principle will be satisfied if the following conditions are imposed:

$$\gamma_{24}=2\gamma_{41}\ ,\quad \gamma_{05}=2\gamma_{53}\ . \tag{67}$$

In this case the lasing transitions $(1\rightarrow 0)$ and $(3\rightarrow 2)$ become indistinguishable as well as the nonactive levels 4 and 5. Again we find a reduction in description. It is interesting to follow what happens with the noise states. The noise dimension of the system is still equal to 4 since each lasing transition makes one noise state and so does each nonactive level. However, because of the indistinguishability of some noise states this system has noise properties identical to those of the conventional (one-photon) three-level system with one nonactive level (Fig. 1). Calculations with expression (60) affirm this result. Indeed, upon taking $\alpha_1=2$, $\alpha_2=1$, and $\alpha_3=2$, which exactly corresponds to conditions (67), we arrive at the result

$$Q=-\tfrac{1}{4}\ . \tag{68}$$

Upon comparing Eq. (68) with Eq. (14) (under the condition $\gamma_{12}\ll\gamma_{02}$) we come to the conclusion that the noise properties of this scheme are identical to those of the three-level one.

E. Effective noise dimension

The above reasoning prompts us to introduce some alternative quantity in order to correctly assess the squeezing capacity of the system. This can be done by the following simple definition:

$$n_{\rm eff}=\frac{n}{n_{\rm ph}}\ , \tag{69}$$

where n is the noise dimension and $n_{\rm ph}$ is the number of coherent photons in the system (number of lasing transitions for the situation in question). The quantity $n_{\rm eff}$ may be called the effective noise dimension. Indeed, exact calculations resulted in Eq. (38) can be generalized for the n-level scheme with $n_{\rm ph}$ lasing photons. The result of such a generalization reads as

$$Q=-\frac{n-n_{\rm ph}}{2n}\ . \tag{70}$$

It follows from Eq. (70) and definition (69) that upon replacing the noise dimension n by the effective noise dimension $n_{\rm eff}$ one can continue to use the expression (38) to assess the squeezing capacity of the system with any number of coherent photons, so that the result reads as

$$Q=-\frac{n_{\rm eff}-1}{2n_{\rm eff}}\ . \tag{71}$$

Let us analyze the effective noise dimension and squeezing capacity for the generation scheme of Fig. 8(a). According to definition (69) the effective noise dimension is as follows:

$$n_{\rm eff}=\tfrac{3}{2}\ . \tag{72}$$

Upon inserting Eq. (72) into Eq. (71) we arrive at the simple result

$$Q=-\tfrac{1}{6}\ , \tag{73}$$

which remarkably coincides with that obtained by the quantum-mechanical consideration above [see text under Eq. (58)].

F. Multiphoton processes

Another remarkable example of the system, the effective noise dimension and the squeezing capacity of which may be easily analyzed, is a multiphoton process. Let us consider for simplicity the two-photon process represented by Fig. 8(c) or Fig. 8(d) (in Sec. III B we have established that these two schemes of lasing are identical from the statistical point of view). In this case there is only one noise state (the lasing transition itself) and the noise dimension of the system is equal to 1. There are two coherent photons (lasing photons) in the system, so that $n_{\rm ph}=2$, therefore for the effective noise dimension we obtain a simple result,

$$n_{\rm eff}=\tfrac{1}{2}\ . \tag{74}$$

Then the general formula (71) yields the following result for the Mandel parameter:

$$Q=\tfrac{1}{2}\ . \tag{75}$$

Again this result coincides with that obtained by a quantum-mechanical calculation (see [27] and Sec. III B) and means 50% super-Poissonian noise. Generalization of the result (74) on three and more photon processes is straightforward. This fractional effective noise dimension agrees well with the elementary reasoning about anticorrelation of different coherent photons (see Sec. III A).

It should be emphasized that the quantity $n_{\rm eff}$ accounts for the total number of coherent lasing photons in the system. It includes the cases of many one-photon lasing transitions [e.g., Figs. 8(a) and 8(b)], multiphoton lasing transition [Fig. 8(d)] or both (Fig. 11). Thus, in order to evaluate the squeezing capacity of the system one has to deal, in general, with the effective noise dimension. The last example (Fig. 11) demonstrates the difference between the effective noise dimension which is equal to 0.8, and the number of the slow atomic variables left after adiabatic elimination of the fast variables, which is equal to 4. Squeezing capacity in the present case is determined only by the effective noise dimension.

G. Minimization principle and quantum correlations

Recently Barnett and Phoenix [25] investigated a quantum correlations issue by using quantum-information

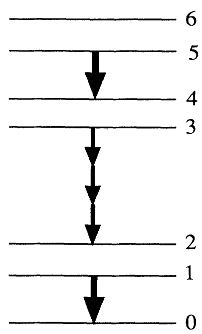

FIG. 11. Example of the combined scheme. There are four noise states (or slow atomic variables) and five coherent photons in the system. Hence, the effective noise dimension $n_{\text{eff}}=0.8<1$. Thus such a scheme cannot produce squeezed light.

theory. They concluded that a squeezed state corresponds to a most-correlated state of the coupled system and that quantum mechanics allows systems to be twice as strongly correlated as classical mechanics. In light of this discussion, the laser noise can probably also be decomposed into classical and purely quantum ones. Along these lines the Poissonian distribution is a result of "classical" noise correlation and can be obtained as a limit in a system of two-level radiating dipoles. In fact, the Poissonian noise of the laser indicates that the total system makes noise as a single two-level atom (see Fig. 2). In this case all "classical" sources of noise (such as a thermal distribution of atomic parameters, etc.) are completely suppressed. The noise of N atoms in a cavity with quality factor q is identical to that of a single atom, but in a cavity with quality qN (see Fig. 2). This result is easily understood since in this case there is only one noise state and the problem of quantum correlations between different noise states does not arise. On the other hand, the sub-Poissonian noise features a purely quantum correlations property [16]. Sub-Poissonian photon statistics can exist only in a system with an effective noise dimension larger than 1. This, in turn, can happen only in a quantum system whose structure is more complicated than that of a two-level atomic dipole. The most squeezed light corresponds to the most quantum-mechanically correlated state of system. This is exactly the state required by the minimization principle. Such a manifestation of quantum correlations differs from the conventional one (when this is the correlation between different systems [25] rather than different levels of the same system), namely, the noises of different levels can coherently interact and the minimization principle selects the most correlated state.

IV. SUMMARY

The dressed-atom approach formulated in paper I has been applied to analyzing the intensity fluctuations in conventionally pumped lasers. A minimization principle that governs the noise reduction in such lasers has been formulated and described comprehensively. A definition of noise states was introduced. It was shown that the squeezing capacity of conventionally pumped lasers is determined by the so-called noise dimension, which is equal to the number of the noise states in the case of one-photon resonance in a single lasing transition. The larger the noise dimension the larger is the squeezing capacity of the system. The validity of the minimization principle has been proven for the various lasing schemes. According to the principle, the squeezing capacity is maximal when all the noise states are equally populated. The minimization principle is generalized for the cases of multiphotons, many lasing transitions, or both. For these cases an effective noise dimension is defined which is equal to the number of noise states divided by the number of the lasing coherent photons.

This minimization principle leads to a reduction of the description of the system. Indeed, in order to evaluate the squeezing capacity of the system one no longer needs a complete quantum-mechanical calculation to treat the fluctuation problem. It is quite enough to assess the number of noise states of the system just from the semiclassical rate equations, which makes the minimization principle helpful in the planning of experiments on squeezing.

ACKNOWLEDGMENTS

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