# Laser-noise suppression in the dressed-atom approach. I. Fluctuations in a regularly pumped laser

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The dressed-atom approach to the fluctuation problem in lasers is described. Such an approach allows one to analyze the fluctuations in various kinds of laser models. The intensity fluctuations in a regularly pumped laser are considered in the framework of the dressed-atom approach. The Fokker-Plank equation for the field density matrix is derived from first principles without heuristic assumptions. An alternative interpretation of the noise suppression in the regularly pumped laser is given. It is based upon the nonstationary picture for atomic variables. In such a regime the statistics of the output laser field may be controlled by the inuput statistics of the atomic initial conditions. It is the regular pump that provides the best input statistics. In the latter case the atoms are essentially nonstationary and coherently oscillate at the Rabi frequency. They are treated in a framework of the time-dependent picture in spite of the short atomic lifetime as compared with the cavity mode lifetime. The nonstationary version of the dressed-atom approach is employed to treat the regular pump case. It is demonstrated that a coherent coupling of the Rabi oscillations in the quadratic dressed-atom terms gives rise to a survival of a linear inverse proportional dependence of the atomic correlation functions upon field intensity. This survival in turn accounts for the anomalously large dressed-atom contribution to the fluctuations. Our results are in quantitative agreement with those of other approaches.

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## I. INTRODUCTION

From the conceptional point of view it seems that there are two ways to generate squeezed light. The first one is an introduction of regularity via pumping with suppressed fluctuations [1–6]. The second way is based upon the intrinsic property of systems with internal degrees of freedom to generate squeezed light [7–13]. In the case of the regular pumping, a deterministic pump results in a noiseless laser field, whereas in the second case, the so-called "conventional pumping," the noise reduction is an intrinsic property of the laser system itself. In the latter case the squeezed light is obtained from incoherent energy, whereas in the regular pumping case the sub-Poissonian property is transferred from one energy source to another.

Noise quenching in a regularly pumped laser is explained on the basis of the standard Scully-Lamb-Haken theory [14,15] in which the stochastic treatment of the pump has been incorporated [2–6]. A phenomenological derivation of the field density matrix master equation is employed. It is also the case with the quantum Langevin approach [4].

The noise suppression in a conventionally pumped laser was predicted by Kazantsev and Surdutovich [16] on the basis of a purely Hamiltonian approach. This result was confirmed by Pokrovsky and Khazanov [17], Lugiato, Casagrande, and Pizutto [18], and Haake and Lewenstein [19]. By using the dressed-atom treatment of the total $N$ atoms + field system it has been demonstrated in Ref. [7] that the noise properties of a three-level system are intrinsically better than those of a two-level one. It has been pointed out, as well, that a unidirectional pumping process minimizes the laser fluctuations. The results were generalized to include multilevel systems by Ralph and Savage [8–10]. They suggested a simple statistical model that relates fluctuations of the photon number with fluctuations of the mean pump recycling time. Hart and Kennedy [11] considered a three-level model with pump depletion. Ritsch *et al.* [12] have been able to identify the mechanism of noise quenching for the unidirectional pumping process. They have established that the multiple recycling the active electrons results in a deterministic pumping process and thus suppresses the stochasticity in the laser system. Ralph and Savage [10] found a simple statistical model for interpretating the multiple recycling in terms of stochastic recycling times and investigated some conditions for experimental realization of squeezed lasers. In Ref. [13] a minimization principle was reported that allows one to evaluate squeezing capacity for all kinds of laser models.

In this paper we analyze the relation between the two different methods of laser noise reduction (regular and conventional pumping). We present a unified approach to both methods of noise reduction which is based upon the dressed-atom treatment [7,16,20,21]. We proceed with the Hamiltonian formulation of the total atoms and field system. As will be demonstrated, the dressed-atom fluctuations in the regularly pumped laser differ from those in the case of the conventionally pumped laser. This difference stems from the oscillatory character (Rabi oscillations) of the atomic quantities evolution and gives rise to a "survival" of a linear inverse proportional dependence of the atomic correlation functions upon field after time averaging. This survival occurs due to the coherent coupling of the Rabi oscillations of different

 

atomic variables, dipole moment and inversion, the oscillation phases of which are shifted by a fixed quantity $\pi/2$. It can be readily seen in the dressed-atom terms which ones are quadratic with respect to the atomic variables. Our results are in full agreement with those of other approaches [1–6]. However, the present consideration has some merits as it (1) indicates a conceptual heredity of the two methods of the noise suppression which is established to be just the different (stationary and unstationary) regimes of the same atomic system, and (2) provides a formal Hamiltonian background for the regular pumping concept.

The setup of the paper is as follows. In Sec. II we describe the dressed-atom treatment of the fluctuations problem. In Sec. III the general dressed-atom concept is employed to treat the regularly pumped laser. The conventional (stationary) multilevel pump will be discussed in the following [27]. In that part we discuss the intrinsic noise quenching property of the systems with internal degrees of freedom. A minimization principle that governs the noise reduction will be formulated. It will be shown that the minimization principle holds true for various laser schemes including many-photon and multiphoton systems.

## II. DRESSED-ATOM APPROACH

### A. Hamiltonian formulation of the fluctuation problem

In this section we present the generalized Hamiltonian formulation of the fluctuations problem. This formulation is applicable to both the conventionally pumped and regularly pumped laser. These two different cases of the pump may be specified just by the proper choice of the atomic relaxation operator. The term "atom" is used here for any possible type of the quantum dipole (electron-hole pair, three-level atom, etc.). The total atoms and field system density matrix $\rho(t)$ obeys the generalized Liouville equation [15–21]

$$\left[\frac{\partial}{\partial t}+\Lambda_A+\Lambda_f+iL_{Af}\right]\rho(t)=0 , \tag{1}$$

where

$$\Lambda_f\Psi=-\kappa([b\Psi,b^\dagger]+[b,\Psi b^\dagger]) , \tag{2}$$

$$\Lambda_A\equiv\sum_a\Lambda_a , \quad L_{Af}\equiv\sum_a L_{af} , \tag{3}$$

$$L_{af}\psi\equiv[(\sigma_a^+ b+\text{H.c.}),\Psi] , \quad \sigma_a^+\equiv|1\rangle\langle 0|_a . \tag{4}$$

The operators $\Lambda_f$ and $\Lambda_a$ describe the linear relaxation of the field and the single atom, respectively; $b$ ($b^\dagger$) is the cavity-mode annihilation (creation) operator, $\sigma_a$ ($\sigma^+$) is the lowering (raising) atomic operator, $\kappa$ is cavity damping constant, and $\Psi$ is an arbitrary operator. Ring laser conditions of the running wave are assumed. The coupling constant in the interaction operator $L_{af}$ is omitted for simplicity. As we noted before, each specific pump may be defined by the proper choice of the atomic system relaxation operator $\Lambda_a$. For instance, in the case of the conventional $n$-level pump it reads as

$$\Lambda_a\Psi=-\sum_{i,j=0}^{n}\gamma_{ij}([\sigma_{ji}\Psi,\sigma_{ij}]+[\sigma_{ji},\sigma_{ij}\Psi]) , \tag{5}$$

$$\sigma_{ji}\equiv|j\rangle\langle i| . \tag{6}$$

Here $\gamma_{ij}$ are the generalized rates of incoherent radiationless transitions between the atomic states. The relaxation in the form (5) does not include any so-called phase-destruction processes. Thus, the relaxation in form (5) corresponds to the minimum value of the dipole moment decay constant. The phase-destruction process may be easily included if necessary.

### B. Dressed atoms as a decoupling procedure

It follows from Eqs. (1)–(5) that the fluctuation problem is a many-particle one and, hence, the atom correlation problem arises formally. Indeed, making a trace of Eq. (1) over atomic variables, one would arrive at a hierarchy of equations for the reduced density matrices $\rho_f$, $\rho_1$, $\rho_{1f}$, $\rho_{12f}$, etc., defined as follows. The field density matrix is

$$\rho_f\equiv\text{Tr}_{\bar{f}}\rho(t) ; \tag{7}$$

the one-atom density matrix is

$$\rho_1\equiv\text{Tr}_{\bar{1}}\rho(t) ; \tag{8}$$

the one-atom–field density matrix is

$$\rho_{1f}\equiv\text{Tr}_{\overline{1f}}\rho(t) ; \tag{9}$$

the two-atom–field density matrix is

$$\rho_{12f}\equiv\text{Tr}_{\overline{12f}}\rho(t) ; \tag{10}$$

and so on. The bar above the quantities $1,2,f$ means that the trace is taken over all variables except those marked with a bar. To obtain a closed set of equations it is necessary to use some decoupling procedure. However, it is known [16–21] that the usage of "pure" $n$-atom–field correlations, which arise from the definitions (1)–(10), does not allow one to limit the calculations to a finite number $n$, since all the $n$ correlations have the same order of magnitude. These difficulties may be overcome by appealing to the dressed-atom concept, because correlations between dressed atoms are actually small and it makes them suitable for a correct perturbation theory. To do this, the following correlation forms are to be introduced [16]:

$$\text{Tr}_{1,2,\ldots,N}\rho(t)\equiv\rho_f(t,z) , \tag{11}$$

$$\text{Tr}_{2,3,\ldots,N}\rho(t)\equiv\rho_f(t,z)r_1(t,z) , \tag{12}$$

$$\text{Tr}_{3,4,\ldots,N}\rho(t)\equiv\rho_f(t,z)r_{12}(t,z) , \tag{13}$$

and so on, $z$ is the field argument of the Glauber $P$ representation (from now on, all field operators are supposed to be transformed into the coherent states representation [23]). The field-dependent quantities $r_1(t,z)$, $r_{12}(t,z)$ are one- and two-atom density matrices describing the dressed atoms. They have nothing to do with the undressed-atom density matrices $\rho_1$ and $\rho_{12}$ which do not depend upon the field variable $z$ altogether. The rigorous

perturbation theory [16] establishes that the dressed-atom–atom correlation form $\delta r_{12} \equiv r_{12} - r_1 r_2$ is very small and can be neglected, *in the adiabatic limit,* in the calculations of the quantum fluctuations. Using the decoupling condition $\delta r_{12} = 0$, one can obtain from (1) the "one-particle" equation,

$$\frac{1}{i}\frac{\partial r_a}{\partial t} + \frac{1}{2}\Lambda_a r_a + [h_a, r_a] = \tilde{\sigma}_a r_a \frac{\partial}{\partial z}\ln\rho_f - \text{H.c.} , \tag{14}$$

where

$$\tilde{\sigma}_a \equiv \sigma_a - \text{Tr}_a(\sigma_a r_a) , \quad \sigma_a^+ \equiv |1\rangle\langle 0|_a , \tag{15}$$

with the equation for the field density matrix $\rho_f$ in the form

$$\frac{\partial \rho_f}{\partial t} = \left\{ \frac{\partial}{\partial z}\left[ \kappa z + \text{Tr}_a \sum_a (\sigma_a r_a) \right] + \text{H.c.} \right\} \rho_f , \tag{16}$$

with $h_a$ being the "classical" part of the interaction Hamiltonian, which does not contain quantum derivatives $\partial/\partial z$ and $\partial/\partial z^*$, i.e.,

$$h_a \equiv h_a(z) = z^* \sigma_a + z \sigma_a^+ . \tag{17}$$

Equation (16) is the basic closed master equation. It differs from that of the standard Scully-Lamb-Haken theory [14,15] due to the difference between $\sigma$ and $\tilde{\sigma}$. Equation (16) takes into account all orders of pure-atom–atom correlations. On the other hand, in terms of the dressed atoms, Eq. (14) is a one-particle equation. Indeed, it does not contain any quantity but those related to the same atom $a$. The left-hand side of Eq. (14) defines classical (i.e., without quantum fluctuations) evolution of the dressed-atom density matrix $r_a$ while its right-hand side (rhs) has a pure quantum origin. It contains only terms with derivatives $\partial/\partial z$ and $\partial/\partial z^*$.

### C. Dressed-atom Hamiltonian

Until now we treated the dressed-atom approach as some decoupling procedure for the total atoms and field density matrix. In this section we present another look at the dressed atoms. Here we show that there exists a dressed-atom Hamiltonian and it may be used as a zero Hamiltonian to build up a correct perturbation theory. Consider the total interaction Hamiltonian

$$H = \sum_a (\sigma_a^+ b + \sigma_a b^\dagger) \tag{18}$$

and select the quantum fluctuations [20–22] $b - \langle b \rangle$, $b^\dagger - \langle b^\dagger \rangle$, $\sigma_a - \langle \sigma_a \rangle$, and $\sigma_a^+ - \langle \sigma_a^+ \rangle$. It results in the following identical form of the Hamiltonian:

$$H = H_0 + \Delta H + C , \tag{19}$$

with

$$H_0 = \sum_a (\langle \sigma_a^+ \rangle b + \langle b \rangle \sigma_a^+) + \text{H.c.} , \tag{20}$$

$$\Delta H = \sum_a [(\sigma_a^+ - \langle \sigma_a^+ \rangle)(b - \langle b \rangle)] + \text{H.c.} , \tag{21}$$

$$C = \sum_a \langle \sigma_a^+ \rangle \langle b \rangle + \text{H.c.} \tag{22}$$

Apart from insignificant $c$-number quantity, $C$, the formulas (19)–(21) define the zero Hamiltonian $H_0$ and the fluctuations Hamiltonian $\Delta H$. The Hamiltonian $H_0$ represents the dressed atoms. The dressed-atom Hamiltonian may be decomposed in two parts [20–22]:

$$H_0 = H_0^{(A)} + H_0^{(f)} , \tag{23}$$

with

$$H_0^{(A)} \equiv \langle b \rangle \sigma_a^+ + \text{H.c.} \tag{24}$$

being the atomic component and

$$H_0^{(f)} \equiv \sum_a \langle \sigma_a^+ \rangle b + \text{H.c.} \tag{25}$$

being the field component. The component $H_0^{(A)}$ describes the atoms situated in external classical field. The Hamiltonian $H_0^{(A)}$ is an operator quantity only with respect to atomic variables and does not contain the field operators. On the other hand, the Hamiltonian $H_0^{(f)}$ is purely a field quantity. It describes the field with the given "mean polarization" $\langle \sigma_a \rangle$ [23] and depends upon field variables only. The energy levels of the dressed-atom Hamiltonian can be easily found because the components $H_0^{(f)}$ and $H_0^{(A)}$ commute as

$$[H_0^{(f)}, H_0^{(A)}] = 0 . \tag{26}$$

It follows from (26) that energy levels of the compound system $H_0$ are a simple sum of the energy levels of the components $H_0^{(f)}$ and $H_0^{(A)}$. Energy levels of both components are well known. The component $H_0^{(A)}$ represents the dynamical Stark atom [24] and the component $H_0^{(f)}$ represents the shifted driven oscillator [23,24].

The regular perturbation theory can be built up formally by treating the term $\Delta H$ as a formal small parameter. This formal treatment resulted in the following master equation [20,21]:

$$\left[ \frac{\partial}{\partial t} + \Lambda_A + \Lambda_f + iL_0 \right] \rho(t) = -\int_{-\infty}^{t} dt' L_1(t) T(t', t) L_1(t') \rho(t') , \tag{27}$$

where

$$L_0 \equiv [H_0, \ldots] , \quad L_1 \equiv [\Delta H, \ldots] , \tag{28}$$

and

$$T(t, t') \equiv \overrightarrow{\exp}\left\{ -i \int_{t'}^{t} L_0(\tau) d\tau \right\} , \tag{29}$$

where $\overrightarrow{\exp}$ is an ordered exponent.

Upon adopting the necessary approximations in rhs of Eq. (27), i.e.,

$$\rho(t) \approx \rho_f(t) \rho_A^{(0)} , \tag{30}$$

$$T(t', t) \approx U_f(t - t') U_A(t - t') ,$$

$$U_A(t) \equiv \exp\{ -(\Lambda_a + iL_0^{(A)})t \} , \tag{31}$$

$$U_f(t) \equiv \exp\{ -(\Lambda_f + iL_0^{(f)})t \} ,$$

with $\rho_A^{(0)}$ being the solution of the zero-approximation equation

$$\left[\frac{\partial}{\partial t}+\Lambda_A+iL_0^{(A)}\right]\rho_A^{(0)}(t)=0 , \tag{32}$$

we arrive at the final diffusion equation with the diffusion term in the form

$$\left[\frac{\partial\rho_f}{\partial t}\right]_{\text{diff}}=-\sum_{i,j=1}^{2} y_i R^{ij}(z,z^*)y_j , \tag{33}$$

where

$$R^{ij}(z,z^*)\equiv\int_0^\infty d\tau\,\mathrm{Tr}_A(x_i U_A(\tau)x_j\rho_A^{(0)}) , \tag{34}$$

$$x_1\Psi\equiv\sum_a(\sigma_a^+-\langle\sigma_a^+\rangle)\Psi , \quad x_2\Psi\equiv\Psi\sum_a(\sigma_a-\langle\sigma_a\rangle) ,$$

$$y_1=\frac{\partial}{\partial z^*} , \quad y_2=-\frac{\partial}{\partial z} .$$

The diffusion term (33) with the correlation forms (34) yields the fluctuation results which are identical to those obtained in the previous section. The details of calculations of the atomic correlation functions $R^{ij}(z,z^*)$ may be found in [21].

We presented this alternative way of discussing field fluctuations in order to emphasize the role of the dressed-atom Hamiltonian $H_0$ (20) and mostly skipped the details of the calculations. In what follows we adhere to the version based upon the decoupling procedure described in the previous section.

## III. THE REGULAR PUMP

The idea of injecting the excited atoms in a laser cavity to improve fluctuation properties of the lasing field dates back to the pioneer works on laser mode fluctuations by Scully and Lamb [14]. Golubev and Sokolov [1] first studied the influence of the statistics of injection on the quantum noise. In recent publications the different approaches to the pump-fluctuation problem were worked out by Haake, Tan, and Walls [2,6], Marte and Zoller [3], Benkert *et al.* [4], and Zhu, Chang, and Wang [5], and a quantitative agreement has been achieved. It had been found that in the case of regular pumping, when atoms are introduced in the cavity regularly at a fixed unfluctuating rate, a drastic reduction of quantum amplitude noise is possible and 50% amplitude squeezing is achievable theoretically. Below we present an alternative, Hamiltonian, approach to the regular pumping problem.

### A. Oscillatory and stationary regimes of the atomic variables

All the atomic quantities oscillate at the Rabi frequency. When the detuning is equal to zero the Rabi frequency is equal to $\omega_R\equiv|gE|/h$, with $g$ being the atom-field coupling constant and $E$ the field amplitude. The Rabi oscillations are damped at the relaxation time of order $\gamma^{-1}$ ($\gamma^{-1}$ is the atomic lifetime). The relaxation time is very short compared to the laser cavity field lifetime for the regimes at which real lasers operate. From a first glance it seems that one can neglect these oscillations and deal just with the stationary atomic quantities. However, the regular pumping is a particular example of the oscillatory behavior of the atomic quantities and also represents an example of the coherent coupling of these oscillations. Here we discuss the stationary and oscillatory regimes of the atomic quantities in order to understand what is the relevant regime to the specific cases of pumping. In the course of this discussion the role of the initial conditions will be clarified. For the sake of simplicity we consider the two-level atomic dipole only. The only evolution of the atomic quantities is governed by the closed set of the Bloch's equations [25]:

$$\frac{\partial P}{\partial t}=-\gamma_\perp P+igz(\rho_a-\rho_b) , \tag{35}$$

$$\frac{\partial\rho_a}{\partial t}=\left[\frac{\partial\rho_a}{\partial t}\right]_R+(igzP^*+\text{c.c.}) , \tag{36}$$

$$\frac{\partial\rho_b}{\partial t}=\left[\frac{\partial\rho_b}{\partial t}\right]_R-(igzP^*+\text{c.c.}) , \tag{37}$$

with $g$ being the coupling constant and $z$ being complex field amplitude. Here the atomic quantities correspond to the atomic density matrix of the form

$$\begin{pmatrix}\rho_a & P\\ P^* & \rho_b\end{pmatrix} . \tag{38}$$

The relaxation terms $(\ )_R$ are determined by the specific case of the pumping and the decay. Consider the case of the conventional pump [Fig. 1(a)]. In this case the relaxation terms have the following form:

$$\left[\frac{\partial\rho_a}{\partial t}\right]_R=\gamma_\uparrow\rho_b-\gamma_\downarrow\rho_a ,$$
$$\left[\frac{\partial\rho_b}{\partial t}\right]_R=\gamma_\downarrow\rho_a-\gamma_\uparrow\rho_b , \tag{39}$$

with $\gamma_\uparrow$ and $\gamma_\downarrow$ being the atomic constants describing the pumping and the decay rates, respectively. The explicit time-dependent solution of Eqs. (35)–(37) with Eqs. (39) reads (in the adiabatic limit, i.e., field amplitude $z$ does not depend on time) as

$$R(t)=\exp^{-\gamma_\parallel t}\{[R(0)-c]\cos(\sqrt{x}\,\gamma_\parallel t)+\frac{1}{\sqrt{x}}[\rho_a(0)-\rho_b(0)-c]\sin(\sqrt{x}\,\gamma_\parallel t)\}+C , \tag{40}$$

$$\rho_a(t)=\frac{1}{2}\exp^{-\gamma_\parallel t}\{[\rho_a(0)-\rho_b(0)-c]\cos(\sqrt{x}\,\gamma_\parallel t)-[R(0)-c]\sqrt{x}\,\sin(\sqrt{x}\,\gamma_\parallel t)\}+\frac{1+c}{2} , \tag{41}$$

$$\rho_b(t)=-\frac{1}{2}\exp^{-\gamma_\parallel t}\{[\rho_a(0)-\rho_b(0)-c]\cos(\sqrt{x}\,\gamma_\parallel t)-[R(0)-c]\sqrt{x}\,\sin(\sqrt{x}\,\gamma_\parallel t)\}+\frac{1-c}{2}\ , \tag{42}$$

$$P\equiv\frac{igz}{\gamma_\perp}R\ ,\quad x\equiv\frac{4|gz|^2}{\gamma_\perp\gamma_\parallel}\ ,\quad \gamma_\parallel=2(\gamma_\uparrow+\gamma_\downarrow)\ ,\quad \Delta_0=\frac{\gamma_\uparrow-\gamma_\downarrow}{\gamma_\uparrow+\gamma_\downarrow}\ ,\quad c\equiv\frac{\Delta_0}{x+1}\ . \tag{43}$$

Here we adopted $\gamma_\perp=\gamma_\parallel$. The solutions (40)–(42) describe the damping oscillations which decay during the atomic lifetime $\gamma_\parallel^{-1}$ to the stationary nonzero values $\rho_a(\infty)$, $\rho_b(\infty)$, and $P(\infty)$. The important issue is that with conventional pumping the atom is in lasing all the time and the oscillatory regime which remembers the initial conditions gives no contribution to the stationary state and can be neglected [Fig. 2]. In fact, with conventional pumping the laser fluctuations are determined by the stationary values of the atomic quantities. Certainly, the memory of the initial conditions is lost in this case.

Consider the case of regular pumping [Fig. 1(b)]. In this case the relaxation terms in (35)–(37) are as follows:

$$\left[\frac{\partial\rho_a}{\partial t}\right]_R=-\gamma_a\rho_a\ ,\quad \left[\frac{\partial\rho_b}{\partial t}\right]_R=-\gamma_b\rho_b\ , \tag{44}$$

with $\gamma_a$ and $\gamma_b$ being the decay constants of the active levels. The explicit time-dependent solution of Eqs. (35)–(37), with the relaxation determined by (44), reads as

$$R(t)=\exp^{-\gamma_\parallel t}\{R(0)\cos(\sqrt{x}\,\gamma_\parallel t)+\frac{1}{\sqrt{x}}[\rho_a(0)-\rho_b(0)]\sin[\sqrt{x}\,\gamma_\parallel t]\ , \tag{45}$$

$$\rho_a(t)=\frac{1}{2}\exp^{-\gamma_\parallel t}\{[\rho_a(0)-\rho_b(0)]\cos(\sqrt{x}\,\gamma_\parallel t)-R(0)\sqrt{x}\,\sin(\sqrt{x}\,\gamma_\parallel t)+\rho_a(0)+\rho_b(0)\}\ , \tag{46}$$

$$\rho_b(t)=\frac{1}{2}\exp^{-\gamma_\parallel t}\{[\rho_b(0)-\rho_a(0)]\cos(\sqrt{x}\,\gamma_\parallel t)+R(0)\sqrt{x}\,\sin(\sqrt{x}\,\gamma_\parallel t)+\rho_a(0)+\rho_b(0)\}\ , \tag{47}$$

with $\gamma_\parallel=\gamma_a+\gamma_b$ and $R$ and $x$ determined by (43). Here we adopted for simplicity $\gamma_a=\gamma_b=\gamma_\perp=\gamma$. The solutions (45)–(47) also describe the damped oscillations. However, in contrast with conventional pumping, these oscillations decay to zero stationary values. Hence, the stationary atomic dipole moment vanishes and, thus, according to this scheme of pumping the atom can effectively interact with the lasing mode only long before it arrives at its stationary state (see Fig. 2). In the stationary state the atom is in the ground state and, thus, is out of lasing altogether. So, for the atom to be active in the lasing process it must be in the oscillatory regime, *in spite of the very short atomic oscillation lifetime as compared with the cavity lasing mode lifetime.* It follows from (45)–(47) that these oscillations critically depend upon the initial conditions $P(0)$, $\rho_a(0)$, $\rho_b(0)$ of the atomic quantities. Since the phase of the dipole moment oscillations is shifted by $\pi/2$ with respect to population oscillations one can imagine the coherent coupling of the oscillations. However, to get this interaction one undoubtedly needs to have some physical process in which the atomic quantities are contained, at least in a product of two terms. Certainly, a linear process cannot give rise to the coupling. We will see below that the dressed-atom terms in fluctuation sources just provide the needed products of the atomic quantities. At this conceptual point the situation resembles the cooperative phenomena in the superfluorescence [26], in which case one encounters the nonlinear equations for the atomic quantities and coherent coupling of the Rabi oscillations is also possible. Thus, we can summarize: (i) The regular pump corresponds to the essentially nonstationary, oscillatory character of the atomic relaxation (see Fig. 2). (ii) These oscillations exhibit the strong sensitivity to the initial conditions [see Eqs.

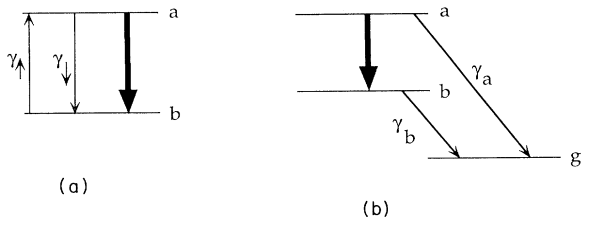


FIG. 1. Schematic illustration of the two different schemes of pumping. (a) Conventional two-level generation. (b) Relaxation of the atomic levels after the atom has been injected in the cavity (regular pumping).

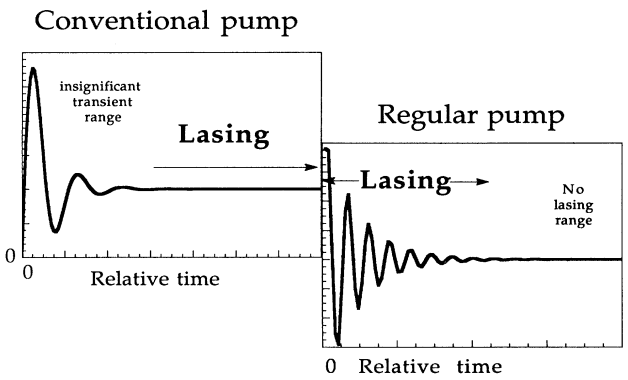


FIG. 2. Evolution of the atomic polarization under two different pumping regimes. Variables are plotted in arbitrary units.

(45)–(47)]. (iii) The interaction of the oscillations is possible, but nonlinear terms, with respect to atomic quantities, are needed.

The dependence of Rabi oscillations upon the initial conditions explicitly reveals a possibility to introduce the statistics from outside. Indeed, $P(0)$, $\rho_a(0)$, and $\rho_b(0)$ quantities may be fluctuating and may have their own statistics. It is clear that this statistics will influence the final field fluctuations. In particular deterministic case when $P(0)$, $\rho_a(0)$, and $\rho_b(0)$ are fixed (degenerate statistics), one might expect the best conditions for the noise suppression. External statistics is also introduced if excitation time $t_j$ fluctuates. In Eqs. (45)–(47) we dealt with the relative atomic time, so that the moment $t=0$ corresponded to $t=t_j$ in a global time scale. The regular excitation, at which $t_{j+1}-t_j=\text{const}$ is also expected to give rise to the best fluctuation properties. From now on, we assume these two conditions to be valid, which is optimal for noise suppression.

*Remarks.* We discussed the nonstationary behavior of the atomic variables under regular pumping conditions. However, one may argue that such behavior is exhibited only by a single atom, so due to averaging over a large number of atoms, macroscopic polarization and population inversion are not sensitive to micro-oscillations [4]. This is true. However, there arises a problem with quantum fluctuations. The point is that the fluctuation terms in quantum equations are not linear with respect to atomic variables (these are dressed-atom terms which are quadratic ones). This nonlinearity does not allow one to obtain quantum equations by simple summations of one-atom equations. Thus the transition from microscopic to macroscopic quantities is a nontrivial one. Moreover, one may envisage a physical situation when macroscopic description is impossible altogether. This is, for example, the case with a single atom in the cavity.

### B. Nonstationary picture of fluctuations: Survival of an inverse proportional dependence of the dressed-atom terms upon field in the nonstationary pumping case

In this section we apply the dressed-atom concept to the nonstationary pumping regime. Before we discuss rigorously the nonstationary fluctuations we investigate the specific features of the dressed-atom terms in this particular case of nonstationary pumping. First of all, it should be noted that all quantities, in the nonstationary regime, have to be averaged over the interaction time (atomic lifetime) and be treated as averaged quantities. The dressed-atom terms are represented by the products of the atomic quantities in fluctuation terms. These products are $PP^*$, $P^2$, $PS$, and $P^*S$. Let us investigate the dependence of these products upon field $x$. In order to avoid unnecessary dealing with complex quantities we use a "real" atomic dipole moment $R$ according to definitions (43).

(i) With stationary pumping when only the stationary terms survive in (40)–(42) we have for the products in question the following formulas:

$$R^2=RS=\Delta_0^2(1+x)^{-2}\equiv\Delta_0^2 f^2 . \tag{48}$$

One can see that at the strong-field limit ($x\gg1$) the following asymptotic behaviors are valid:

$$R\sim S\sim\frac{1}{x} , \quad R^2\sim RS\sim\frac{1}{x^2} . \tag{49}$$

The inverse quadratic dependence of the dressed-atom terms upon field accounts for the vanishing of the dressed-atom contribution at the strong-field limit for the conventional (stationary) two-level pumping. These quadratic terms are unable to compete with the linear ones at this limit.

(ii) With nonstationary pumping result (49) is crucially changed because of the time averaging. We assume for simplicity the initial conditions [see Eqs. (45)–(47) and (40)–(42)] to be as follows:

$$\rho_a(0)=1 , \quad \rho_b(0)=0 , \quad R(0)=0 , \tag{50}$$

which corresponds to injection of atoms in a pure excited state. In this case nonstationary solutions (45)–(47) yield

$$R(t)=\exp^{-\gamma_\parallel t}\frac{1}{\sqrt{x}}\sin(\sqrt{x}\,\gamma_\parallel t) , \tag{51}$$

$$S(t)\equiv\rho_a(t)-\rho_b(t) \tag{52}$$

$$=\exp^{-\gamma_\parallel t}\cos(\sqrt{x}\,\gamma_\parallel t) . \tag{53}$$

Upon time integration one obtains the asymptotic dependencies

$$\overline{R(t)}\sim\overline{S(t)}\sim\frac{1}{x} , \quad \overline{R^2(t)}\sim\overline{R(t)S(t)}\sim\frac{1}{x} , \tag{54}$$

with the definition

$$\overline{\Psi(t)}\equiv\int_0^\infty dt\,\Psi(t) \tag{55}$$

used.

One can see that on integration over time the products in question $\overline{R^2}$ and $\overline{RS}$ have the same asymptotic field dependence as the linear terms $\overline{R}$ and $\overline{S}$, namely, $1/x$. This survival of inverse linear dependence upon field in the dressed-atom terms is characteristic for nonstationary pumping. By contrast with conventional (stationary) pumping the dressed-atom contribution does not vanish in this case at the strong-field limit. It follows from our simple consideration that this survival of inverse linear dependence is a consequence of a coherent interaction of Rabi oscillations of the atomic quantities. This interaction becomes possible because of the presence of products of the atomic quantities in the fluctuation terms. In what is followed we present a rigorous consideration of the nonstationary fluctuations (nonstationary pumping).

### C. Rigorous formulation of the fluctuations problem for the nonstationary pumping regime

We start from the basic equations (14)–(16) which have to be adjusted to the specific pumping [Fig. 1(b)]. On being rewritten for matrix elements these equations read as

$$\frac{\partial\rho_f}{\partial t}=\left\{\frac{\partial}{\partial z}\left[\kappa z+i\sum_{j=0}^{\infty}P_j(t)\right]+\text{H.c.}\right\}\rho_f , \tag{56}$$

$$\frac{\partial P_j}{\partial t}+\gamma_\perp P_j-izS_j=i\left\{\frac{\partial}{\partial z}\frac{P_j-1-S_j}{2}+P_j\Lambda_j\right\}\ln\rho_f\ , \tag{57}$$

$$\frac{\partial S_j}{\partial t}+\gamma_1 S_j-\gamma_2(K_j-1)+(2izP_j^*+\text{c.c.})=i\Lambda_j(1+S_j)\ln\rho_f\ , \tag{58}$$

$$\frac{\partial K_j}{\partial t}+\gamma_1(K_j-1)-\gamma_2 S_j=i\Lambda_j K_j\ln\rho_f\ , \tag{59}$$

$$P_j\equiv(r_j)_{ab}\ ,\quad S_j\equiv(r_j)_{aa}-(r_j)_{bb}\ ,\quad K_j\equiv(r_j)_{gg}\ , \tag{60}$$

$$\Lambda_j\equiv-\frac{\partial}{\partial z}P_j-\text{H.c.}\ ,\quad \gamma_1\equiv\gamma_a+\gamma_b\ ,\quad \gamma_2\equiv\gamma_a-\gamma_b\ , \tag{61}$$

where $K_j$ is the ground-state population.

For the sake of simplicity we adopt $g=1$. According to our numeration of atoms, the atom with the number $j=0$ is supposed to be injected in a cavity at the moment $t$. The time gap $\tau$ between the consecutive atom's arrivals is independent of $j$ number (regular pumping assumption). Since each atom is situated under the same conditions which are physically identical we can introduce the unified atomic functions $P$, $S$, and $K$ so that

$$P_j(t)=P(j\tau)\ ,\quad S_j(t)=S(j\tau)\ ,\quad K_j(t)=K(j\tau)\ . \tag{62}$$

The time $t'=j\tau$ represents the relative atomic time so that $t'=0$ corresponds to the moment of injection. It is convenient, therefore, to discuss the atomic equations (57)–(59) by using the relative atomic time $t'$ and formulate the initial conditions with this time $t'$. The time scale we are interested in (of order $1/\gamma$) is much longer than the time gap $\tau$. Therefore, we can change the summation in Eq. (56) into an integration

$$\sum_{j=0}^{\infty}P_j(t)=\sum_{j=0}^{\infty}P(j\tau)\cong\frac{1}{\tau}\int_0^{\infty}P(t')dt'\equiv\bar{P}\ . \tag{63}$$

Upon introducing the relative atomic time $t'$, using Eq. (63) and making some evident incidental arrangements, we arrive at the final set of equations and initial conditions

$$\frac{\partial\rho_f}{\partial t}=\left\{\frac{\partial}{\partial\eta}\eta\left[2\kappa-\frac{2}{\gamma_\perp}\bar{R}\right]\right\}\rho_f\ , \tag{64}$$

$$\frac{1}{\gamma_\perp}\frac{\partial R}{\partial t}+R-S=\alpha\ ,\quad R(0)=0\ , \tag{65}$$

$$\frac{\partial s}{\partial t}+\gamma_1 S-\gamma_2 K'+\gamma_\parallel xR=\beta\ ,\quad S(0)=1\ , \tag{66}$$

$$\frac{\partial K'}{\partial t}+\gamma_1 K'-\gamma_2 S=\theta\ ,\quad K'(0)=-1\ , \tag{67}$$

$$\alpha\equiv\frac{1}{2}\frac{\partial}{\partial\eta}\left[K'-S+\frac{x}{m}R^2\right]\ln\rho_f\ ,$$

$$\beta\equiv\frac{2}{\gamma_\perp}\frac{\partial}{\partial\eta}\eta R(1+S)\ln\rho_f\ ,$$

$$\theta\equiv\frac{2}{\gamma_\perp}\frac{\partial}{\partial\eta}\eta R(1+K')\ln\rho_f\ ,\quad R\equiv-i\frac{\gamma_\perp}{z}P\ ,\quad K'\equiv K-1\ ,$$

$$\gamma_\parallel\equiv\frac{4\gamma_a\gamma_b}{\gamma_a+\gamma_b}\ ,\quad m\equiv\frac{\gamma_\perp}{\gamma_\parallel}\ ,\quad x\equiv\frac{4\eta}{\gamma_\perp\gamma_\parallel}\ ,\quad z\equiv\sqrt{\eta}e^{i\phi}\ .$$

We have fixed the initial conditions in atomic equations (65)–(67). In contrast to conventional (stationary) pumping the initial conditions will influence the laser field output fluctuations. The method described below allows one to deal with any possible initial conditions in principle. For the sake of simplicity we fix those corresponding to the atom injected in a pure excited state [$R(0)=0$, $S(0)=1$, and $K(0)=0$]. The closed set of equations is to be reduced to the Fokker-Plank equation for the field density matrix. This problem can be solved by finding the quantity $\bar{R}$ as it is readily seen from (64).

### D. Classical solutions

In this section we neglect, for the time being, the fluctuations, i.e., we put $\alpha=\beta=\theta=0$ in atomic equations (65)–(67). First, we shall obtain the explicit time-dependent solutions for the following two relevant cases: (i) $\gamma_a=\gamma_b$, $x\gg1$, (ii) $\gamma_a\ll\gamma_b$, $x\gg1$. Then we shall obtain exact formulas for the time-averaged quantities $\bar{R}$, $\bar{S}$, and $\bar{K'}$.

(i) $\gamma_a=\gamma_b=\gamma$ (equal decay constants for both active levels). In this case the classical equations read as

$$\frac{1}{\gamma_\perp}\frac{\partial R}{\partial t}+R-S=0\ ,\quad R(0)=0\ , \tag{68}$$

$$\frac{1}{\gamma_\perp}\frac{\partial S}{\partial t}+S-xR=0\ ,\quad S(0)=1\ . \tag{69}$$

Taking for $\gamma_\perp$ its minimal value $\gamma_\perp^{\min}=\gamma_a+\gamma_b=2\gamma$ one has the following explicit solutions:

$$R=\frac{1}{\sqrt{x}}e^{-2\gamma t}\sin(2\sqrt{x}\,\gamma t)\ , \tag{70}$$

$$S=e^{-2\gamma t}\cos(2\sqrt{x}\,\gamma t)\ . \tag{71}$$

(ii) $\gamma_a\ll\gamma_b$ (disparity between the decay constants), $x\gg1$. In this limit case the classical equations read as

$$\frac{1}{\gamma_\perp}\frac{\partial R}{\partial t}+R-S=0\ , \tag{72}$$

$$\frac{1}{\gamma_1}\frac{\partial S}{\partial t}+S+K'+yR=0\ ,\quad y\equiv x\frac{\gamma_\parallel}{\gamma_1}\approx4x\frac{\gamma_a}{\gamma_b}\ , \tag{73}$$

$$\frac{1}{\gamma_1}\frac{\partial K'}{\partial t}+K'+S=0\ . \tag{74}$$

Again we put $\gamma_\perp=\gamma_\perp^{\min}=\gamma_1$. On assuming $y\gg1$ we have an explicit solution of Eqs. (72)–(74):

$$R(t)=e^{-\gamma_b t}\left\{\frac{1}{y}+\frac{1}{\sqrt{y}}\sin(\sqrt{y}\,\gamma_b t)-\frac{1}{y}\cos(\sqrt{y}\,\gamma_b t)\right\}, \tag{75}$$

$$S(t)=e^{-\gamma_b t}\left\{\frac{1}{\sqrt{y}}\sin(\sqrt{y}\,\gamma_b t)+\cos(\sqrt{y}\,\gamma_b t)\right\}, \tag{76}$$

$$K'(t)=e^{-\gamma_b t}\left\{-1-\frac{1}{y}-\frac{1}{\sqrt{y}}\sin(\sqrt{y}\,\gamma_b t)+\frac{1}{y}\cos(\sqrt{y}\,\gamma_b t)\right\}. \tag{77}$$

(iii) The exact expressions for the general case can be obtained for the time-averaged quantities $\bar{R}$, $\bar{S}$, and $\overline{K'}$. After integrating the classical equations over time we have (the initial conditions are taken into account)

$$\bar{R}-\bar{S}=0\,, \tag{78}$$

$$\frac{1}{\tau}+\gamma_1\bar{S}-\gamma_2\overline{K'}+\gamma_\parallel x\bar{R}=0\,, \tag{79}$$

$$-\frac{1}{\tau}+\gamma_1\overline{K'}-\gamma_2\bar{S}=0\,. \tag{80}$$

This set of equations results in the following solution

$$\bar{R}=\bar{S}=\frac{1}{2\gamma_a\tau}f\,,\quad f\equiv(1+x)^{-1}\,, \tag{81}$$

$$\overline{K'}=-\frac{1}{\gamma_1\tau}+\frac{\gamma_2}{\gamma_1}\frac{f}{2\gamma_a\tau}\,. \tag{82}$$

At the "no fluctuations" limit the field Eq. (64) can be rewritten, with the help of Eq. (81), in the form

$$\frac{\partial\rho_f}{\partial t}=2\kappa\frac{\partial}{\partial\eta}\eta(1-\xi f)\rho_f \tag{83}$$

with $\xi$ the lasing parameter

$$\xi\equiv\frac{1}{2\kappa\gamma_a\tau\gamma_\perp} \tag{84}$$

from which one can find a stationary solution for field. It reads as

$$x=\xi-1\,. \tag{85}$$

An explicit stationary solution of Eq. (83) is represented by a $\delta$ function,

$$\rho_f^{(0)}=\delta(x-(\xi-1))\,. \tag{86}$$

Incorporating the fluctuations (taking into account the diffusion terms) in the theory gives rise to nonzero width of the distribution $\rho_f(x)$. It will be done in the next section.

### E. Fluctuations

The next step is a calculation of fluctuations. In order to do this we have to calculate $\bar{R}$ in Eq. (64) with one more order of the perturbation theory. In other words, we have to solve the lower Eqs. (65)–(67) taking into account the small terms $\alpha$, $\beta$, and $\theta$. Upon making a time integration of Eqs. (65)–(67), with proper account of the initial conditions, we arrive at the following set of equations:

$$\bar{R}-\bar{S}=\bar{\alpha}\,, \tag{87}$$

$$\frac{1}{\tau}+\gamma_1\bar{S}-\gamma_2\overline{K'}+\gamma_\parallel x\bar{R}=\bar{\beta}\,, \tag{88}$$

$$-\frac{1}{\tau}+\gamma_1\overline{K'}-\gamma_2\bar{S}=\bar{\theta}\,. \tag{89}$$

According to our scheme of perturbation theory the small quantities in this set of equations $\alpha$, $\beta$, and $\theta$ are treated as constant terms calculated with zero-order functions $R_0$, $S_0$, and $K_0'$ (i.e., classical solutions). This algebraic set of equations is to be resolved and the result for $\bar{R}$ is to be inserted in Eq. (64). We skip this straightforward procedure and present here only the final Fokker-Plank equation:

$$\frac{\partial}{\partial t}\rho_f=2\kappa\frac{\partial}{\partial\eta}\eta\left[1-\xi f+D\frac{\partial}{\partial\eta}\right]\rho_f\,, \tag{90}$$

with the diffusion coefficient in the form

$$D=-\gamma_a\tau\left\{\overline{K'}-\bar{S}+\left[1+\frac{\gamma_2}{\gamma_1}\right]x\bar{R}+x\left[\frac{m+1}{m}\overline{R^2}+\frac{\gamma_2}{\gamma_1}\overline{K'R}\right]\right\}. \tag{91}$$

The Mandel parameter is ordinarily determined by the formula

$$Q=\frac{\xi D}{x}\,. \tag{92}$$

Formulas (91) and (92) solve the fluctuation problem in principle but certain work is still to be done. One has to evaluate the second moments $\overline{R^2}$ and $\overline{K'R}$ [the first moments $\bar{R}$, $\bar{S}$, and $\overline{K'}$ have been presented by (81) and (82)]. The moments in question, $\overline{R^2}$ and $\overline{K'R}$ (dressed-atom terms), will be calculated (1) with the help of the explicit expressions for $R(t)$ and $K'(t)$ functions taken for two different limit situations $\gamma_a=\gamma_b$ and $\gamma_a\ll\gamma_b$, and (2) with the help of some moment consideration for the general case of arbitrary $\gamma_a$ and $\gamma_b$. As for the linear terms in Eq. (91) their contribution to $Q$ is equal to zero because of the equality

$$\overline{K'}-\bar{S}+\left[1+\frac{\gamma_2}{\gamma_1}\right]x\bar{R}=0\,, \tag{93}$$

which is valid at the strong-field limit [see expressions (81) and (82)]. Thus, the Mandel parameter $Q$ is determined solely by the dressed-atom terms.

### F. Calculation of the dressed-atom terms (i.e., quadratic moments $\overline{R^2}$ and $\overline{K'R}$)

(i) $\gamma_a=\gamma_b=\gamma$. In this case we need only moment $\overline{R^2}$ since $\gamma_2=0$ [see Eq. (91)] and moment $\overline{K'R}$ gives no contribution. Upon integrating the explicit expression (70)

one obtains

$$\overline{R^2}=\frac{1}{\tau}\int_0^\infty\left[\frac{1}{\sqrt{x}}e^{-2\gamma t}\sin(2\sqrt{x}\,\gamma t)\right]^2 dt=\frac{1}{8\gamma\tau x} \tag{94}$$

and the final expression for the Mandel parameter becomes

$$Q=-\gamma_a\tau x\frac{m+1}{m}\overline{R^2}=-\frac{1}{4}\ . \tag{95}$$

Here we adopted for $m$ its minimal value $m=1$. Formula (95) indicates 25% squeezing [2–6].

(ii) $\gamma_a \ll \gamma_b$. In this case both moments $\overline{R^2}$ and $\overline{K'R}$ are significant for the fluctuation results. Elementary integration over time can be easily performed in this case, too. Upon integrating the products of functions (75)–(77) one obtains (for first nonvanishing order of small quantity $y^{-1}$, $y\equiv 4x\gamma_a/\gamma_b$)

$$\overline{R^2}=\frac{1}{4}\frac{1}{\gamma_\parallel\tau x}\ ,\quad \overline{K'R}=-\frac{7}{4}\frac{1}{\gamma_\parallel\tau x}\ , \tag{96}$$

and, finally, an expression for the Mandel parameter

$$\begin{aligned}Q&\approx-\xi\gamma_a[\overline{R^2}-\overline{K'R}]\\&=-\xi\frac{\gamma_a}{\gamma_\parallel x}\left[\frac{1}{4}+\frac{7}{4}\right]\approx-\frac{1}{2}\ .\end{aligned} \tag{97}$$

Here we again adopted the strong-field limit. The result (97) indicates 50% squeezing in full agreement with other works [2–6].

(iii) Now we consider the general case of arbitrary $\gamma_a$ and $\gamma_b$. In this case it is very convenient not to solve explicitly Eqs. (65)–(67) but to treat them like a moment problem. On multiplying the classical (i.e., with $\alpha=\beta=\theta=0$) version of Eqs. (65)–(67) by proper function and integrating over time, taking into account the initial conditions, one arrives at a close set of algebraic equations. On solving this set of equations (see the Appendix) one obtains the following expressions for the moments in question:

$$\overline{R^2}=\frac{1}{\gamma_\parallel\tau}\frac{a_{22}+0.5a_{12}}{a_{11}a_{22}-a_{12}a_{21}}\ , \tag{98}$$

$$\overline{K'R}=-\frac{1}{\gamma_\parallel\tau}\frac{a_{21}+0.5a_{11}}{a_{11}a_{22}-a_{12}a_{21}}\ , \tag{99}$$

where

$$a_{11}\equiv 3a_2\left[\frac{x}{a_1}+2\right]\ ,\quad a_{12}\equiv-a_1\left[\frac{x}{a_1}+4+2\left[\frac{a_2}{a_1}\right]^2\right]\ , \tag{100}$$

$$a_{22}\equiv 3a_2\ ,\quad a_{21}\equiv-a_1\left[2\left[\frac{x}{a_1}+1\right]+\left[\frac{a_2}{a_1}\right]^2\right]\ , \tag{101}$$

$$a_1\equiv\frac{\gamma_1}{\gamma_\parallel}\ ,\quad a_2\equiv\frac{\gamma_2}{\gamma_\parallel}\ .$$

At the strong-field limit ($x\gg 1$) Eqs. (98)–(99) [together with (91)–(93)] yield the general formula for the Mandel parameter,

$$Q=-\frac{1}{2}\frac{\gamma_b}{\gamma_a+\gamma_b}\ . \tag{102}$$

This formula is in full agreement with other approaches [2–6].

## IV. CONCLUSIONS

In this part of the paper we introduced a classification of the laser pumping according to the time regime of the atomic variables. It has been shown that the oscillatory pumping regime features an ability to introduce fluctuations from the outside through the initial conditions. In the particular case when the initial values of atomic variables are fixed (degenerated statistics), one might expect the best conditions for noise suppression. Another way to introduce an external statistics is through the statistics of excitation (or injection) times. The regular excitation, i.e., under the condition $t_{j+1}-t_j=\text{const}$, is also expected to give rise to the best fluctuations properties. This method of noise suppression is known as regular pumping [1–6].

On the other hand, the stationary pumping regime is insensitive to initial conditions but in this case noise suppression is also possible [7–13] through the coherent interaction of the internal degrees of freedom and will be discussed in the following paper [27].

We presented a general approach to consider both pumping regimes. In the framework of this dressed-atom approach the two different pumping regimes are just two limits of one and the same physical system (stationary and nonstationary pictures). In this paper we discussed in detail the nonstationary picture of fluctuations. We emphasize that our results are obtained without any heuristic assumptions but are based upon the rigorous treatment of the total many atoms + field Liouville equation. The crucial difference in the results, as compared to the conventionally pumped laser, stems from the quadratic terms $\overline{R^2}$ and $\overline{RK'}$. These terms grow anomalously large due to the essentially nonstationary (oscillatory) character of the atomic variable dynamics. Because of this oscillatory character and the presence of the products of the atomic quantities in dressed-atom terms, this interaction of the very-short-lifetime Rabi oscillation becomes possible. Mathematically, this interaction is manifested in a survival of the linear inverse proportional dependence of fluctuations upon the field (instead of the inverse quadratic one in the case of a conventionally pumped laser). Our treatment is also applicable to the case of the micromaser ($\gamma_a=\gamma_b=0$). In the latter case, one has to deal with the real summation in Eq. (56) instead of integration (63) and with explicit time-dependent solutions $R(t)$ and $S(t)$ averaged over a time within the interaction period.

Finally, our calculations (the limit $\gamma\tau\to 0$ is taken) are completely applicable to the opposite limit of $\gamma\tau\gg 1$ (low rate of injection) as well. In the latter case the integral in Eq. (63) means an averaging over nonstationary states of the single atom.

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## APPENDIX: CALCULATION OF $\overline{R^2}$ and $\overline{K'R}$ BY APPEALING TO THE MOMENT TREATMENT OF THE CLASSICAL EQUATIONS

The classical version of basic Eqs. (65)–(67) reads as

$$\frac{1}{\gamma_\perp}\frac{\partial R}{\partial t}+R-S=0\ ,\quad R(0)=0\ , \tag{A1}$$

$$\frac{1}{\gamma_\parallel}\frac{\partial S}{\partial t}+a_1S-a_2K'+xR=0\ ,\quad S(0)=1\ , \tag{A2}$$

$$\frac{1}{\gamma_\parallel}\frac{\partial K'}{\partial t}+a_1K'-a_2S=0\ ,\quad K'(0)=-1\ , \tag{A3}$$

$$a_1\equiv\frac{\gamma_1}{\gamma_\parallel}\ ,\quad a_2\equiv\frac{\gamma_2}{\gamma_\parallel}\ .$$

Upon multiplying Eqs. (A1)–(A3) by proper functions and taking into account the initial conditions we arrive at the following closed set of linear equations:

$$x_1-x_2=0\ , \tag{A4}$$

$$a_2x_2-(a_1+m)x_3+mx_5=0\ , \tag{A5}$$

$$-xx_3+a_2x_4-2a_1x_5+a_2x_6=\frac{1}{\gamma_\parallel\tau}\ , \tag{A6}$$

$$[m+(a_1+x)]x_1-a_2x_3+mx_4=0\ , \tag{A7}$$

$$xx_1+a_1x_4-a_2x_5=\frac{1}{2\gamma_\parallel\tau}\ , \tag{A8}$$

$$a_2x_5-a_1x_6=-\frac{1}{2\gamma_\parallel\tau}\ , \tag{A9}$$

with the following notations:

$$\begin{aligned}&x_1\equiv\overline{R^2}\ ,\quad x_2\equiv\overline{RS}\ ,\quad x_3\equiv\overline{RK'}\ ,\\&x_4\equiv\overline{S^2}\ ,\quad x_5\equiv\overline{SK'}\ ,\quad x_6\equiv\overline{(K')^2}\ .\end{aligned} \tag{A10}$$

Elimination of $x_2$, $x_4$, $x_5$, and $x_6$ yields the reduced set of equations for the moments in question:

$$a_{11}x_1+a_{12}x_3=\frac{1}{\gamma_\parallel\tau}\ , \tag{A11}$$

$$a_{21}x_1+a_{22}x_3=-\frac{1}{2\gamma_\parallel\tau}\ , \tag{A12}$$

the solution of which is presented in Eqs. (98)–(101).


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