Quasiclassical analysis of laser cooling by velocity-selective coherent population trapping

E. Korsunsky, D. Kosachiov, B. Matisov, and Yu. Rozhdestvensky
Department of Theoretical Physics, State Technical University of St. Petersburg, 195251 St. Petersburg, Russia

L. Windholz and C. Neureiter
Institute of Experimental Physics, Technical University of Graz, Petersgasse 16, A-8010, Graz, Austria
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Laser cooling based on velocity-selective coherent population trapping is investigated theoretically. Quasiclassical treatment is used to identify the conditions for optimal cooling and to study the dynamics of the cooling process in detail. For different cooling schemes we give the analytical expressions for the temperature of the cold atoms, which can be much lower than the Doppler cooling limit. The effect of the degree of incompleteness of coherent population trapping on dynamics and the limits of laser cooling are revealed.

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I. INTRODUCTION

The phenomenon of coherent population trapping (CPT) provides a very effective mechanism for superdeep laser cooling of atoms. The striking example is the experiment [1] by Aspect *et al.*, where the ^{4}He atoms were cooled by CPT to a one-dimensional effective temperature of 2 μK, two times lower than the so-called recoil limit $T_R = 4$ μK, which is determined by the one-photon recoil energy R: $T_R = R/k_B = \hbar^2 k^2 / 2Mk_B$ (k is the wave number of the applied laser field, M is the atom mass, k_B is the Boltzmann constant).

The physical foundations of using CPT for cooling of atoms are the following: In multilevel quantum systems interacting with coherent electromagnetic radiation, special superpositional, noncoupling states $|\psi_{NC}\rangle$, which are not coupled with the rest of the system, appear under certain conditions. In the case of atoms moving in the field of some light waves, the states $|\psi_{NC}\rangle$ can be localized in well-defined regions of the momentum space (known as velocity-selective coherent population trapping [1,2]). Atoms, initially rather smoothly distributed in the momentum space, can be accumulated in these regions by means of spontaneous relaxation, which leads to formation of a narrow momentum distribution.

The width of a so-formed distribution decreases when the time of interaction of atoms with the applied radiation increases [2,3]. In the steady state it is determined by the degree of incompleteness of the coherent population trapping, i.e., by the part of the quantum system population, which is not trapped in $|\psi_{NC}\rangle$ and which participates in the process of interaction with the laser fields. So, in case of a three-level Λ system (Fig. 1), the degree of incompleteness of CPT is equal to the ratio of the relaxation rate Γ of coherency between the states $|1\rangle$ and $|2\rangle$ (transversal relaxation rate) composing the superposition $|\psi_{NC}\rangle$ to the spontaneous relaxation rate γ of the excited state $|3\rangle$ [4,5]. The value of Γ is determined in general by the correlation degree of the exciting fields [6], by collisional and transit-time broadenings, etc.

Thus, in real physical situations, the vector $|\psi_{NC}\rangle$ is determined in a state space with an accuracy of the ratio Γ/γ. Therefore, there exists a limit of laser cooling by CPT, which is determined by Γ/γ. Since, however, the parameter Γ/γ can be made in experiment very small ($\Gamma/\gamma \simeq 10^{-7}$ for a slow beam of Λ atoms excited by well-correlated laser fields [7]), the effective temperature of cooled atoms can achieve values much lower than T_R.

Because of the stochastic nature of spontaneous emission, the process of populating the state $|\psi_{NC}\rangle$, hence the process of formation of the narrow momentum distribution, is the diffusion in a momentum space [2,3,8]. Such a cooling is of rather limited effectiveness. It is possible, however, to build the configuration of laser cooling (i.e., an atomic excitation scheme, directions and polarizations of light waves, detunings of laser waves) so that an additional process appears, which "pushes" atoms diffusing in a momentum space to $|\psi_{NC}\rangle$. Such a process is performed by the radiation force [8–10], which can have a Doppler origin [8], or be a specific CPT force with a

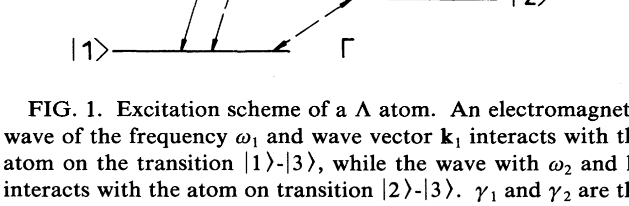

FIG. 1. Excitation scheme of a Λ atom. An electromagnetic wave of the frequency ω_1 and wave vector $\mathbf{k}_1$ interacts with the atom on the transition $|1\rangle$-$|3\rangle$, while the wave with ω_2 and $\mathbf{k}_2$ interacts with the atom on transition $|2\rangle$-$|3\rangle$. γ_1 and γ_2 are the spontaneous relaxation rates, and Γ is the transversal relaxation rate.

characteristic narrow dip in its momentum dependence [9,10].

The aim of this paper is to investigate the dynamics of one-dimensional laser cooling of atoms by CPT caused by both momentum diffusion and radiation force. It is known (see, e.g., [2]) that in the cooling process the atomic internal degrees of freedom are established into the steady state faster than the translational ones. This fact allows the study of the laser cooling dynamics at times $t \gg \tau$ (τ is the relaxation time of the internal variables) by use of kinetic equations for an atomic Wigner function $\mathbf{w}(\mathbf{r},\mathbf{p},t)$ [11], where $\mathbf{r}$ and $\mathbf{p}$ are the classical coordinate and momentum of an atom, respectively.

In order to derive the equation for $\mathbf{w}(\mathbf{r},\mathbf{p},t)$, we use the quasiclassical approach when it is possible to neglect the wave nature of atomic motion in laser light, i.e., if the atomic coherence length $\hbar/\delta p$ (δp is the width of momentum distribution of atoms) is small compared with the laser wavelength $\lambda=2\pi/k$, or if $\hbar k/\delta p \ll 1$. Then one can consider the atomic motion in laser light as a Brownian motion described by a Fokker-Planck equation. Such an approach has some advantages over approaches based on a full quantum description of atomic motion. First of all, it becomes possible to derive analytical expressions for the radiation force and the momentum diffusion tensor, to investigate effectiveness and limits of laser cooling by CPT, time scales of cooling processes, dependence of the effective temperature of an atomic ensemble on interaction parameters, and so on.

However, there are some restrictions in the approach because of the condition $\hbar k/\delta p \ll 1$. So, it is impossible to study the structures of momentum distributions having widths $\delta p \leq \hbar k$ (such as in experiment [1]). Consequently, the temperature limit of the quasiclassical theory is the temperature T_R. We emphasize that T_R is not the limit for the real mechanism of laser cooling by CPT. Physical phenomena do not depend on the method of description. We believe, therefore, that the general conclusions about the dynamics of laser cooling by CPT, which can be drawn from the quasiclassical theory, must be valid in the whole region of temperatures, including $T<T_R$.

Laser cooling by CPT in the framework of a quasiclassical approach was studied in [9,12]. However, the investigations in [9,12] are not complete. They do not take into account many important effects. So, the relaxation rate Γ of the coherency of state $|\psi_{\mathrm{NC}}\rangle$ is not taken into account. The role of momentum diffusion in the cooling process is also not investigated.

In the present paper a consistent quasiclassical theory of laser cooling by CPT is developed. We study the dynamics of formation of atomic momentum distribution due to both momentum diffusion and radiation force arising under CPT conditions. The effectiveness of cooling is shown to be improved considerably by means of a friction force, the value of which is greater for a more asymmetrical scheme of levels used for the cooling of atoms. We find magnitudes of interaction parameters, for which the temperature of atoms cooled by CPT can be much lower than the Doppler limit $T_D=\hbar\gamma/k_B$. The role of relaxation of the coherency of $|\psi_{\mathrm{NC}}\rangle$ as a factor limiting the degree of cooling by CPT is demonstrated. We consider some specific schemes of interaction of an atom with the field and obtain the analytical expressions for radiation force, dynamical friction coefficient, momentum diffusion coefficient, and temperature of the cooled atoms.

II. QUASICLASSICAL APPROACH

In the present paper the mechanism of laser cooling by coherent population trapping is investigated for a Λ atom (Fig. 1) interacting wit the field of two traveling waves with the frequencies ω_m $(m=1,2)$:

$$\mathbf{E}(\mathbf{r},t)=E_1\mathbf{e}_1\cos(\omega_1 t-\mathbf{k}_1\mathbf{r})+E_2\mathbf{e}_2\cos(\omega_2 t-\mathbf{k}_2\mathbf{r}) , \qquad (2.1)$$

where $\mathbf{e}_m$ are the polarization unit vectors, $\mathbf{k}_m$ with $|\mathbf{k}_m|=k_m=\omega_m/c$ are the wave vectors, and E_m are the amplitudes of the light waves.

We assume that the upper level $|3\rangle$ in the Λ atom decays to the lower ones with the rates γ_m for channels $|3\rangle$-$|m\rangle$ $(m=1,2)$, which are allowed dipole transitions, while the transition $|1\rangle$-$2\rangle$ is dipole forbidden. Moreover, we take into account the damping rate Γ of the coherency between the level $|1\rangle$ and $|2\rangle$.

Such a system is known [13] to be the simplest quantum system in which the population trapping is possible. So, it can be easily shown [12,14] that the state $|\psi_{\mathrm{NC}}\rangle$ noncoupled to the rest of the system arises in Λ atoms, the center-of-mass velocity $\mathbf{v}_0$ of which satisfies the two-photon resonance condition

$$\Delta_1-\mathbf{v}_0\mathbf{k}_1=\Delta_2-\mathbf{v}_0\mathbf{k}_2 , \qquad (2.2)$$

where $\Delta_m=\omega_m-\omega_{3m}$ are the detunings and ω_{3m} are the frequency distance between the states $|3\rangle$ and $|m\rangle$ $(m=1,2)$.

In a momentum representation the wave function of the trap state is a superposition of the two ground-state wave functions, each multiplied by a δ function [14]:

$$|\psi_{\mathrm{NC}}(\mathbf{p})\rangle=(\Omega_2/\Omega_0)\delta(\mathbf{p}-(M\mathbf{v}_0-\hbar\mathbf{k}_1))|1\rangle$$
$$-(\Omega_1/\Omega_0)\delta(\mathbf{p}-(M\mathbf{v}_0-\hbar\mathbf{k}_2))|2\rangle , \qquad (2.3)$$

where $\Omega=\langle 3|\hat{d}E_m|m\rangle/2\hbar$ are the Rabi frequencies of transitions $|3\rangle$-$|m\rangle$ $(m=1,2)$, $\hat{d}$ is the atomic dipole momentum operator, and $\Omega_0^2=|\Omega_1|^2+|\Omega_2|^2$ determines the sum intensity of laser waves.

The state $|\psi_{\mathrm{NC}}\rangle$, being a superposition of two low long-lived states, is populated by optical pumping through the intermediate state $|3\rangle$ [4,5]. In this paper we consider the dynamics of filling the trap state, hence the dynamics of the momentum distribution narrowing (since the trap state is localized in momentum space at points $\mathbf{p}_1=M\mathbf{v}_0-\hbar\mathbf{k}_1$ and $\mathbf{p}_2=M\mathbf{v}_0-\hbar\mathbf{k}_2$) by use of a quasiclassical approach.

A quasiclassical approach to laser cooling implies the analysis of kinetic equations, such as the Fokker-Planck equation (FPE) for the atomic Wigner function. The derivation of the FPE and its analysis are given here for the case of Λ atoms interacting with the field of two waves propagating along the z axis in opposite directions (as shown in Fig. 1). To describe the dynamics of Λ atoms, we start from equations for the atomic density matrix in a Wigner representation [10,11]:

$$
\begin{aligned}
i\frac{d}{dt}\rho_{11}^{0} &= -\Omega_1 \exp(-ikz+i\Delta_1 t)\rho_{31}^{+}\text{c.c.} \\
&\quad + i\gamma_1 \int \Phi_{31}(\mathbf{n})\rho_{33}(\mathbf{p}+\mathbf{n}\hbar\omega_{31}/c)d\mathbf{n} , \\
i\frac{d}{dt}\rho_{12}^{0} &= -\Omega_1 \exp(-ikz+i\Delta_1 t)\rho_{32}^{+} \\
&\quad + \Omega_2 \exp(-ikz-i\Delta_2 t)\rho_{13}^{-} - i\Gamma\rho_{12}^{0} , \\
i\frac{d}{dt}\rho_{13}^{0} &= -\Omega_1 \exp(-ikz+i\Delta_1 t)(\rho_{33}^{+}-\rho_{11}^{-}) \\
&\quad + \Omega_2 \exp(ikz+i\Delta_2 t)\rho_{12}^{+} - i\gamma\rho_{13}^{0} , \\
i\frac{d}{dt}\rho_{22}^{0} &= -\Omega_2 \exp(ikz+i\Delta_2 t)\rho_{32}^{-} + \text{c.c.} \qquad (2.4) \\
&\quad + i\gamma_2 \int \Phi_{32}(\mathbf{n})\rho_{33}(\mathbf{p}+\mathbf{n}\hbar\omega_{32}/c)d\mathbf{n} , \\
i\frac{d}{dt}\rho_{23}^{0} &= -\Omega_2 \exp(ikz+i\Delta_2 t)(\rho_{33}^{-}-\rho_{22}^{+}) \\
&\quad + \Omega_1 \exp(-ikz+i\Delta_1 t)\rho_{21}^{-} - i\gamma\rho_{23}^{0} , \\
i\frac{d}{dt}\rho_{33}^{0} &= -\Omega_1 \exp(ikz-i\Delta_1 t)\rho_{13}^{-} \\
&\quad - \Omega_2 \exp(-ikz-i\Delta_2 t)\rho_{23}^{+} + \text{c.c.} - 2i\gamma\rho_{33}^{0} ,
\end{aligned}
$$

where $d/dt = \partial/\partial t + \mathbf{v}(\partial/\partial \mathbf{r})$ represents a convective derivative, and we have introduced

$$
\begin{aligned}
&2\gamma = \gamma_1 + \gamma_2 , \\
&\rho_{\alpha\beta}^{\pm} = \rho_{\alpha\beta}(\mathbf{r}, \mathbf{p} \pm \hbar\mathbf{k}/2, t) , \\
&\rho_{\alpha\beta}^{0} = \rho_{\alpha\beta}(\mathbf{r}, \mathbf{p}, t) .
\end{aligned}
$$

For simplicity, we have also supposed in (2.4) that the wave numbers of the waves are nearly the same for both transitions in the Λ system: $k_1 \simeq k_2 = k$. The functions $\Phi_{3m}(\mathbf{n})$ determine the relative probability of the photon emission in the $\mathbf{n}$ direction when the upper state decays spontaneously into the channels $|3\rangle$-$|m\rangle$ $(m=1,2)$. However, for our purposes the spherical symmetry approximation is sufficient for $\Phi_{3m}(\mathbf{n})$: $\Phi_{3m}(\mathbf{n}) \cong 1/4\pi$.

Then, after the substitution of the off-diagonal density matrix elements

$$
\begin{aligned}
&\rho_{13} = \rho_{13}\exp(-ikz+i\Delta_1 t) , \\
&\rho_{23} = \rho_{23}\exp(ikz+i\Delta_2 t) , \qquad (2.5) \\
&\rho_{12} = \rho_{12}\exp[2ikz - i(\Delta_1-\Delta_2)t] ,
\end{aligned}
$$

we expand in (2.4) the density matrix elements $\rho_{\alpha\beta}^{\pm}$ around a point $\mathbf{p}$ in power series of the parameter

$$\hbar k/\Delta p \ll 1 , \qquad (2.6)$$

where Δp is the characteristic momentum scale of the variation of the density-matrix elements.

In the presence of coherent population trapping in the system, the quantity $\Delta p \cong M\Gamma'/k$ is determined by the width $\Gamma' = \Gamma + \Omega_0^2/\gamma$ of the narrow coherent trapping resonance [15], and therefore (2.6) takes the form

$$\frac{\hbar k}{\Delta p} \cong \frac{R}{\hbar\gamma}(\gamma/\Gamma') \cong \xi(\gamma/\Gamma') \ll 1 , \qquad (2.7)$$

where $\xi = R/\hbar\gamma \ll 1$ for strong dipole transitions under consideration.

The trapping of population in the state $|\psi_{\text{NC}}\rangle$ is nearly complete (i.e., when the population of $|\psi_{\text{NC}}\rangle$ is close to 1), when $\Omega_0^2 \gg \Gamma\gamma$ [4,5,15]. Under this condition, expression (2.7) can be written as follows:

$$\xi(\gamma/\Omega_0)^2 \ll 1 . \qquad (2.8)$$

For typical values of ξ, $\xi = R/\hbar\gamma \cong \alpha^{-1}m/M \cong 10^{-3}$, where α is the fine-structure constant and m is the electron mass, the intensity of exciting waves is limited by the condition $\Omega_0^2 \gg 10^{-3}\gamma^2$, which corresponds, e.g., for transitions $3S$-$3P$ of the sodium atom, to laser radiation intensities $I \gg 10^{-4}$ W/cm^2.

The condition (2.7) determines the intensities of exciting waves, for which the expansion of elements $\rho_{\alpha\beta}(\mathbf{r}, \mathbf{p} \pm \hbar\mathbf{k}/2, t)$ in a power series of the photon momentum $\hbar\mathbf{k}$ is valid:

$$
\begin{aligned}
\rho_{\alpha\beta}(\mathbf{r}, \mathbf{p} \pm \hbar\mathbf{k}/2, t) &\simeq \rho_{\alpha\beta}(\mathbf{r}, \mathbf{p}, t) \\
&\quad \pm (\hbar k/2)\frac{\partial}{\partial p_z}\rho_{\alpha\beta}(\mathbf{r}, \mathbf{p}, t) + \cdots . \qquad (2.9)
\end{aligned}
$$

In this case the description of the dynamics of a Λ atom is possible using a single kinetic equation for the distribution function $w = \sum_{m=1}^{3} \rho_{mm}(\mathbf{r}, \mathbf{p}, t)$. In the following analysis we consider, following the Bogolubov technique [11], the density-matrix elements $\rho_{\alpha\beta}(\mathbf{r}, \mathbf{p}, t)$ to be functionals of the atomic distribution function w:

$$\rho_{\alpha\beta}(\mathbf{r}, \mathbf{p}, t) \simeq \rho_{\alpha\beta}^{(0)} w(\mathbf{r}, \mathbf{p}, t) \pm \rho_{\alpha\beta}^{(1)} \hbar k \frac{\partial}{\partial p_z} w(\mathbf{r}, \mathbf{p}, t) + \cdots , \qquad (2.10)$$

where $\rho_{\alpha\beta}^{(n)}$ are unknown functions of the z component p_z of the atomic momentum $\mathbf{p}$ and can be determined by substituting expansion (2.10) into the reduced set of equations (2.4). Considering the second order in photon momentum $\hbar k$, we get the equation of Fokker-Planck type for the classical atomic distribution function w:

$$
\begin{aligned}
\left[\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}}\right] w(\mathbf{r}, \mathbf{p}, t) &= -\frac{\partial}{\partial p_z}(F_z w) \\
&\quad + \sum_{j=x,y,z} \frac{\partial^2}{\partial p_j^2}(D_{jj} w) , \qquad (2.11)
\end{aligned}
$$

where F_z is the z component of the radiation force and D_{jj} are the elements of the momentum diffusion tensor.

The radiation force and the diffusion tensor in Eq. (2.11) completely determine the dynamics of Λ atoms in the field (2.1). So, one can obtain the cold-atom temperature by use of the Einstein formula for the Brownian motion

$$T = D_{zz}(p_z=0)/\beta k_B M , \qquad (2.12)$$

where D_{zz} is the magnitude of the z component of the diffusion tensor at $p_z=0$, and

$$\beta=-\left.\frac{\partial F_z}{\partial p_z}\right|_{p_z=0} \tag{2.13}$$

is the dynamic friction coefficient determining the rate of narrowing of the momentum distribution. Then, the width of the momentum distribution centered at $p_z=0$ is expressed through (2.12): $\delta p=(2k_BTM)^{1/2}$. Note that in the quasiclassical description of laser cooling we use, the width δp cannot be less then the photon momentum $\hbar k$. Otherwise, the condition (2.6) is broken.

In this connection we emphasize the following circumstance. The state $|\psi_{\rm NC}\rangle$ is localized in the momentum space in two points, $\mathbf{p}_1=M\mathbf{v}_0-\hbar\mathbf{k}_1$ and $\mathbf{p}_2=M\mathbf{v}_0-\hbar\mathbf{k}_2$ (2.3). Therefore, the accumulation of atoms in $|\psi_{\rm NC}\rangle$ leads to the formation of the momentum distribution with two peaks centered at $\mathbf{p}_1$ and $\mathbf{p}_2$. However, the quasiclassical approach we use is limited by the condition (2.6), and so the two peaks cannot be resolved. In our theory the momentum distribution of atoms trapped in $|\psi_{\rm NC}\rangle$ includes a single peak centered at $\mathbf{p}_0=M\mathbf{v}_0$ (see the next section).

The derivation of the FPE for the case $k_1\neq k_2$ is analogous to that given above, except for the density-matrix elements $\rho^{\pm}_{\alpha\beta}$ being expanded around a point $\mathbf{p}$ in power series of the small parameters $\hbar k_1/\Delta p$ and $\hbar k_2/\Delta p$. Expressions for F_z and D_{zz} for arbitrary parameters are given in the Appendix. The additional analysis of laser cooling by CPT will be performed for various specific types of Λ systems.

III. COOLING OF ATOMS BY COHERENT POPULATION TRAPPING

In this section we consider the laser cooling by CPT for various types of double resonance in Λ systems. The main results and conclusions are obtained for the case of double optical resonance at $k_1\simeq k_2$. The results for the other types of Λ systems are the straightforward generalizations of the case $k_1\simeq k_2$.

A. Optical-optical double resonance ($k_1\simeq k_2$)

Such a scheme of atoms interacting with the field can be easily realized by optical excitation of the alkali-metal atoms from hyperfine structure sublevels of the ground state to the first excited state (the excitation of D_1 or D_2 lines). Otherwise, the lower levels of the Λ system can be generated by the Zeeman sublevels of one of the hyperfine levels of the ground state. Both transitions $|m\rangle$-$|3\rangle$ ($m=1,2$) in such a system are optical, and the magnitudes of the wave-vector moduli can be considered to be approximately the same: $|\mathbf{k}_1|\cong|\mathbf{k}_2|=k$.

Using the technique discussed above, we obtain both the light pressure force acting on the Λ atom and the z component of the momentum diffusion tensor for times $t\gg\gamma^{-1}$:

$$F_z=\hbar k\gamma\eta\Omega^2 aL^{-1}\ , \tag{3.1}$$

$$D_{zz}=2\hbar^2k^2\gamma\Omega^2aL^{-1}\ , \tag{3.2}$$

where the small nonadiabatic addition related to the statistic of reemitted photons are not taken into account [11]. In expressions (3.1) and (3.2) we denote

$$\eta=(\gamma_1-\gamma_2)/\gamma\ ,$$
$$a=4(kv_z)^2+2\Omega^2\Gamma/\gamma\ ,$$
$$L=4(kv_z)^4+4\Delta\eta(kv_z)^3+4(\Delta^2+\gamma^2+\Omega^2)(kv_z)^2$$
$$-2\Omega^2\Delta\eta(kv_z)+4\Omega^2G^2\ ,$$
$$G^2=\Delta^2\Gamma/2\gamma+\Omega^2\ ,$$

where $v_z=p_z/M$ is the z component of the velocity of the atom's center of mass, and we assume that the frequency detunings $\Delta_m\equiv\Delta$ as well as the Rabi frequencies $\Omega_m\equiv\Omega$ are the same for both transitions $|m\rangle$-$|3\rangle$ ($m=1,2$). We also assume the rate Γ of transversal relaxation to be much less than the natural width of the upper state: $\Gamma\ll\gamma$. The latter is always valid for a Λ atom excited by the correlated fields [6]. In addition, the condition for the light wave intensities $\Omega^2\gg\Gamma\gamma$ is considered to be fulfilled.

One can see from (3.1) that in this case the radiative force influencing the Λ atom is not equal to zero only for the asymmetrical scheme where $\eta\neq0$, i.e., $\gamma_1\neq\gamma_2$. Otherwise, for $\gamma_1=\gamma_2$ the cooling of an atomic ensemble is performed only due to momentum diffusion in a momentum space (the experiment [1] by Aspect *et al.* on subrecoil cooling of ^{4}He atoms by CPT was carried out precisely in case of $\gamma_1=\gamma_2$). The words "cooling due to diffusion" can seem somehow strange. However, a more accurate analysis shows that the momentum diffusion under the CPT conditions is a such that during a certain period of time the momentum distribution of atoms narrows considerably, and that can be interpreted as cooling.

1. Cooling due to momentum diffusion

Let us consider the FPE (2.11) for the case of $\gamma_1=\gamma_2$ (i.e., when $F_z=0$). We also assume the distribution function w to be spatially uniform: $w(\mathbf{r},\mathbf{p},t)=w(p_z,t)$. Then the FPE describing one-dimensional motion of atoms along the z axis can be written in the following form:

$$\frac{\partial w}{\partial t}-\left[\frac{\partial D_{zz}}{\partial p_z}\right]\frac{\partial w}{\partial p_z}=\frac{\partial}{\partial p_z}\left[D_{zz}\frac{\partial w}{\partial p_z}\right]+\left[\frac{\partial^2 D_{zz}}{\partial p_z^2}\right]w\ , \tag{3.3}$$

where D_{zz} is determined by Eq. (3.2) for $\gamma_1=\gamma_2$.

Formally, one can consider Eq. (3.3) as an equation of transfer of atoms with density w in a momentum space. Then the second term on the left-hand side of Eq. (3.3) corresponds to the convective transfer, the first term on the right-hand side corresponds to the diffusion proper with the effective diffusion coefficient D_{zz}, and the second term on the right-hand side of Eq. (3.3) plays the role of the linear source term. Note that the direction of the convective transfer is determined by the sign of the derivative $(\partial D_{zz}/\partial p_z)$, while the sign of the second derivative $(\partial^2D_{zz}/\partial p_z^2)$ determines the nature of the source term, viz. there is a source in point $p=p'$ at $(\partial^2D_{zz}/\partial p_z^2)_{p=p'}>0$ and a sink at $(\partial^2D_{zz}/\partial p_z^2)_{p=p'}<0$.

The characteristic shape of the dependence D_{zz} on momentum p_z of atoms is shown in Fig. 2. In the range of a momentum p_z close to zero we have $(\partial^2 D_{zz}/\partial p_z^2)>0$, and $(\partial D_{zz}/\partial p_z)<0$ at $p_z<0$ and $(\partial D_{zz}/\partial p_z)>0$ at $p_z>0$. At the same time, the value of diffusion is small for small values of momenta: $D_{zz}(p_z=0)=\hbar^2 k^2\gamma\Omega^2(\Gamma/\gamma)/G^2$ (since $\Gamma/\gamma \ll 1$; see Fig. 2 also). Therefore, at the initial stage of momentum distribution evolution, the influence of diffusion proper in the zero-momentum range is small. On the contrary, atoms are washed away from the range of momenta close to $|p_z^*|$ where the coefficient D_{zz} is maximum (see Fig. 2). As a result, there is a stream of atoms directed to $p_z=0$ in momentum space, giving rise to the rapid increase in the number of atoms with $p_z=0$ and to the narrowing of the momentum distribution. However, since $D_{zz}(p_z=0)\neq 0$ [note that $D_{zz}(p_z=0) \sim\Gamma/\gamma$], the diffusion proper becomes predominant for certain time and the momentum peak centered at $p_z=0$ is washed out.

Let us evaluate the characteristic time scales of source τ_s and diffusion τ_D in the range of small velocities of atoms. The value of parameter τ_s can be estimated as

$$\tau_s \approx \left[\left. \frac{\partial^2 D_{zz}}{\partial p_z^2} \right|_{p_z=0} \right]^{-1} \simeq G^4/(4\omega_R^2\gamma\Omega^2) , \tag{3.4a}$$

where $\omega_R=R/\hbar$ is the atomic recoil frequency. The time τ_D of diffusion is equal to $\tau_D \approx (\Delta p_z)^2/[D_{zz}(p_z=0)]$, where Δp_z is the characteristic momentum scale. In the case under consideration, the value of Δp_z is determined by the width Γ' of the narrow dip in the dependence $D_{zz}/(p_z)$ (Fig. 2): $\Delta p_z \simeq M\Gamma'/k \simeq 2M\Omega^2/\gamma k$ [15]. Hence

$$\begin{aligned} \tau_D &\approx (\Delta p_z)^2/[D_{zz}(p_z=0)] \\ &\simeq 2\Omega^2 G^2/(\omega_R^2\gamma^2\Gamma)=\tau_s 8\Omega^4/(G^2\Gamma\gamma) . \end{aligned} \tag{3.4b}$$

Deriving the formulas (3.4), we suppose $\Gamma \ll \gamma$ and $\Omega^2 \gg \Gamma\gamma$, $\Omega^2 \geq \Delta^2\Gamma/\gamma$. Therefore, $\tau_D \gg \tau_s$. So, for the cooling of, e.g., the sodium atoms on transitions $3S$-$3P$,

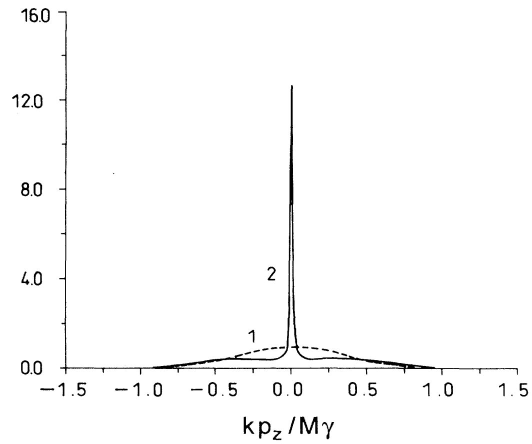

FIG. 3. Time evolution of the atomic momentum distribution w under the action of momentum diffusion. The width of initial distribution $\delta p_z(t=0)=M\gamma/k$ corresponds to the Doppler cooling. Other parameters are the same as in Fig. 2. 1, Initial distribution; 2, distribution after the interaction time $t=5\times10^{-4}$ sec.

we have $\tau_s \approx 10^{-5}$ sec, $\tau_D \approx 10^{-3}$ sec for $\Omega=0.1\gamma$, $\Gamma=10^{-3}\gamma$, $\gamma\approx 10$ MHz, and $\Delta=0$. Thus, on interaction times τ such that $\tau_s \leq \tau \ll \tau_D$ one can cool the Λ atoms due to momentum diffusion in the sense that the momentum distribution of atoms is strongly narrowed.

The picture of momentum distribution evolution described above is illustrated by Figs. 3 and 4, which were generated by solving numerically Eq. (3.3). A high narrow momentum peak centered at $p_z=0$ increases for times $t \leq t^*$ (Fig. 4). The height of the peak increases by 20 times, and the width decreases by 35 times compared to the initial distribution (for the parameters of Fig. 2). This leads to a momentum distribution having a width substantially lower than that determined by the Doppler limit of laser cooling. On the other hand, for times $t>t^*$ the distribution width increases and the height decreases slowly (at the scale of Fig. 4, these changes are so small that it would not be visible), i.e., the peak is washed out with the rate τ_D^{-1} ($\ll \tau_s^{-1}$).

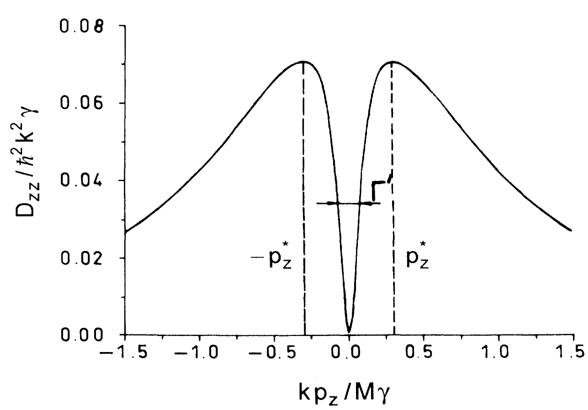

FIG. 2. Dependence of the momentum diffusion coefficient D_{zz} on the atomic momentum p_z in the case of Na ($\omega_R\approx 3\times10^{-3}\gamma$). The width of the narrow dip Γ' is equal to $M\Omega^2/\gamma k$ [15]. Rabi frequencies $\Omega_1=\Omega_2=0.3\gamma$, detunings $\Delta_1=\Delta_2=0$, and $\Gamma=10^{-3}\gamma$.

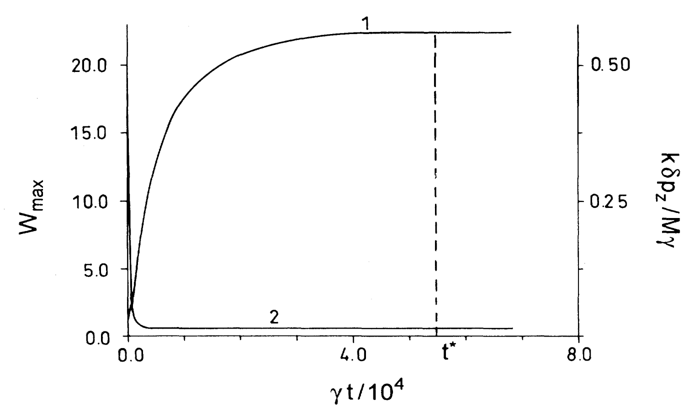

FIG. 4. Time evolution of the height w_{max} of the distribution peak of Fig. 3 (curve 1) and of width δp_z (curve 2) of the peak. Parameters are the same as in Fig. 2.

The value of time t^* and the minimal value of width $\delta p_{\min}$ are found from the analytical solution of FPE (3.3), which will be presented elsewhere [16]:

$$t^* = \Omega^4/(48\omega_R^2\gamma^2\Gamma) , \tag{3.5a}$$

$$\delta p_{\min} = M\Omega(\Gamma/2\gamma)^{1/2}/2k . \tag{3.5b}$$

Expressions for t^* and $\delta p_{\min}$ as well as for τ_S and τ_D show that the rate Γ of coherence of state $|\psi_{NC}\rangle$ determines both the character of the evolution and the limit of cooling by CPT.

2. Radiation force under CPT conditions

In the more general case, when $\gamma_1 \neq \gamma_2$, the Λ atom excited by counterpropagating light waves with $k_1 \simeq k_2$ is subjected to the special radiation force (3.1) ("CPT force") [3,9,10]. For a proper choice of interaction parameters, the CPT force improves considerably the efficiency of Λ atoms cooling due to the action of two factors. First, the force maintains atoms in the low-momentum region, leading to a constant pumping rate in the trap state $|\psi_{NC}\rangle$ by diffusion in the momentum space. Second, the CPT force causes a strong damping of atomic motion.

Consider the momentum dependence of radiation force (3.1) in Fig. 5 for different values of detunings Δ. Note that the force F_z is of constant sign in the whole range of momentum variation. The dependence includes a sharp narrow dip which is near the "resonance" momentum $p_0 = Mv_0$ determined by the condition (2.2). The minimal value of the force $F_z^{\min}$ at the bottom of the dip is small compared to the maximum force $F_z^{\max}$: $|F_z^{\min}| = |F_z(p_z=0)| = \hbar k|\eta|\Gamma \ll |F_z^{\max}| = \hbar k|\eta|\gamma$ (for $\Delta=0$). Such a structure of the force leads to a strong deceleration of atoms with $p_z<0$ at $\eta>0$ (or $p_z>0$ at $\eta<0$) and to acceleration of atoms with $p_z>0$ at $\eta>0$ (or $p_z<0$ at $\eta<0$) with a dynamical friction coefficient β (2.13) having an anomalously high value in the region of the dip.

As a result, an atomic momentum distribution is formed having a width much lower than that obtained by Doppler cooling. The peak is centered at a point near the bottom on the left (right) slope of dip at $\eta>0$ ($\eta<0$). At the same time, the so-generated narrow peak is shifted towards the bottom of the dip with a rate determined by $|F_z|$. The time evolution of momentum distribution of Na atoms under the action of force (3.1) is given in Fig. 6.

Sub-Doppler laser cooling of atoms taking into account the CPT force was studied for the first time in Ref. [9]. However, it was investigated only for the case of $\Delta=0$, when the center of the force dip is at the point $p_z=0$. Then the friction coefficient β is equal to zero at the point $p_z=0$ [see Eq. (4.5)]. Therefore, the only possibility in this case is the monochromatization of an atomic beam with nonzero average velocity $\langle v_z\rangle$.

When the detuning $|\Delta|$ increases, the center of the coherent trapping resonance is shifted to regions of positive momenta of atoms [see the inset of Fig. 5(a)]. Hence, here is $\beta(p_z=0)\neq 0$, permitting cooling of the atomic ensemble with $\langle v_z\rangle=0$. One can obtain an explicit expres-

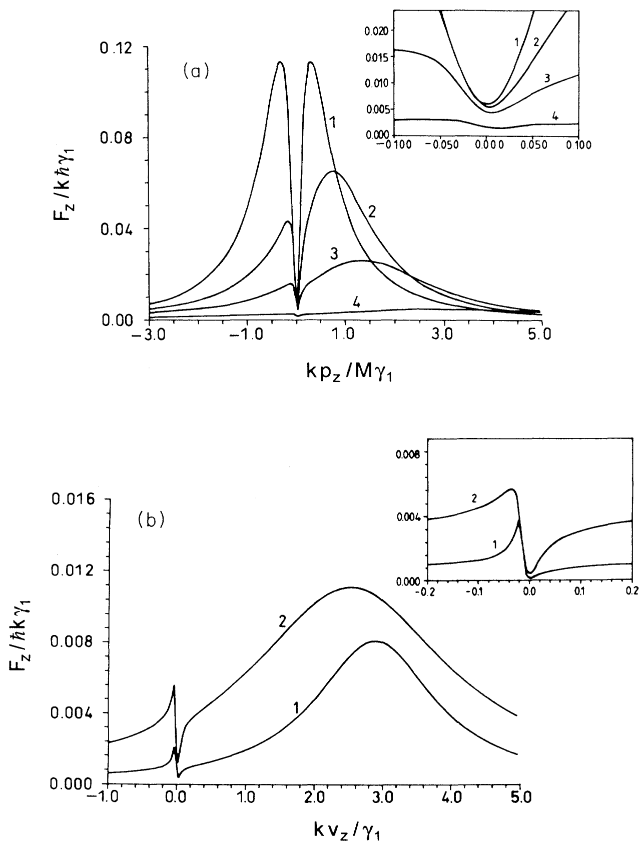

FIG. 5. Momentum dependence of the light pressure force affecting the Λ atom with $k_1=k_2$ (Na). (a) For different common detunings of light waves; 1, $\Delta=0$; 2, $\Delta=-\gamma_1$; 3, $\Delta=-2\gamma_1$; 4, $\Delta=-5\gamma_1$ with fixed laser intensity $\Omega_1=\Omega_2=0.3\gamma_1$, $\Gamma=10^{-2}\gamma_1$, $\eta=1.33$. (b) For different Rabi frequencies: 1, $\Omega_1=0.1\gamma_1$, $\Omega_2=0.3\gamma_1$; 2, $\Omega_1=0.2\gamma_1$, $\Omega_2=0.3\gamma$ with fixed common detuning $\Delta=-3\gamma_1$, $\Gamma=10^{-2}\gamma_1$, $\eta=1.33$. The inset shows the region of small momenta.

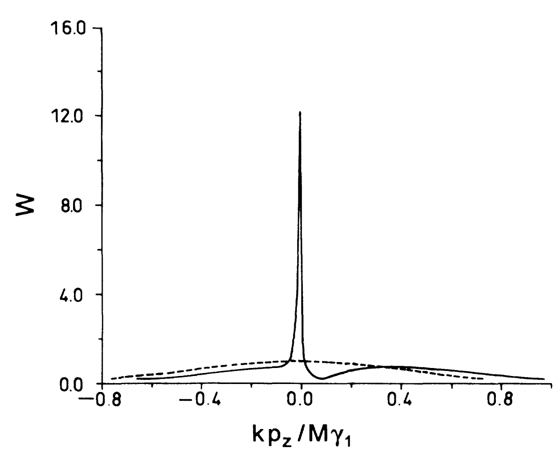

FIG. 6. Time evolution of the momentum distribution of Na atoms ($k_1=k_2$) under the action of the force F_z. 1, initial distribution with the width $\delta p_z(t=0)=M\gamma/k$ corresponding to the temperature T_D; 2, momentum distribution after the interaction time $t=3\times10^{-5}$ sec, $\Omega_1=\Omega_2=0.3\gamma_1$, $\Delta_1=\Delta_2=-2\gamma_1$, $\eta=1.33$, and $\Gamma=10^{-3}\gamma_1$.

sion for β at the point $p_z=0$ by expanding the force (3.1) into power series of the momentum p_z up to the first order: $F_z=F_z^0+\beta p_z$. Here the force $F_z^0=\hbar k\Gamma\eta\Omega^2/(2G^2)$ induces the motion of an atomic ensemble as a whole, the coefficient β is equal to

$$\beta=\omega_R\eta^2\Omega^2\Delta\Gamma/2G^4 , \tag{3.6}$$

and the detuning Δ must be negative in order to cool the Λ atoms. The time $\tau_{\mathrm{fr}}=|\beta|^{-1}$ is the characteristic time of momentum distribution narrowing due to friction force.

The time scales of the evolution under the joint action of radiation force and momentum diffusion can be estimated in the same way as in the case of pure diffusion cooling (see the previous section). Writing the FPE in the form analogous to Eq. (3.3), we obtain the following:

(i) The time of effective source action (at point $p=0$)

$$\tau_s\simeq\left[-\frac{\partial F_z}{\partial p_z}+\frac{\partial^2 D_{zz}}{\partial p_z^2}\bigg|_{p_z=0}\right]^{-1}$$
$$\simeq\frac{G^4}{\omega_R^2\gamma\Omega^2}[1+\eta^2(\Gamma/8\gamma)(\Delta^2/G^2+|\Delta|/\omega_R)]^{-1} , \tag{3.7a}$$

(ii) The time of diffusion

$$\tau_D\simeq(\Delta p_z)^2(D_{zz}|_{p=0})^{-1}\simeq\frac{2\Omega^2G^2}{\omega_R^2\gamma^2\Gamma} , \tag{3.7b}$$

(iii) The time of "convective force" action (or time of drift of the whole atomic ensemble)

$$\tau_{\mathrm{CF}}\simeq(\Delta p_z)\left[F_z-\frac{\partial D_{zz}}{\partial p_z}\bigg|_{p_z=0}\right]^{-1}$$
$$\simeq\frac{4G^2}{\omega_R\gamma\Gamma|\eta|}(1+\omega_R|\Delta|/G^2)^{-1} . \tag{3.7c}$$

At $\Gamma\ll\gamma$, $\Omega^2\gg\Gamma\gamma$, and $\Omega^2\geq\Delta^2\Gamma/\gamma$, the time of peak formation is much smaller than the times of its washing out and drift: $\tau_S\ll\tau_D,\tau_{\mathrm{CF}}$. For example, we have $\tau_S\simeq5\times10^{-6}$ sec, $\tau_D\simeq10^{-3}$ sec, and $\tau_{\mathrm{CF}}\simeq10^{-4}$ sec for Na atoms $|\Delta|=|\Delta_{\mathrm{opt}}|=(2\Omega^2\gamma/\Gamma)^{1/2}\simeq4.5\gamma$ (such a choice of $|\Delta|$ will be clear from the following discussion), for $\Omega=0.1\gamma$, $\Gamma=10^{-3}\gamma$, $\eta=1.33$, $\gamma\simeq10$ MHz, and $\omega_R\simeq25$ kHz. Thus, in an interaction time t, $\tau_S\leq t\leq\tau_{\mathrm{CF}},\tau_D$ one can obtain a very narrow momentum distribution.

We emphasize that the momentum distribution does not become a δ function when t becomes infinite because of the incompleteness of population trapping in $|\psi_{\mathrm{NC}}\rangle$, which is characterized by the parameter Γ/γ. Note that the expressions for the "convective force" $(F_z-\partial D_{zz}/\partial p_z)$ at point $p_z=0$, as well as for the diffusion coefficient D_{zz} at point $p_z=0$, contain the factor Γ/γ. So, the smaller Γ/γ, the smaller the "convective force" and diffusion coefficient, and the larger the times τ_{CF} and τ_D. At the same time, there is inverse dependence on Γ/γ for the source term (though the dependence is much weaker than for "convective force" and diffusion proper), i.e., the smaller Γ/γ, the more insensitive the source and the smaller τ_S. It is also important to note that the existence of radiation force F_z improves the efficiency of momentum distribution narrowing compared to cooling due only to diffusion.

Let us obtain the temperature T of atoms cooled by CPT, substituting expressions (3.2) and (3.6) into the formula (2.12):

$$T=T_D\frac{2(\Delta^2\Gamma/2\gamma+\Omega^2)}{\eta^2\gamma|\Delta|} . \tag{3.8}$$

The dependence of T on the detuning $|\Delta|$ is depicted in Fig. 7. It can be seen that for large values of $|\Delta|$ the temperature of the atoms increases slightly linearly, while for $|\Delta|\to0$ the temperature rises rapidly due to the fact that friction near the zero momenta is absent for $\Delta=0$.

As follows from (3.8), there exists a wide region of detunings where T is much less than the Doppler limit T_D. The minimum value of temperature

$$T_{\min}=T_D\frac{4\Omega}{\eta^2\gamma}(\Gamma/2\gamma)^{1/2} \tag{3.9}$$

is achieved at a detuning

$$|\Delta_{\mathrm{opt}}|=(2\Omega^2\gamma/\Gamma)^{1/2} .$$

For example, for Na atoms cooled by CPT on transitions $3S$-$3P$, the minimum temperature $T_{\min}$ is equal to $T_{\min}\simeq3\times10^{-6}$ K at a detuning $|\Delta_{\mathrm{opt}}|\simeq4.5\gamma$ for $\Omega=0.1\gamma$, $\eta=1.33$, $\Gamma=10^{-3}\gamma$, and $\gamma\simeq10$ MHz (while $T_D\simeq240$ μK and $T_R\simeq1.2$ μK). Thus, the joint action of diffusion and friction force under CPT conditions is shown to produce the deep cooling of atoms.

As regards the expressions (3.8) and (3.9), we remind the reader that they are obtained in a quasiclassical approach and are valid only in the range $T>T_R$. This circumstance limits the values of laser field intensities and the value of Γ [see condition (2.7)].

B. Cooling of Λ atoms with $k_1\neq k_2$

Here we discuss Λ atoms with substantially different wave numbers of the applied fields: $k_1\neq k_2$. Remember

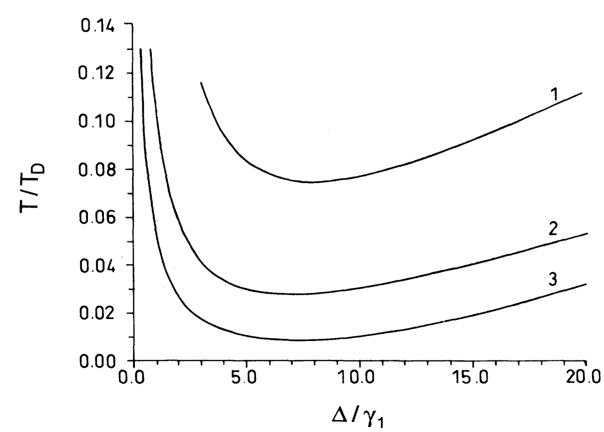

FIG. 7. Dependence of the temperature T of the cold Na atoms on the common detunings $|\Delta|$ for the following: 1, $\Omega_1=\Omega_2=0.3\gamma_1$; 2, $\Omega_1=0.2\gamma_1$, $\Omega_2=0.3\gamma_1$; 3, $\Omega_1=0.1\gamma_1$, $\Omega_2=0.3\gamma_1$. $\Gamma=10^{-2}\gamma_1$ and $\eta=1.33$.

that the velocity-selective CPT phenomenon appears in a Λ system for arbitrary wave numbers k_1 and k_2. Our general conclusions obtained in Sec. III A concerning the role of the parameter Γ/γ are valid in the case of $k_1 \neq k_2$ as well.

Let us discuss the case of double resonance in a Λ atom (Fig. 1), where the wave numbers of the optical waves are substantially different: $k_1 \neq k_2$. This excitation scheme can be realized, for example, in the thallium atom, where the states $|1\rangle$, $|2\rangle$, and $|3\rangle$ are the fine-structure states $6^2P_{1/2}$, $6^2P_{3/2}$, and $7^2S_{1/2}$, respectively [17]. One has to excite such a Λ system by two laser waves with $k_1 = 1.68 \times 10^5$ cm^{-1} and $k_2 = 1.18 \times 10^5$ cm^{-1}.

Since $k_1 \neq k_2$, from (2.2) it follows that the CPT is velocity selective both in the case of counterpropagating waves $\mathbf{k}_1 \uparrow \downarrow \mathbf{k}_2$ as well as in the case of copropagating waves $\mathbf{k}_1 \uparrow \uparrow \mathbf{k}_2$. This fact allows one to carry out the transversal cooling (collimation) of an atomic beam in the field of two copropagating light waves (note that this is not possible for $k_1 = k_2$).

By using the technique discussed in Sec. II, we obtain both the light pressure force F_z and the z component D_{zz} of the momentum diffusion tensor, for example of copropagating waves:

$$F_z = \hbar(k_1\gamma_1 + k_2\gamma_2)2\Omega^2 aL^{-1} , \tag{3.10}$$

$$D_{zz} = \hbar^2(k_1^2 + k_2^2)2\gamma\Omega^2 aL^{-1} , \tag{3.11}$$

where

$$2\gamma = \gamma_1 + \gamma_2 ,$$

$$a = 2\epsilon\Omega^2 + x^2 ,$$

$$L = \sum_{m=0}^{4} c_m x^m ,$$

$$x = qv_z , \quad q = k_1 - k_2 ,$$

$$c_0 = 8\gamma\Omega^2 G^2 ,$$

$$c_1 = 2\gamma\Omega^2\Delta\eta ,$$

$$c_2 = 2\gamma(\gamma^2 + \Delta^2) + 2\Omega^2(k_1\gamma_2 - k_2\gamma_1)/q ,$$

$$c_3 = -2\Delta(k_1\gamma_1 + k_2\gamma_2)/q ,$$

$$c_4 = (k_1^2\gamma_1 + k_2^2\gamma_2)/q^2 ,$$

$$\epsilon = \Gamma/\gamma , \quad \eta = (\gamma_1 - \gamma_2)/\gamma .$$

Here we suppose for simplicity that the Rabi frequencies $\Omega_1 = \Omega_2 \equiv \Omega$ as well as the detunings $\Delta_1 = \Delta_2 \equiv \Delta$ are the same for both transitions in the Λ atom. Moreover, we assume that $\epsilon \ll 1$ and $\Omega^2 \gg \Gamma\gamma$.

On the whole, the momentum dependencies of the force F_z (3.10) and of the diffusion coefficient D_{zz} (3.11) only differ weakly from the ones examined in Sec. III A. Owing to such a shape of force and diffusion, the narrow momentum distribution of atoms is formed in the zero-momentum region. The effectiveness of laser cooling by CPT can be evaluated as before by the temperature (2.12). In the case under consideration we have

$$T = T_D \frac{2(k_1^2 + k_2^2)(2\Omega^2 + \Delta^2\Gamma/\gamma)}{(k_1\gamma_1 + k_2\gamma_2)|\Delta\eta q|} . \tag{3.12}$$

The dependence of the temperature on the detuning in the case of $k_1 \neq k_2$ is analogous to that for $k_1 = k_2$. It is characteristic that here a temperature below the Doppler limit T_D can be obtained. The minimum value $T_{\min}$ is reached for a detuning $|\Delta_{\mathrm{opt}}| = \Omega(2\gamma/\Gamma)^{1/2}$:

$$T_{\min} = T_D \frac{8(k_1^2 + k_2^2)\Omega}{(k_1\gamma_1 + k_2\gamma_2)|\eta q|} (\Gamma/2\gamma)^{1/2} . \tag{3.13}$$

In the case of Tl atoms, $T_{\min} \simeq 30$ μK at $|\Delta_{\mathrm{opt}}| = 5\gamma$ for $\Omega = 5 \times 10^{-2}\gamma$, $\Gamma = 2 \times 10^{-4}\gamma$, $\gamma_1 = 62.5$ MHz, and $\gamma_2 = 70.5$ MHz ($T_D = 3150$ μK, $T_{R1} \simeq 0.33$ μK, and $T_{R2} \simeq 0.17$ μK). We note finally that for the systems with $k_1 \neq k_2$, the longitudinal deceleration of atomic beams using a chirping scheme is possible.

IV. DISCUSSION

We have considered two simple excitation schemes of atoms for which laser cooling by CPT can be used. It was shown that these schemes permit the cooling to effective one-dimensional temperatures substantially below the Dopler limit. The same is valid for cooling by use of CPT in three-level atoms with double radio-optical resonance [5]. This fact opens the prospects of sub-Doppler laser cooling for a wide class of quantum objects, viz. for atoms and ions of alkali metals K, Na, Rb, and Cs, alkaline-earth metals Zn, Cd, and Hg [5], as well as for elements Br, Tl, Ba, etc., and for some types of molecules.

Note also the possibility of using the CPT mechanism as a very effective tool to master the translation of atoms. There are many physical applications of laser cooling by CPT, such as collimation and compression as well as longitudinal deceleration of an atomic beam, localization and channeling in a standing light wave [5,10]. The experimental realization of the application is in progress.

The technique discussed above is the one-dimensional cooling. However, it is possible to carry out laser cooling by CPT in two and three dimensions [2,3,8,18]. In spite of more complicated atomic and laser configurations then those we have considered above, the physical mechanism is the same. At the same time, the complexity of the configurations leads to an excessive complexity of theoretical description of cooling dynamics. We believe, therefore, that the quasiclassical approach presented here is rather useful just for two- and three-dimensional cooling by CPT. We also suppose that some results of our study (such as the effect of degree Γ/γ of incompleteness of CPT on the cooling dynamics) are valid in two- and three-dimensional cooling, too.

V. CONCLUSIONS

We have presented the quasiclassical theory of laser cooling by velocity-selective coherent population trapping. The process of laser cooling by CPT is based on accumulation of atoms in the state $|\psi_{\mathrm{NC}}\rangle$ noninteracting with the exciting field, which leads to their localization in strictly defined regions of a momentum space. We have shown that the dynamics of accumulation of atoms in $|\psi_{NC}\rangle$ can be described by the Fokker-Planck equation as being due to the two processes: (i) the redistribution in the momentum space by means of diffusion, and (ii) the narrowing of momentum distribution under the action of radiation friction force (in the case of asymmetrical schemes). These two supplement each other, providing effective cooling down to superlow temperatures. This is in contrast to other schemes of laser cooling, such as Doppler cooling, stimulated molasses, and polarization gradient cooling, where momentum diffusion plays a destructive role, heating an atomic ensemble at any time.

Quasiclassical description has allowed us to find the analytical expressions for the temperature of cooled atoms as well as to investigate the time scales of atomic evolution. It was shown that the incompleteness of CPT influences considerably the cooling dynamics, leading to the appearance of a "damping" stage of the evolution (when primarily formed narrow momentum distribution is washed out) and, in general, to the imperfection of cooling by CPT. Both characteristic time scales and temperature are determined by the degree Γ/γ of incompleteness of CPT.

APPENDIX

We give here the common expression for the light pressure force $\mathbf{F}$ and for the momentum diffusion coefficient D_{zz} in a Λ system when the Rabi frequencies Ω_m, the detunings Δ_m, the wave vectors $\mathbf{k}_m$, the rate Γ of the transverse relaxation, and the mixing rate γ_0 between the levels $|1\rangle$ and $|2\rangle$ are arbitrary:

$$\mathbf{F}=\hbar(\mathbf{k}_1\gamma_1+\mathbf{k}_2\gamma_2)\rho_{33} ,$$

$$D_{zz}=\hbar^2(k_1^2+k_2^2)\gamma\rho_{33} ,$$

$$\rho_{33}=A/B ,$$

$$A=\gamma\gamma_0a_0(\Omega_1^2a_2+\Omega_2^2a_1-2\Omega_1\Omega_2b)+2\Omega_1^2\Omega_2^2a_0^2\gamma^2 ,$$

$$B=2\gamma\gamma_0d_0+\gamma a_0[\Omega_1^2(\gamma_2+3\gamma_0)a_2+\Omega_2^2(\gamma_1+3\gamma_0)a_1 +2\Omega_1\Omega_2(\gamma-3\gamma_0)b]+6\Omega_1^2\Omega_2^2a_0^2\gamma^2 ,$$

$$d_0=a_1a_2-b^2 ,$$

$$a_1=a_0(\gamma^2+\alpha_1^2)+\Omega_2^2[\Omega_0^2+\epsilon(\gamma^2-\alpha_1^2)-2\alpha\alpha_1] ,$$

$$a_2=a_0(\gamma^2+\alpha_2^2)+\Omega_1^2[\Omega_0^2+\epsilon(\gamma^2-\alpha_2^2)+2\alpha\alpha_2] ,$$

$$b=\Omega_1\Omega_2[\Omega_0^2+\epsilon(\gamma^2+\alpha_1\alpha_2)-\alpha^2] ,$$

$$a_0=\epsilon\Omega_0^2+\alpha^2+\epsilon^2\gamma^2 ,$$

$$\alpha_1=\Delta_1-\mathbf{k}_1\mathbf{v} ,$$

$$\alpha_2=\Delta_2-\mathbf{k}_2\mathbf{v} ,$$

$$\alpha=\alpha_1-\alpha_2 ,$$

$$\Omega_0^2=\Omega_1^2+\Omega_2^2 ,$$

$$2\gamma=\gamma_1+\gamma_2 ,$$

$$\epsilon=\Gamma/\gamma .$$

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