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Logical reversibility in quantum-nondemolition measurements

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We show that it is possible to make a logically reversible quantum-nondemolition measurement of an optical-field photon number, where the measurement preserves all the information contained in the premeasurement wave function. The nonunitary dynamics of the proposed scheme is analyzed using a Monte Carlo wave-function approach.

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It is generally believed that irreversibility is an inherent property of quantum measurements. An implication of this property is that the premeasurement density operator cannot be reproduced from that of the postmeasurement, as some information about the system is lost during the measurement process itself. In a recent paper, however, Ueda and Kitagawa [1] have shown that the nonunitary evolution during a quantum measurement does not necessarily imply "logical irreversibility," i.e., a loss of information about the system that is being measured. A measurement can be logically reversible if it is unsharp and sensitive to the vacuum field fluctuations. The first requirement implies that the measurement should leave some (arbitrarily small) uncertainty in the quantity that is being measured. Ueda and Kitagawa [1] have analyzed a quantum counter and shown that the information of the initial density matrix is preserved during the counting and, in principle, can be extracted. Such a counter, however, does not measure a conserved quantity. It is therefore not clear from their work whether a quantum measurement that does not alter the measured observable could be reversible.

In this paper, we show that it is possible to make a logically reversible quantum-nondemolition (QND) measurement of an optical- (coupling) field photon number using a *nonunitary* atom-field interaction. The proposed measurement system is based on an electromagnetically induced transparency [2–4], where the absorption (scattering) rate of a probe field is a function of the applied coupling electromagnetic field intensity. The system is sensitive to vacuum field fluctuations [5] and performs an unsharp QND measurement of this coupling field.

We analyze the QND measurement scheme using the wave-function approach to dissipative quantum systems [6,7], and show that each atomic scattering event (and absence of it) results in a "partial collapse" of the coupling-field wave function. If the scattered photons are actually detected, then the QND measurement is logically reversible. The measurement gives us photon-number information, without destroying the coherence terms (off-diagonal elements of the initial density matrix in the photon-number basis). Unlike Ref. [1], the scheme presented here measures a photon-number operator, which in this case, is a conserved quantity. We therefore show that it is possible to make repetitive (logically reversible) measurements of this observable.

Figure 1 shows a three-state closed Λ system that exhibits coherent population trapping [2] at the two-photon resonance ($\omega_2+\omega_c=\omega_1+\omega_p$). The metastable state $|2\rangle$ is coupled to an upper state $|3\rangle$ by the coupling field at ω_c. We will assume that the interaction takes place inside a lossless optical ring cavity, which is tuned to ω_c ($\approx\omega_3-\omega_2$). In the absence of the probe field, the upper states $|2\rangle$ and $|3\rangle$ are not populated. It is assumed that only the spontaneous decay on the $|3\rangle$-$|1\rangle$ transition (Γ_{31}) is significant in the time scale of the measurement. In the interaction region, the atomic beam is perpendicular to the coupling and probe fields. To the extent that the incoherent rates in and out of state $|2\rangle$ and the cavity decay rate for the coupling-field mode ω_c are zero, the coupling-field photon number does not experience gain or loss. The probe-field intensity, however, is not a constant of motion, as the atomic medium scatters the probe photons. The scattering rate, or equivalently, the (weakly coupled) probe-field transmission coefficient through the medium, is a (different) function of n_c for (almost) any given probe frequency.

The basic idea behind the quantum-nondemolition measurements is to be able to monitor an observable without perturbing it; that is, to eliminate the back-action noise usually associated with a quantum measure-

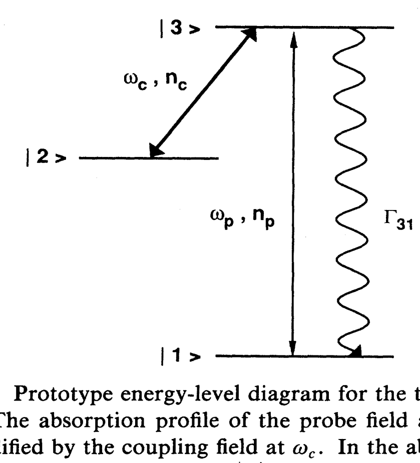

FIG. 1. Prototype energy-level diagram for the three-state Λ system. The absorption profile of the probe field at frequency ω_p is modified by the coupling field at ω_c. In the absence of incoherent rates in and out of state $|2\rangle$, the coupling field does not experience gain or loss.

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ment [8]. Several QND schemes have been proposed [8–10] and demonstrated [11]. The proposals for QND measurements in the optical domain are generally based on a unitary nonresonant nonlinear interaction [9] and can only be used to measure a large number of photons. The system described in this Rapid Communication is ideally lossless for the field to be measured and allows one to use *resonant* interactions: Optical photon numbers down to (and including) $n_c=0$ can be measured with the proposed scheme [10]. To the best of our knowledge, this is the first example of a QND scheme that utilizes dissipative atom-field coupling.

Our physical model is based on the recently developed treatment of the system-reservoir interactions using Monte Carlo wave functions (MCWF's) [6]. In a recent paper, Dalibard, Castin, and Molmer [7] have shown that a non-Hermitian evolution for an atomic wave function combined with the "gedanken measurements" of the photon-emission events is equivalent to the density-matrix approach that is commonly used to describe the dissipative processes. We will use a simple extension of the model presented in Ref. [7], where the "system" consists of the coupling field, the atom(s), and the probe field. We will in addition assume that the scattered photons are actually measured by a fast detector.

The effective Hamiltonian for the coupling-field–atom(s)–probe-field system of Fig. 1 in the interaction picture is

$$\hat{H}_{\rm eff}=i\hbar g_p(\hat{a}_p^\dagger\hat{\sigma}_{13}-\hat{\sigma}_{31}\hat{a}_p)+i\hbar g_c(\hat{a}_c^\dagger\hat{\sigma}_{23}-\hat{\sigma}_{32}\hat{a}_c) -(i/2)\hbar\Gamma_{31}\hat{\sigma}_{33}\ . \quad (1)$$

Here, $\hat{\sigma}_{nm}(0)=|n\rangle\langle m|$ $(m,n=1,2,3)$. g_c and g_p denote the (real) coupling coefficients of the coupling and probe modes, respectively [12]. This effective, non-Hermitian interaction Hamiltonian neglects the coupling-field–reservoir coupling (i.e., lossless cavity) and assumes that the radiation field reservoir is in the vacuum state $|0\rangle$. In the analysis, we also assume that the probe field is in a number state, as this assumption simplifies the expressions.

We consider the time evolution of the system in the interaction picture when the coupling field is initially in a superposition of photon-number states

$$|\varphi_c\rangle=\sum_{n_c}c_{n_c}|n_c\rangle\ . \quad (2a)$$

We assume that a probe field in a photon-number state $|n_p\rangle$ is turned on in a time that is short compared to the radiative decay time, i.e., no scattering takes place during the turn-on. We note again that this assumption is introduced for simplicity, and is not a requirement for the properties to be obtained. The combined atom-field-reservoir (AFR) wave function at time t is then

$$|\Psi_{\rm AFR}(t)\rangle=\sum_{n_c}c_{n_c}[a_{1n_c}(t)|1,n_c,n_p\rangle+a_{2n_c}(t)|2,n_c+1,n_p-1\rangle +a_{3n_c}(t)|3,n_c,n_p-1\rangle]\otimes|0\rangle\ . \quad (2b)$$

Following Dalibard, Castin, and Molmer [7], we evolve the system in time until $t+dt$, where we check whether a spontaneous-emission (or scattering) event has occurred. Here, dt is taken to be much smaller than all the relevant time scales and reciprocal detunings. Just before the measurement, the state of the combined system-reservoir unit is [7]

$$|\Psi_{\rm AFR}(t+dt)\rangle=|\Psi^{(0)}(t+dt)\rangle+|\Psi^{(1)}(t+dt)\rangle\ , \quad (3a)$$

where

$$|\Psi^{(0)}(t+dt)\rangle=\sum_{n_c}c_{n_c}[a_{1n_c}(t+dt)|1,n_c,n_p\rangle +a_{2n_c}(t+dt)|2,n_c+1,n_p-1\rangle +a_{3n_c}(t+dt)|3,n_c,n_p-1\rangle]\otimes|0\rangle\ , \quad (3b)$$

$$|\Psi^{(1)}(t+dt)\rangle=\sum_{\mathbf{k},n_c}c_{n_c}\beta_{\mathbf{k}n_c}(t+dt)|1,n_c,n_p-1\rangle\otimes|1_{\mathbf{k}}\rangle\ . \quad (3c)$$

Here, $\beta_{\mathbf{k}n_c}(t+dt)$ denotes the probability amplitude of the state $|1,n_c,n_p-1\rangle\otimes|1_{\mathbf{k}}\rangle$ and $\mathbf{k}$ denotes the wave vector and the polarization of the reservoir mode. The time evolution of the probability amplitudes a_{in_c} $(i=1,2,3)$ is given by the $\hat{H}_{\rm eff}$ of Eq. (1).

Before proceeding with the general scheme, we first consider the case where the coupling field is initially in a photon-number state, i.e., $c_{n_c}=\delta_{n_c,n_{ci}}$. At time $t+dt$, the measurement of the emitted photons is introduced: The probability dp of a spontaneous emission occurring in the time interval $(t,t+dt)$ is compared to a random number ϵ that is uniformly distributed between 0 and 1. Here, $dp=\langle\Psi^{(1)}|\Psi^{(1)}\rangle=\Gamma_{31}dt|a_3(t)|^2$. If $\epsilon>dp$, then we assume that a spontaneous photon is not emitted, and the state of the system reservoir is projected into

$$|\Psi_{\rm AFR}(t+dt^+)\rangle=(1/\sqrt{1-dp})[a_1(t+dt)|1,n_{ci},n_p\rangle +a_2(t+dt)|2,n_{ci}+1,n_p-1\rangle +a_3(t+dt)|3,n_{ci},n_p-1\rangle]\otimes|0\rangle\ . \quad (4a)$$

If $\epsilon<dp$, then a photon is assumed to be emitted (and/or detected) at $t+dt$, and we get

$$|\Psi_{\rm AFR}(t+dt^+)\rangle=|1,n_{ci},n_p-1\rangle\otimes|0\rangle\ . \quad (4b)$$

The physical implication of Eq. (4) is that spontaneous emission (scattering) of a photon always collapses the wave function of the coupling field into the original photon-number state $|n_{ci}\rangle$. Failure to emit a spontaneous photon preserves the superposition created by the resonant atom-field interactions. Although this superposition has a nonzero amplitude in the $|2,n_{ci}+1,n_p-1\rangle$ state, the coupling field is never projected into a state other than $|n_{ci}\rangle$. This is another way of saying that the changes in the coupling-field photon number are *virtual;* that is, they do not imply a loss or gain to the coupling field. Provided that the interactions with the probe field are turned on and off adiabatically, all the atomic population is returned to state $|1\rangle$, and the state of the coupling field before and after the measurement is unchanged. Moreover, as we shall see, the scattering events carry information about the state $|n_c\rangle$, as the rate at which they occur is in general a unique function of (n_c+1). We therefore claim that the system performs a QND measurement as (i) the coupling-field wave function is always projected into the initial photon-number state, and (ii) the scattering rate carries information about n_c.

We now consider the evolution of the system when the coupling field is in an arbitrary superposition state [Eq. (2a)]. First, we assume that the result of the measurement at $t+dt$ is that no scattering took place. The postmeasurement wave function in this case is given by

$$|\Psi_{\rm AFR}(t+dt^+)\rangle=(1/\sqrt{1-dp})|\Psi^{(0)}(t+dt)\rangle , \quad (5)$$

where

$$dp=\langle\Psi^{(1)}|\Psi^{(1)}\rangle=\Gamma_{31}dt\sum_{n_c}|c_{n_c}|^2|a_{3n_c}(t)|^2$$

and $|\Psi^{(0)}(t+dt)\rangle$ is as given in Eq. (3b).

There are two implications of Eq. (5): (i) The system remains in a pure state following the measurement; (ii) the "no scattering" result of the measurement leads to a redistribution of the probability amplitudes for each atom-field state that constitutes the superposition. In order to see how the result of the measurement modifies the state of the coupling field, we could hypothetically assume that the probe field is turned off adiabatically right after the measurement. In this limit, all the amplitude in the upper states $|2\rangle$ and $|3\rangle$ is returned to lower state $|1\rangle$ and the magnitudes of the postmeasurement probability amplitudes c'_{n_c} are given by

$$|c'_{n_c}|=|c_{n_c}|\frac{[|a_{1n_c}(t+dt)|^2+|a_{2n_c}(t+dt)|^2+|a_{3n_c}(t+dt)|^2]^{1/2}}{\sqrt{1-dp}} \quad (6)$$

Equation (6) implies that the amplitudes of the coupling-field states $|n_c\rangle$ corresponding to low-loss (i.e., low scattering rate) atom-field states are enhanced at the expense of the states that create high loss to the probe field. This can be considered as a redistribution or partial collapse of the coupling-field wave function. This redistribution, however, is reversible and no information is lost, as the coherence terms are preserved [Eq. (5)].

We now consider the case when a scattering or spontaneous emission takes place in the time interval $(t,t+dt)$. The state immediately after the measurement is

$$|\Psi_{\rm AFR}(t+dt^+)\rangle=(1/\sqrt{dp})|\Psi^{(1)}(t+dt)\rangle , \quad (7)$$

where $|\Psi^{(1)}(t+dt)\rangle$ is as given in Eq. (3c). The state of the system after the detection could be found by tracing the density operator over the reservoir states and by assuming that the scattered photon is destroyed during the measurement. The wave function of the (uncoupled) coupling field after the measurement is given by

$$|\varphi_c(t+dt^+)\rangle=\sum_{n_c}c_{n_c}[a_{3n_c}(t+dt)\sqrt{\Gamma_{31}dt}/\sqrt{dp}]|n_c\rangle . \quad (8)$$

Therefore, the coupling field remains in a pure state and the measurement *modifies the probability amplitudes* so as to favor the high-loss states without destroying the coherence terms. We keep track of all the relevant information, and as a result the system performs a *logically reversible QND measurement.* This is the principal result of this Rapid Communication. In Eq. (8), the postmeasurement state vector (in the photon-number basis) of the coupling field is obtained from the initial one by a diagonal matrix, whose entries $[a_{3n_c}(t+dt)\sqrt{\Gamma_{31}dt}/\sqrt{dp}]$ are nonzero and uniquely determined by system parameters for each n_c. By inverting this matrix, we can obtain the premeasurement state vector from that of the postmeasurement. The unsharp photon-number measurement described here gives information about the magnitude of the initial probability amplitudes, without destroying the information on their relative phase.

We have so far described the short-time dynamics of the measurement scheme, using the formalism developed by Dalibard, Castin, and Molmer [7]. We have seen that each scattering event, or the absence of it, redistributes the probability amplitudes of the coupling field in the photon-number basis [13]. The property that causes the distinction between different photon-number states is given by how much loss they create to the probe field, or equivalently the scattering rate that they determine. Individual photoemission events, when considered alone, perform a (very) unsharp measurement of the coupling-field photon number. The interplay of a large number of scattering and/or no-scattering events, however, eventually "collapses" the wave function into a certain number state [14]. Figure 2 shows the result of a sample computer simulation that verifies this result: In this run with $\Gamma_{31}=g_c=\omega_{31}-\omega_p$, $\omega_{32}=\omega_c$, and $g_p\sqrt{\langle n_p\rangle}=0.32\Gamma_{31}$, we observed that after 170 scattering events (occuring in 2500 Γ_{31}^{-1}), the initially Poissonian distribution (with $\langle n_c\rangle=4$) of the coupling field collapsed into the $n_c=3$ state with $|c_3|^2=0.999$ [14].

The extension of the analysis for a probe field in a superposition of photon-number states (i.e., coherent state) is lengthy but straightforward. In the assumed traveling probe-field geometry, the reversible measurement is complete when the transmitted probe photon number is also measured along with the scattered photons [14].

Although the actual collapse is caused by the scattering events, (partial) information on the photon-number distribution can be gathered by measuring the absorption

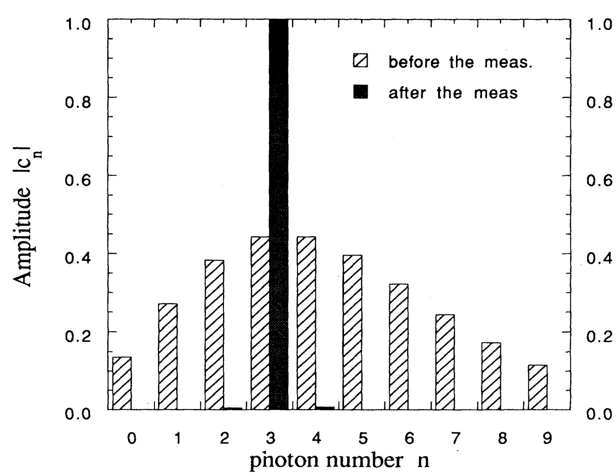

FIG. 2. Numerical simulation of the measurement process. The parameters used are $\Gamma_{31}=g_c=\omega_{31}-\omega_p$, $\omega_{32}=\omega_c$, and $g_p\sqrt{\langle n_p\rangle}=0.32\Gamma_{31}$. Initially, the distribution is Poissonian with $\langle n_c\rangle=4$. After 170 scattering events, the probability for the state $n_c=3$ state is $|c_3|^2=0.999$.

(transmission) of the probe field through the medium, in a given time interval. As the information on the photon emission times is lost, this measurement alone is not sufficient to obtain logical reversibility. After tracing over the reservoir parameters, we obtain the following equation for the time evolution of the reservoir averaged probe-field number operator $\hat{n}_p(t)$ in the weak-probe-coupling limit ($\langle\hat{\sigma}_{11}\rangle \approx \langle\hat{\sigma}_{11}+\hat{\sigma}_{22}+\hat{\sigma}_{33}\rangle$):

$$d\hat{n}_p/dt = -\hat{\kappa}_{\rm abs}(\omega_p,\hat{n}_c)\hat{\sigma}_{11}\hat{n}_p \ , \tag{9a}$$

$$\hat{\kappa}_{\rm abs}(\omega_p,\hat{n}_c) = g_p^2 {\rm Re}\{[\hat{\mathbb{1}}+g_c^2\hat{a}_c\hat{a}_c^\dagger]^{-1}[2i\Delta\omega_{21}\hat{\mathbb{1}}]\} \ , \tag{9b}$$

where $\Delta=(\Gamma_{31}/2)+[i\Delta\omega_{31}][i\Delta\omega_{21}]$, $\Delta\omega_{21}=\omega_{21}+\omega_c-\omega_p$, and $\Delta\omega_{31}=\omega_{31}-\omega_p$. Equation (9) states that the probe field experiences single-photon loss, which is modified by the presence of a coherence between the (bare) states $|1,n_c\rangle$ and $|2,n_c+1\rangle$, induced by the coupling field [2]. The eigenstates of the (scattering) operator $\hat{\kappa}(\omega_p,\hat{n}_c)$ are the same as those of $\hat{a}_c\hat{a}_c^\dagger(|n_c\rangle)$, with eigenvalues $\kappa(\omega_p,n_c)=\langle n_c|\hat{\kappa}(\omega_p,\hat{n}_c)|n_c\rangle$ [14]. Therefore, by detecting $\hat{\kappa}(\omega_p,\hat{n}_c)$ we measure the operator $\hat{n}_c+1$. The fact that the atom-field system is in a superposition of $|n_c\rangle$ and $|n_c+1\rangle$ states does not cause any error in measurement, provided that the upper atomic-state population (in states $|2\rangle$ and $|3\rangle$ combined) does not exceed unity. Moreover, the operator $\hat{\kappa}(\omega_p,\hat{n}_c)$ is sensitive to the vacuum field, as a result of the vacuum Rabi splitting [5,12].

The photon-number operator $\hat{n}_c$ under assumed resonant excitation conditions (Fig. 1) does not satisfy $[\hat{n}_c(t),\hat{H}(t)]=0$. The entangled atom-field system, as we have seen, evolves into a superposition of $|1,n_c\rangle$ and $|2,n_c+1\rangle$ states. As each scattering event reprojects the entangled state into $|1,n_c\rangle$, the photon-number operator $\hat{n}_c(t)$ is not a constant of motion. The changes in $\hat{n}_c(t)$, however, are *virtual* and the coupling-field mode does not experience real loss or gain.

The measurement error $\Delta n_c^{\rm meas}$ in this QND scheme is predominantly determined by the random deletion noise arising from the stochastic nature of the atomic scattering process. In the limit of detection times that are much longer than the mean scattering time $\Gamma_{\rm scatt}^{-1}$, the effect of the random deletion noise can become arbitrarily small. The error due to the probe-field photon-number fluctuations can be effectively eliminated by measuring the number of scattered photons, along with the transmitted probe photon number, in the weak atom-probe interaction limit. A detailed analysis of the QND measurement scheme will be published elsewhere [14].

In the presence of the neglected nonideal effects, such as $\Gamma_{\rm cav}>0$ or $\Gamma_{32}>0$, the coupling field will experience gain or loss. In this case, the measurement should be completed in a time scale determined by $\Gamma_{\rm cav}^{-1}$ and Γ_{32}^{-1}; i.e., if the (unsharp) photon-number measurement requires N scattering events, then $(N/\Gamma_{\rm scatt}) \ll \Gamma_{\rm cav}^{-1},\Gamma_{32}^{-1}$. The cooperative effects between the atoms [12] are also neglected in this work.

The experimental demonstration of a QND measurement using the scheme analyzed here presents no conceptual difficulties and should be technically feasible [14]. This paper demonstrates that such a measurement would be logically reversible if the scattered photons are actually detected by a fast (compared to the relevant atomic time scales) detector. It was not our aim here to prescribe a method by which reversibility can be experimentally demonstrated.

The measurement scheme described here gives us information about the magnitude of the probability amplitudes. The information about the (relative) phase of the probability amplitudes could in principle be obtained by a logically reversible "coherence-QND measurement," which follows the photon-number measurement described here. In fact, a sequence of logically reversible (unsharp) QND measurements of noncommuting operators may be used to "measure" the initial wave function of a single quantum system. If possible, such a measurement will contribute to our understanding of the significance of a wave function.

We have proposed a method for making quantum-nondemolition measurements that is based on the electromagnetically induced transparency. Provided that the scattered photons are detected, the measurement scheme performs a logically reversible quantum-nondemolition measurement, where one obtains photon-number information without destroying any information contained in the premeasurement wave function. A QND measurement that is logically reversible raises the question of whether physical reversibility in quantum measurements could be possible as well.

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