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Low-energy scattering in the $p+p\mu$ system via the Faddeev approach: Virtual-state effects

Andrei A. Kvitsinsky
Department of Mathematical and Computational Physics, Institute for Physics, University of St. Petersburg, St. Petersburg 198904, Russia

Chi-Yu Hu
Department of Physics, California State University at Long Beach, Long Beach, California 90840
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The Faddeev approach in the total-angular-momentum representation is applied to study s-wave $p+p\mu$ scattering at very low energies. It is shown that the threshold behavior of phase shifts and cross sections is strongly affected by a virtual state leading to a significant amplification of cross sections near $k=0$.

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Low-energy scattering in mesic atomic systems is involved in a broad range of problems of muon physics, in particular in muon-catalyzed fusion. Previously, most calculations of such processes have been done within the framework of the adiabatic and the adiabatic representation methods (see Refs. [1–3] and references therein), and the nonadiabatic coupled rearrangement channel method due to Kino and Kamimura [4] was developed during the last couple of years. Recently, a method was developed [5,6] to treat the three-body Coulomb problem that is based on the modified Faddeev equations [7] in the total-angular-momentum representation [8]. In this paper, we apply this method to study the s-wave elastic scattering

$$p+p\mu\to p+p\mu \tag{1}$$

at very low energies (≤ 10 eV) of the incident proton.

An interesting feature of this system is that it has a virtual state located very close to the elastic threshold, with the same quantum numbers as the $pp\mu$ ground state. A tiny decrease of the mass ratio m_μ/m_p moves the virtual state into the discrete spectrum and a new bound state appears [1,2]. It is natural to expect that the existence of this state can considerably affect low-energy scattering leading to a drastic increase of the elastic cross section near the threshold. Although this can hardly be of practical importance for the nuclear fusion in hydrogen-isotope mixtures (for the nuclear constant of the reaction $p+p\to d+e^+ +\nu$ is very small compared to that of $d+t\to{}^4\mathrm{He}+n$), the phenomenon may be rather of interest in other problems, among them weak transitions $p\mu\to n+\nu$ in hydrogen. In this case, the additional contribution of the three-body channel $p+p\mu\to p+n+\nu$ can be important in dense mixtures [9].

The first numerical estimations of the effect have been made in Ref. [9], where the s-wave Jost function of (1) was calculated via an adiabatic two-state approximation. However, the zero-energy limit is a rather difficult case for the adiabatic method, and the two-state approximation can only provide a qualitative picture. The goal of this work is to study the problem in a more accurate manner on the basis of the modified Faddeev equations (MFE's) in the total-angular-momentum representation, which is the projection of the MFE's in the subspace with fixed total angular momentum L. In this approach, we are able to do the calculation in coordinate space without any truncation. A full description of our method is given in Refs. [5,6,10]. Let us outline briefly its major points.

The modified Faddeev equations for a three-body Coulomb system are built up along a cutoff procedure due to Merkuriev [7]. Namely, the Coulomb potentials V are divided into short- and long-range parts,

$$\begin{aligned} V^{(s)}(x,y)&=V(x)\zeta(x,y)\ , \\ V^{(l)}(x,y)&=V(x)[1-\zeta(x,y)]\ , \end{aligned} \tag{2}$$

where x,y belong to one of the three sets of standard Jacobi coordinates for a three-body system. The cutoff function ζ vanishes asymptotically within the so-called three-body region (where $x\sim y\to\infty$) and goes to one outside of this region (where $x\ll y\to\infty$). We exploit the form of the cutoff proposed in Ref. [11]:

$$\zeta(x,y)=2\Big/\left[1+\exp\left[\frac{(x/x_0)^\nu}{y/y_0+1}\right]\right], \tag{3}$$

where one must take $\nu>2$, and other parameters are arbitrary (see discussion in Ref. [11]). The reason for introducing the cutoff is to bring back compactness to the Faddeev equations that are not compact in their original form for scattering states of charged particles.

The MFE with total angular momentum $L=0$ takes the form

$$\begin{aligned} &(H_{\mathrm{as}}+V_\alpha^{(s)}-E)\Psi_\alpha=-V_\alpha^{(s)}\sum_{\beta\,(\neq\alpha)}\Psi_\beta,\quad \alpha,\beta=1,2,3 \\ &H_{\mathrm{as}}=H_0+\sum_\alpha V_\alpha^{(l)}\ , \end{aligned} \tag{4}$$

$$\begin{aligned} H_0&=-\frac{1}{x_\alpha^2}\partial_{x_\alpha}x_\alpha^2\partial_{x_\alpha}-\frac{1}{y_\alpha^2}\partial_{y_\alpha}y_\alpha^2\partial_{y_\alpha}+\left[\frac{1}{x_\alpha^2}+\frac{1}{y_\alpha^2}\right]l_\theta^2\ , \\ l_\theta^2&=-\csc(\theta_\alpha)\partial_{\theta_\alpha}\sin(\theta_\alpha)\partial_{\theta_\alpha}\ , \end{aligned} \tag{5}$$

with the total wave function $\Psi=\sum_\alpha \Psi_\alpha$. Let the two protons be labeled 1 and 2 and the muon be labeled 3. The third set of the Jacobi vectors, or the Faddeev component ψ_3, involves the repulsive proton pair. It is convenient to choose $\zeta_3 \equiv 0$ for the corresponding cutoff function; then the third component of Eq. (4) becomes

$$(H_0+V_3+V_1^{(l)}+V_2^{(l)}-E)\Psi_3=0 .$$

It is clear that the only solution for this homogeneous equation is $\psi_3 \equiv 0$. With this restructuring Eq. (4) becomes

$$\left[H_0+V_\alpha+V_3+\sum_{\beta\,(\neq\alpha)} V_\beta^{(l)}-E\right]\Psi_\alpha = -V_\alpha^{(s)}\sum_{\beta\,(\neq\alpha)}\Psi_\beta, \quad \alpha,\beta=1,2 . \tag{6}$$

By further application of the symmetry of this system, the number of the Faddeev components of the total wave function Ψ can be reduced to one, using the same functional form for ζ_1 and ζ_2

$$\Psi=[1+p(\mathcal{P})]\Phi ,$$

where Φ is the Faddeev component including the in state of (1), $p=\pm 1$ is parity with respect to the proton interchange, and $\mathcal{P}$ is the operator of the proton permutation. The Faddeev component obeys the equation [4]

$$(H_0+V_1+V_2^{(l)}+V_3-E)\Phi=-pV_1^{(s)}\mathcal{P}\Phi , \tag{7}$$

where H_0 is the s-wave kinetic energy operator given in Eq. (5), V_1 stands for the Coulomb potential of the $p\mu$ atom, $V_1^{(s)}$ is its short-range part (2), $V_2^{(l)}$ is the long-range part of the potential between the incident proton and μ, and V_3 denotes the repulsive potential between protons.

Equation (7) is a three-dimensional partial differential equation in the internal space, which is parametrized by the Jacobi coordinates x, y, and $\theta=\arccos(x,y)$. Below the excitation thresholds, the following asymptotic condition fixes solution to the scattering problem:

$$\Phi(x,y,\theta)|_{y\to\infty} \sim \frac{\varphi_0(x)}{ky}[\sin ky+\tan\delta(k^2)\cos ky] , \tag{8}$$

where φ_0 is the atomic wave function, δ is the (s-wave) phase shift and, k is the momentum, $k^2=2m(E-\varepsilon_0)$, where m is the reduced mass of p and $p\mu$, and ε_0 is the atomic ground-state energy.

In terms of the phase shifts, the s-wave elastic cross section for each parity state ($p=\pm 1$) is given by

$$\sigma_{\rm el}^{\pm}(k^2)=\frac{4\pi}{k^2}\sin^2\delta^{\pm}(k^2) ,$$

and the spin-weighted result is

$$\sigma_{\rm el}=\tfrac{1}{4}\sigma^+ +\tfrac{3}{4}\sigma^- . \tag{9}$$

The cross-section of the conversion of the $p\mu$ atom from the ortho to the para state equals

$$\sigma_c=\frac{\pi}{4k^2}\sin^2(\delta^+-\delta^-) . \tag{10}$$

TABLE I. Phase shifts (in rad), elastic and ortho-para conversion cross sections (in units of πa_μ^2) near threshold.

Units of	Phase shift		Cross section	
k (a_μ^{-1})	δ^+	δ^-	$\sigma_{\rm el}$	σ_c
0	0	0	900	270
0.01	0.279	−0.0356	797	239
0.02	0.483	−0.0759	582	176
0.03	0.617	−0.121	421	126
0.04	0.680	−0.168	299	87.9
0.05	0.713	−0.217	227	64.3
0.06	0.722	−0.266	179	48.4
0.07	0.716	−0.314	146	37.5
0.08	0.700	−0.363	124	29.8
0.09	0.664	−0.420	109	24.1
0.10	0.632	−0.494	102	20.4

To solve the problem (7),(8) numerically, we make use of the triquintic spline expansion [5] of the Faddeev component and a collocation procedure with three-point Gauss-quadrature points per subinterval of each variable. The resulting algebraic equation is solved by direct matrix inversion. A typical grid providing an accuracy of a few percent consists of $24\times15\times15$ collocation points in y, x, and θ, respectively.

The results of calculations in the low-energy region are presented in Table I, where the mesic atom length unit is used, $a_\mu=\hbar^2/e^2m_\mu=2.56\times10^{-11}$ cm. The zero-energy cross sections correspond to the values of the $p+p\mu$ scattering lengths obtained in Ref. [6] by means of solving Eq. (4) at zero energy ($E=\varepsilon_0$): $a_+=-29.4a_\mu$, $a_-=3.45a_\mu$. These results agree to within 2% with an earlier calculation by Bracci *et al.* using the adiabatic method [3]. Note that a large negative value of a_+ is itself an indication of the presence of a virtual state near threshold. With a decrease of the mass ratio m_μ/m_p, the singlet ($p=+1$) scattering length goes through $-\infty$ when a new bound state appears and then becomes positive.

Figure 1 shows the threshold behavior of the phase shifts. Again, the structure of the singlet phase shift, with a maximum near origin, is typical for the case when there is a virtual state on the negative axis of the second sheet of energy [12]. When the virtual state moves toward threshold, the position of the maximum of δ^+ tends to zero and $\delta^+(0)$ reaches $\pi/2\,(\mathrm{mod}\pi)$ just before the effective potential becomes strong enough to bind a new bound state, in accordance with the Levinson theorem.

The influence of the virtual state on the elastic and conversion cross sections (9),(10) near threshold is clearly seen in Fig. 2. Both cross sections undergo a significant increase by a factor of ~ 10 when k goes from $0.1a_\mu^{-1}$ ($E-\varepsilon_0=6$ eV) to zero.

Note that the cross sections are large compared to the

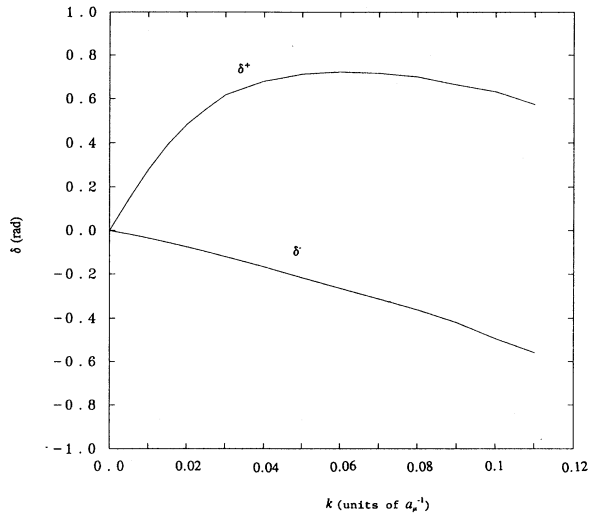

FIG. 1. Threshold behavior of the singlet ($p = +1$) and triplet ($p = -1$) phase shifts (modπ).

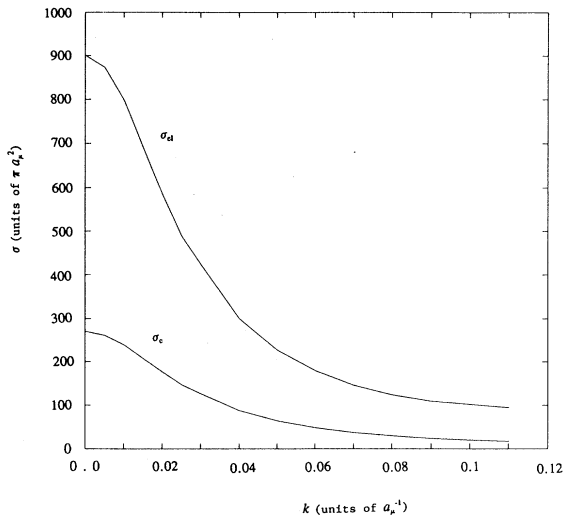

FIG. 2. Elastic and ortho-para conversion cross sections near threshold.

characteristic geometric value πa_μ^2, due to an important contribution of the long-range polarization potential.

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