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Restless optical vortex

C. O. Weiss, H. R. Telle, and K. Staliunas*
Physikalisch-Technische Bundesanstalt Braunschweig, Braunschweig, Germany

M. Brambilla
Università di Milano, Milano, Italy
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We describe a phenomenon of nonlinear optics that occurs in the framework of transverse effects and optical vortices in lasers. We find that in a class-B laser vortices can never be at rest, but move, starting from an initial symmetry breaking of the direction of motion, continuously. In a cylindrically symmetric laser this motion will normally be about the laser axis, while in a constant background the motion is toruslike.

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The phenomenon of "optical vortices" or "phase singularities" has recently been predicted [1], observed experimentally in lasers [2], and studied in some detail [3,4], in particular with respect to the particlelike behavior of the vortices and their interaction. Equally, the analogies of laser vortices with superfluid vortices, magnetic flux lines in superconductors, and with normal fluid vortices have been studied [5]. These studies have predominantly treated cases of class-A lasers; i.e., lasers with material variables that are eliminable, since these lasers' equation shows the most direct relation to the complex Ginzburg-Landau equation, which permits us to explain the above analogies.

In an attempt to test the analogy of "optical vortex crystals," i.e., ordered arrays of vortices observed in lasers [6,3], with real crystals or Abrikosov lattices in superconductors, we searched for lattice vibrations ("phonons") in the vortex crystals. The search failed for class-A lasers (predictably since all time derivatives are negative [7] and the system is consequently overdamped). We looked then to class-B lasers where we felt the chances for finding lattice vibrations were higher—given the existence of relaxation oscillations in these lasers.

A numerical study revealed a somewhat surprising phenomenon: not only were arrays of vortices found to show vibrations in the lattice, but it was also found that this vibration is (i) undamped and (ii) accompanied by a rotation of the array as a whole. For illustration Fig. 1(a) shows the arrangement of vortices studied numerically. The structure consists of two positive and two negative vortices forming a quadrangle (a.k.a. the "optical leopard" [3]). The pattern Fig. 1(a) was photographed with a Na_2 laser (a class-A laser, which does not show the motion of vortices discussed) tuned to the transverse mode family $q=2$ [8]. We expected to find a damped oscillation of the mutual distances of the vortices following a perturbation of a laser parameter (e.g., the laser pump). Instead, the motion shown in Fig. 1(b) is found numerically. In the calculations it is assumed that only the mode family $q=2$ is excited (as it can be realized, e.g., in a CO_2 laser, a typical class-B laser). The motion consists of an oscillation in the radial direction accompanied by an initially accelerated angular motion. The angular motion in particular appeared difficult to explain. A closer consideration, however, reveals that this motion is a special case of what we would like to call "wandering" vortices characteristic of class-B lasers.

This phenomenon is explained as follows: Fig. 2(a) shows the optical field in the vicinity of the vortex. Near the vortex center the field is small or zero, resulting in the absence of stimulated emission here. As the inversion does not decay in the absence of an optical field (since the corresponding relaxation rate γ is small in a class-B laser) an inversion maximum builds up at the location of the vortex. This "excess" inversion forms an unfavorable state of the laser, since the attainment of a state of minimum free energy requires that this inversion be radiated away by stimulated emission (i.e., the laser tends to avoid, as usual, the "spatial hole burning"). One way to achieve this would evidently be to build up a field at the vortex core; however, as the vortex is stabilized by topological constraints (as any defect), this is impossible. In principle the vortex could transform into a dark line. In an infinitely extended background field the energy required to transform the vortex is infinite. For a laser of finite dimensions the energy required is finite [14]. It reaches zero only at the laser threshold. The option the laser has to reduce the excess inversion is not to destroy the vortex, but to displace it, which does not require energy, so that the high field outside the vortex core comes to coincide spatially with the inversion peak, which can then be reduced by the stimulated emission of that field.

Evidently, after being displaced, the vortex builds up immediately a new peak of "excess" inversion at its new core location, which then calls for a further displacement of the vortex. It is clear, then, that the vortex motion has to go on forever. Figure 2(b) depicts the relative location of field vortex and inversion maximum while the vortex is moving. We note that this motion has at its roots the topological stability of the defect and can thus only occur in a greater than two-dimensional system. It can also only occur in class-B lasers since it requires the slow spontaneous decay of the inversion. Due to the spatial symmetry of the vortex there is no preferred direction for the initial vortex motion. This means that the motion has to start with a symmetry breaking.

In the case of Fig. 1, where circular symmetry of the laser is assumed and where radial restoring forces and drift forces [5] give a stable distance of the vortices from the laser axis, the essential motion is azimuthal. Whether the motion occurs right- or left-handedly depends on the initial symmetry breaking. Both can occur. It appears that the oscillation of the vortices in the radial direction in Fig. 1(b) is driven by the azimuthal motion.

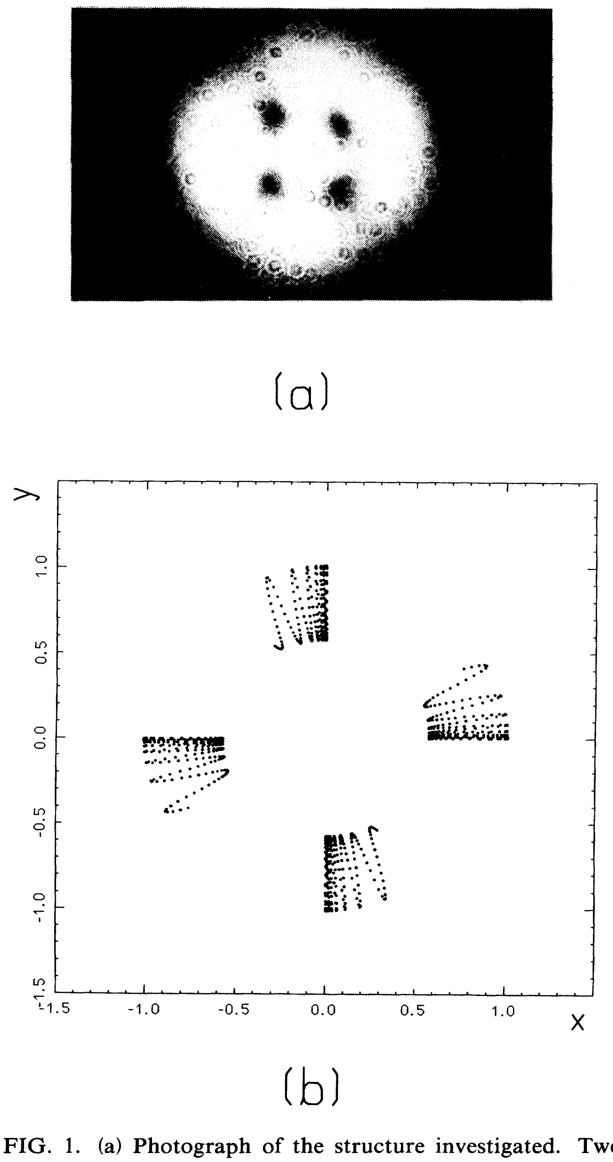

FIG. 1. (a) Photograph of the structure investigated. Two vortices with positive topological charge and two negative ones form a quadrangle as radiated by a Na_2 laser [3]. (b) Location of the zeros of the four vortices calculated for equidistant points in time. An azimuthally accelerated oscillatory motion is observed (mode family $q=2$). The parameters used for the calculation are $\gamma_\perp=3.3$, $\gamma_\parallel=0.033$, and $D_0=6$. $\gamma_\perp,\gamma_\parallel$ are normalized to the resonator decay time κ, D_0 is the pump above threshold, and the pump profile is plane.

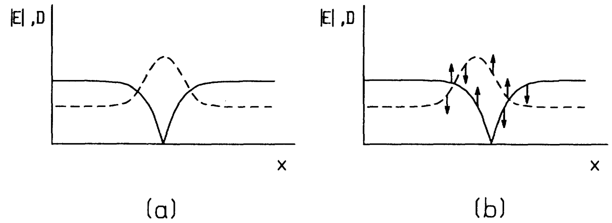

FIG. 2. Electric-field modulus (solid line) and population inversion (dashed line) corresponding to a stationary (a) and moving to the right (b) vortex, on the axis along the direction of vortex motion. The changes of field and inversion connected to vortex motion are shown by arrows.

From the above one can make predictions on the behavior of vortices in a class-B laser of homogeneous transverse distribution of parameters. Here, the vortex is free to move in two dimensions. Thus its motion has to start with a symmetry breaking that chooses a direction out of all the possible directions of motion (angle $0-2\pi$). One can also predict that the direction of motion will change under external influences; thus the direction can undergo a random walk in the presence of noise in the course of time in the same way as the phase diffuses in lasers. Effectively this means that, under the influence of noise, the vortex will "meander" around the plane. Therefore we believe that the "restless" vortex reported here is related to the "meandering vortex" described in [9], where the equations bear some similarity to class-B laser equations and to Winfree's "meandering rotor" [10].

We have studied the vortex motion in an infinite two-dimensional plane numerically. The equations are

$$\frac{\partial E}{\partial t}=(D-D_{\rm th})(1+i\beta)E+i\nabla^2E\ ,$$
$$\frac{\partial D}{\partial t}=-\gamma_\parallel\left[D\left(+\frac{|E|^2}{D_{\rm th}}\right)-D_0\right]\ . \qquad (1)$$

Here $E(\mathbf{r},t)$ and $D(\mathbf{r},t)$ are the two-dimensional envelopes of electric field and population inversion, D_0 is the population inversion in the absence of radiation, $D_{\rm th}=1+\beta^2$ is the threshold population inversion, γ is the population inversion decay rate, and β is the detuning from resonance (both $\gamma_\parallel$ and β are normalized to the photon decay rate κ); the spatial coordinates were normalized to make the diffraction constant d in the diffraction term equal to 1. As the polarization relaxation for class-B lasers is much faster than that of the other variables ($\gamma_\perp \gg 1 \gg \gamma_\parallel$), the polarization is adiabatically eliminated from the Maxwell-Bloch equations to obtain (1).

The system (1) was numerically integrated. The pump function D_0 was homogeneous in space, and periodic boundary conditions were used (care was taken to ensure that the boundaries did not influence the observed dynamical behavior). The vortex was set in motion by a small perturbation of the field phase in the vicinity of the vortex core.

Figure 3 represents the trajectory of the electromagnetic field zero. It is seen from Fig. 3 that the direction, in

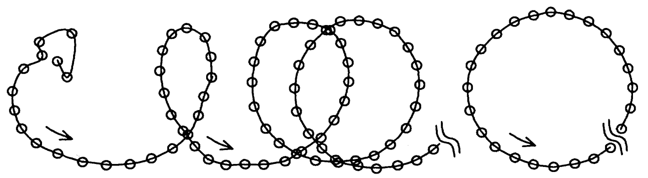

FIG. 3. Vortex motion obtained by numerical integration of Eq. (1). The circles represent the positions of the electric-field zeros at equidistant time intervals $\Delta t = 5$. The trajectory was interrupted after 80 snapshots and continued after 200 snapshots (divided by eight circle periods). The parameters are $\gamma_{\parallel} = 0.05$, $D_0 = 2$, and $\beta = 0$. The circle radius is $r = 13$.

which the vortex starts to move, is given by an initial symmetry breaking. After a long transient of toroidal motion the vortex trajectory is found to become circular. Toroidal motion of the vortices has also been observed in the mode expansion calculations of the laser [11]. Apparently the rotary element of the motion is related to the fact that the vortex itself is a rotating object. (The vortex with the opposite topological charge circles in the opposite direction.) The difference between Figs. 3 and 1(b) results primarily from the finite and circular geometry of a realistic laser shown in Fig. 1(b) and the realistic restriction to the excitation of one mode family. However, for parameters different from Fig. 1(b), toroidal motion similar to Fig. 3 can also be obtained [11] in this mode family.

Figure 4 shows the electric field and population inversion configurations near the core of the vortex moving along a circular trajectory. It is characteristic that the shapes of the population inversion peak and the electric field well are significantly "squeezed" in the direction of motion, and their centers are displaced with respect to one another along the same direction.

The detailed characteristics of the circular motion will be reported elsewhere. Here we note that the radius of the trajectory circle is comparable to the radius of the vortex core (usually slightly larger) and the frequency of the circular motion is proportional to and of the same order of magnitude as the frequency of the laser relaxation oscillations. (Our numerical integrations show circular motion frequency being by a factor of 7 or 8 smaller than the relaxation oscillation frequency.)

While the radius of the vortex circle was found to be uniquely determined by the system parameters (mainly by β, $\gamma_{\parallel}$, and D_0), the circle location was very sensitive to external perturbations, and even slight perturbations of the field or of the boundary conditions caused the vortex circle to drift. This could explain the slow angular component of the toroidal motion in [11]. (We suspect that this is the reason why in [9] "meandering" of the vortex was reported instead of a circular motion.)

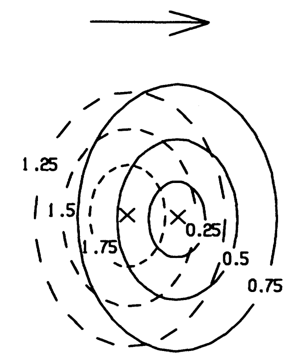

FIG. 4. Iso-lines of the electric field (solid lines) and population inversion (dashed lines) of a moving vortex. The parameters are as in Fig. 3. The direction of motion is indicated by the arrow. The approximate half axes of the 0.5-level line ellipse of the electric field are 5 and 7.5; the distance between the electric field and population inversion ellipse centers is $\Delta x = 4$.

Finally we mention possible relations of the vortex motion described here with similar observations in other fields. In [12] it is mentioned that an entire spiral flow pattern in a circular convection cell is set in uniform rotation by the presence of one defect in the spiral pattern. Slowly rotating intensity patterns have been observed in CO_2 lasers—typical representations of class-B lasers [13]—and it was even observed that the rotation of the pattern sometimes reversed itself after the laser emission was blocked.

We think that the speed of vortex motion in the class-B laser will attain an asymptotic value of the order of the ratio of vortex radius and the "hot" (that is, in the presence of stimulated emission) laser medium inversion lifetime. We mention for distinction that the wandering vortex described here is unrelated to the phenomenon of the "spinning vortex" in a recently reported class-A laser [11,4]. The latter is a motion related to a mode spacing and occurs consequently on a nanosecond time scale.

The phenomenon of the "restless vortex" appears interesting to us since it constitutes a clear case in which dynamics occurs not because the system "knows what it wants" (e.g., to reach a stable fixed point) but because the system "knows only what it does not want" (namely, for the vortex to remain at its current location). We think that it thus particularly simply captures the essence of chaotic system dynamics.

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*Permanent address: Vilnius University, Vilnius, Lithuania.

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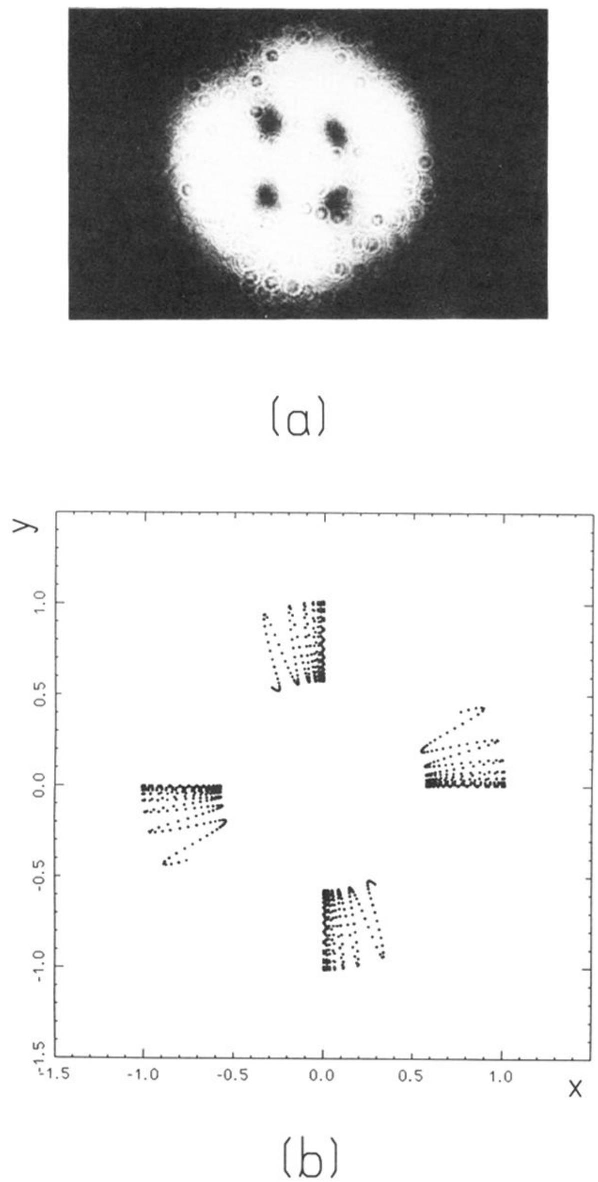

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