Significance of an experiment of the Greenberger-Horne-Zeilinger kind

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Restrictive conditions on the class of allowed physical theories are drawn from the assumption that the predictions of quantum theory are valid for an experiment of the kind proposed by Greenberger, Horne, and Zeilinger [in *Bell's Theorem, Quantum Theory and Conceptions of the Universe,* edited by M. Kafatos (Kluwer Academic, Dordrecht, 1989)]. It is shown that no theory can be compatible with the following four conditions. (1) The choices to be made by the three experimenters can be treated, in this context, as three independent free variables. (2) For each of the six possible local measurements under consideration, if that local measurement were to be performed, then exactly one of the alternative possible outcomes of this measurement must be selected as the actual outcome. (3) For each triad of measurements in a certain set of possible triads, if that triad were to be performed, then the corresponding triad of selected outcomes must satisfy the correlation condition predicted by quantum theory. (4) For each of the six possible local measurements, if that local measurement were to be performed, then the selected outcome must, according to the theory, be independent of which two experiments will later, in some frame of reference, be performed in the other two regions.

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I. ARGUMENT

Greenberger, Horne, and Zeilinger [1] have proposed spin-correlation experiments of a new kind. They are generalizations of the Einstein-Podolsky-Rosen–Bohm (EPR-Bohm) [2] experiments analyzed by Bell [3], and they allow a significant simplification of earlier arguments [4] pertaining to the apparent existence in nature of some sort of influence that acts over spacelike intervals. The importance of these experiments in this connection ensures that they will eventually be performed. The aim of this work is to elucidate their theoretical significance.

The essential difference between the (GHZ)-type spin-correlation experiments and the earlier EPR-Bohm experiments is that they involve more than two particles. The simplest (GHZ) experiment involves three spin-$\frac{1}{2}$ particles, which are first prepared in a particular spin state ψ, and are then allowed to travel to three space-time regions 1, 2, and 3, which are mutually spacelike separated. In each region, there is a Stern-Gerlach device, and an experimenter who makes, within that region, a choice between two alternative possible directions, say north or east, for the orientation of the Stern-Gerlach device in his region. In the ideal case, where the detectors are 100% efficient (and where an array of detectors is positioned to detect and exclude the cases where the particles do not enter the three devices), there will be in each region, for whichever one of the two orientations, north or east, is chosen in that region, a detection event. This event will occur in either one or the other of two properly positioned detection devices. A detection event in the first detection device is represented here by the number $+1$, and a detection event in the second detection device is represented by the number -1. Thus, if the cases where the three particles do not enter the three devices are excluded, then the choices of the measurements to be performed and the values of the subsequently observed results can be represented by a pair of triads (d_1,d_2,d_3) and (r_1,r_2,r_3), where each d_i is either N (for north) or E (for east), and each r_i is either $+1$ or -1.

It is shown in the Appendix that, for a certain choice of the initial spin state ψ, quantum theory makes the following predictions:

(1) If $(d_1,d_2,d_3)=(N,N,N)$, then $r_1r_2r_3=-1$.
(2) If $(d_1,d_2,d_3)=(N,E,E)$, then $r_1r_2r_3=+1$.
(3) If $(d_1,d_2,d_3)=(E,N,E)$, then $r_1r_2r_3=+1$.
(4) If $(d_1,d_2,d_3)=(E,E,N)$, then $r_1r_2r_3=+1$.

In other words, if the experimenters in each of the three regions all choose to orient their devices so that they point north, then either all three results r_i will be -1, or one of the three results r_i will be -1 and the other two will be $+1$; and if, alternatively, the experimenters in two of the three regions choose to orient their devices to point east and the other experimenter chooses to orient his device to point north, then either all three results r_i will be $+1$, or just one of the three results will be $+1$ and the other two results will be -1. The predictions of quantum theory for the remaining four combinations of choices of orientations of devices are not used in the argument. The labeling of the two alternative possible orientations of the devices by the words "north" and "east" is simply for physical clarity: the only important requirement is that these two directions be perpendicular to each other.

The essential advantage of this new experimental arrangement over the EPR-Bohm experiment considered earlier is that in the new situation the only predictions that need be considered are predictions that hold with certainty (i.e., with probability unity); no use is made of predictions that become certain only in a limit where the number of repetitions of the measurement becomes infinite. The need to deal with the infinite number of measurements implicit in the meaning of statistical predictions rendered the earlier arguments less direct.

In the context of the experimental setup described above, we may formulate Theorem 1. The following four conditions cannot be satisfied conjunctively.

(1) The choices to be made by the three experimenters concerning which measurements will be performed in the three regions can be treated as three independent free variables.

(2) For each of the three regions, and for each of the two alternative possible measurements that might be performed in that region, if that measurement were to be performed, then nature must select some single result for that measurement.

(3) For each of the four alternative possible combinations of measurements *NNN*, *NEE*, *ENE*, and *EEN*, if that combination of measurements were to be performed, then the three selections that nature would, by virtue of assumption 2, be required to make under those conditions must conform to the corresponding prediction of quantum theory.

(4) For each of the three regions, and for each of the two alternative possible measurements that might be performed in that region, if that measurement were to be performed, then the selection that nature would, by virtue of assumption 2, be required to make under that condition must yield a result that is independent of which of the two alternative possible measurements will be performed in the other two regions.

The physical meaning of these four conditions is now explained. For each of the three regions R_i, only one of the two alternative possible measurements in that region can actually be performed. However, physical theories generally give predictions for various alternative possible situations that might arise in nature or be created in the laboratory. Indeed, it is precisely this feature of physical theories that makes them useful: they give predictions about what will happen under various alternative possible conditions that human beings might choose to create. This feature is common to classical theory and quantum theory. Assumption (1) makes explicit the facts that (i) we are considering a class of theories that allows us to contemplate the various alternative possible experimental conditions that experimenters might choose to create, and (ii) the choices by the three experimenters as to which measurements they will perform are treated as independent free variables.

In a theory, such as quantum theory, that accommodates stochastic elements, the condition that the choices to be made by the experimenters can be treated as free variables poses no problem at all; these three choices can be taken to be independent stochastic variables. In a deterministic context, condition (1) is essentially the assumption that completely whimsical choices by the three experimenters are sufficiently independent of the contemporaneous state of the physical system under consideration to be idealized as three independent free variables.

The second assumption injects the theoretical idea that, for each of the three regions R_i, *if* some particular measurement were to be performed in R_i, *then* nature must select some single result for that particular measurement. Of course, a single result always appears to any individual human observer under the conditions of this experiment, and communicating human observers agree, in general, about which results have appeared. However, there is an interpretation of quantum theory, namely Everett's relative-state interpretation [4], in which this appearance of one single result in each individual human mind is illusory. According to that interpretation, no single result is selected by nature. Instead, all of the conceivable possible results occur in the fullness of nature, even though only one single result appears in any individual human consciousness. Assumption (2) explicitly rules out such a scenario; it asserts that, *under the condition that a particular measurement be performed,* nature must select one single result for this particular measurement.

The third assumption is that, for any one of the four specified combinations of three measurements, *if* that combination were to be created by the experimenters, *then* the triad of selections that, by virtue of assumption (2), nature is required to make under that set of conditions, must yield a triad of results that conforms to the predictions of quantum theory.

The fourth assumption expresses the idea that *no influence whatever can act over a spacelike interval* (i.e., faster than the speed of light). Since the three regions are mutually spacelike separated (which means that nothing can travel directly from any point in any one of the regions to any point in another region without traveling faster than light), the demand that *no influence whatever* can act over a spacelike interval entails that, for each of the three regions R_i, the free choices to be made by the experimenters in the two regions $R_j \neq R_i$ can have no influence whatever on nature's selection in R_i. But if the choices to be made by these two faraway experimenters can have no influence whatever on nature's selection in R_i, then, as far as this selection in R_i is concerned, it is exactly as if those two faraway choices did not exist. However, no selection can yield one result or another depending upon the outcome of a choice that does not exist. Hence for each of the three regions R_i, and for any selection that nature is required to make for the result of a measurement in that region, that selection must yield a result that is independent of which choices are freely made by the experimenters in the two regions $R_j \neq R_i$.

None of these four conditions contravenes the orthodox philosophy of quantum theory. Indeed, Bohr [5] affirms that "The freedom of experimentation, presupposed in classical physics, is of course retained and corresponds to the free choice of experimental arrangements for which the mathematical structure of the quantum mechanical formalism offers the appropriate latitude." Dirac [6] uses the idea that nature chooses the result of a quantum measurement. Heisenberg [7], when describing "what happens in an atomic event," says that the "observation itself . . . *selects* of all possible events the actual one that has taken place." Finally, it is a widely held belief among quantum physicists that the property of classical relativistic physics that no influence whatever can act over a spacelike interval can be carried over to quantum theory.

In certain other generalizations of Bell's theorem, there is, in addition to conditions analogous to some of those mentioned above, an assumption of *realism* that directly contravenes accepted quantum ideas. This extra assumption involves the introduction of either the assumption that the outcomes of all of the alternative possible measurements are *simultaneously determinate* [8], or the notion of a *hidden variable,* linked to the assumption that, for each separate value of this hidden variable, the outcome selected in one region (or perhaps merely the probability of this outcome) is independent of the outcome (or perhaps merely the probability of the outcome) of the measurement in a spacelike separated region [9]. This assumption of "outcome independence" goes far beyond the idea of no action over spacelike intervals. Quantum systems, when decomposed as fully as is empirically possible, and indeed even into elemental systems of $J=0$, display outcome *dependence:* the outcomes in the different regions are often strongly correlated. The notion that this feature of the quantum systems can be undone by decomposing the system into nonquantal components is altogether alien to orthodox quantum philosophy, and hence from the orthodox viewpoint, very dubious. Contradictions obtained by using it would seem merely to confirm orthodox thinking, nothing more.

The following proof of Theorem 1 refers to Fig. 1. The experimental conditions are supposed to be such that, in each of the three regions R_i, the experimenter in that region will make a choice between *precisely two* alternative possible measurement conditions; he will necessarily, in his region, choose either a particular condition labeled N or a particular condition labeled E. If the measurement condition E were to be created by the experimenter in region 1, then, by virtue of condition (2) of the theorem, nature would be required to select some result for this measurement. Let us consider the various possibilities for the result of this selection.

One conceivable possibility for the result of this selection is $x(=+1$ or $-1)$. If measurement condition E were to be created in region 1, and if the selection that nature would then be required to make in region 1 were to yield the result x, in the circumstance that the choices

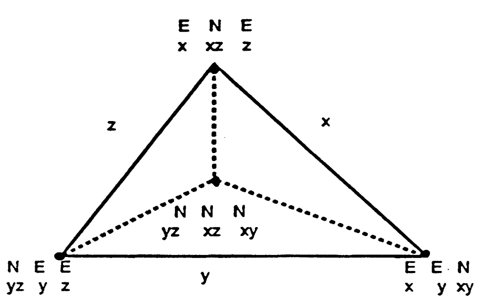

FIG. 1. Diagram indicating the four alternative possible combinations of measurements under consideration, and some possibilities for the results that might be selected by nature under those alternative possible combinations of conditions.

to be made (later, in some frame of reference) by the experimenters in regions 2 and 3 were to create conditions N and E in those two regions, respectively, then, by virtue of condition 4 of the theorem, this same selection in region 1 must yield the result x also in case the choices to be made *(later)* by the experimenters in regions 2 and 3 were to create conditions E and N in those two regions, respectively. This restriction on the allowed possibilities is represented by the x's in the first positions on the two ends of the x line in Fig. 1.

If the experimenter in region 2 were to create the measurement condition E, then, by virtue of condition (2) of the theorem, nature would be required to select a result in region 2. One conceivable possibility for the result of this selection is $y(=+1$ or $-1)$. If condition E were to be created in region 2, and if the selection that nature would then be required to make in region 2 were to yield this result y, in the circumstance that the choices to be made *(later,* in some frame of reference) by the experimenters in regions 1 and 3 were to create conditions N and E in those two regions, respectively, then, by virtue of condition 4 of the theorem, this same selection in region 2 must yield the result y also in case the choices to be made (later) by the experimenters in regions 1 and 3 were to create, instead, the conditions E and N in those two regions, respectively. This restriction on the allowed possibilities is represented in Fig. 1, together with the analogous restriction associated with the conceivable possibility $z(=+1$ or $-1)$ for the result selected under condition E in region 3.

If the combination of measurements NEE were to be chosen, and the results in regions 2 and 3 were to be y and z, respectively, then, by virtue of prediction (2) of quantum theory and condition (3) of the theorem, the result selected in region 1 must by yz. This restriction on the possibilities for nature's selection under condition N in region 1 is indicated in Fig. 1.

If the combination of measurements ENE were to be chosen, and the results in regions 1 and 3 were to be x and z, respectively, then, by virtue of prediction (3) of quantum theory and condition (3) of the theorem, the result selected in region 2 must be xz. This restriction on the possibilities for nature's selection under condition N in region 2 is indicated in Fig. 1.

If the combination of measurements EEN were to be chosen, and the results in regions 1 and 2 were to be x and y, respectively, then, by virtue of prediction (4) of quantum theory and condition (3) of the theorem, the result selected in region 3 must by xy. This restriction on the possibilities for nature's selection under condition N in region 3 is indicated in Fig. 1.

If, under the conditions NEE, nature's selection in region 1 were to yield the result yz, then, by virtue of condition (4) of the theorem, nature's selection in region 1 must yield the result yz also in case the choices to be made (later, in some frame of reference) by the experimenters in regions 2 and 3 were to create, instead, the conditions N and N in those two regions. This restriction on the allowed possibilities is represented in Fig. 1.

If, under the conditions ENE, nature's selection in region 2 were to yield the result xz, then, by virtue of condition (4) of the theorem, nature's selection in region 2 must yield the result xz also in case the choices to be made (later, in some frame of reference) by the experimenters in regions 1 and 3 were to create, instead, the condition N and N in those two regions. This restriction on the allowed possibilities is represented in Fig. 1.

If, under the conditions EEN, nature's selection in region 3 were to yield the result xy, then, by virtue of condition (4) of the theorem, nature's selection in region 3 must yield the result xy also in case the choices to be made (later, in some frame of reference) by the experimenters in regions 1 and 2 were to create, instead, the conditions N and N in those two regions. This restriction on the allowed possibilities is represented in Fig. 1.

The possibilities x, y, and z introduced at the beginning of the proof were conceivable possibilities for nature's selections under conditions E in regions 1, 2, and 3, respectively. The four conditions of the theorem entail that if, under the triad of conditions (E,E,E), the triad of possibilities (x,y,z) were to be selected by nature, then, under the triad of conditions (N,N,N), nature's selection must yield the triad of results $(r_1,r_2,r_3)=(yz,zx,xy)$. But in this case, $r_1r_2r_3=x^2y^2z^2=1$, which contradicts prediction 1 of quantum theory. Consequently, the four conditions of the theorem cannot be satisfied simultaneously with this particular initial set of possibilities x, y, and z. However, the value for each of the three quantities x, y, and z can be allowed to range independently over its full set of possibilities, $+1$ and -1. For each of the conceivable possible combinations, a contradiction ensues. Consequently, there is no way of satisfying simultaneously the four conditions of the theorem. Q.E.D.

The crucial feature of this argument is that it rests solely on the four conditions enumerated in the theorem. In particular, the theorem and its proof involve no supposition, tacit or otherwise, that results of unperformed measurements are determinate within nature, or that results of performed measurements are predetermined.

In accordance with normal usage, and with the locality idea being examined here, a measurement is considered to be a local affair; a combination of three measurements, one in each of the three mutually spacelike separated regions, is considered to be precisely that: a combination of *three* measurements, not one.

Two key conditions of the theorem are, first, that, *if* a measurement were to be performed, *then* nature must select a result for this measurement, and, second, that any such required selection must yield a result that is independent of the choices to be made (later, in some frame of reference) by experimenters in spacelike separated regions. The proof of the theorem is based upon a consideration of the various conceivable possible triads of results (x,y,x) of the E-type measurements. For each such triad, the four conditions of the theorem logically entail the possibilities for the outcomes of the triad of measurements (N,N,N) to a triad of results of the form (xy,zy,zx). However, for every conceivable possible initial triad (x,y,x), the corresponding triad (xy,zy,zx) is incompatible with the predictions of quantum theory for the set of conditions (N,N,N).

It might seem at first that, if a result is selected for a measurement *only* under the condition that this measurement be performed, then no logically valid argument could involve conjunctively the possibilities for the results of any two measurements that cannot be performed conjunctively. However, the locality condition, condition (4), provides the necessary logical linkage between measurements that cannot be performed conjunctively. This condition is a conjunction of six individual locality conditions, one for each of the six edges of the tetrahedron shown in Fig. 1. The predictions of quantum theory provide four more conditions, one for each of the four vertices of this tetrahedron. These ten conditions constitute an interlocked set of conditions that is not self-consistent. However, none of these ten individual conditions involves the results of any pair of mutually incompatible measurements: each of the four predictions of quantum theory involves the results of a triad of mutually consistent measurements, whereas such of the individual locality conditions involves the result of only one single measurement. Consequently, each of these ten conditions can be formulated without introducing the idea of the results of an unperformed measurement. On the other hand, quantum theory, like classical theory, is basically a logical conjunction of assertions, each pertaining to one of the alternative possible conditions that we might choose to create; this conjunctive structure is precisely what makes these theories useful. Likewise the locality condition, which expresses the idea that no influence whatever can act over a spacelike interval, is a conjunction of six conditions, one for each of the six local conditions that the experimenters might choose to create. In this way, we are provided with a conjunction of ten conditions, each of which is formulated without any use of the concept of a result of an unperformed measurement. Yet this conjunction of ten conditions, which expresses logically the conjunction of the demands that the predictions of quantum theory be valid and that no influence of any kind can act over a spacelike interval, is logically inconsistent.

The significance of this result for science is this: It shows any physical theory that accepts the idea that whimsical choices by experimenters can be treated as free variables; that eschews the Everett idea that, contrary to appearances, all the possible results of a quantum measurement really occur; and that reproduces the predictions of quantum theory in this GHZ experiment, cannot consistently affirm also the no faster-than-light-influence condition embodied in assumption (4).

II. DISCUSSION

The argument given above is fundamentally the same as an argument I gave many years ago [10], but with all the details filled in, and adapted to the GHZ experiment, which simplifies the logic. Redhead [11] criticized that earlier argument, suggesting that assumption (4) makes sense only in a deterministic context. I shall rebut that claim by constructing a counterexample. The counterexample will be a simple indeterministic theory in which assumption (4) expresses the condition of no faster-than-light influence.

To construct this indeterministic theory, I start with a local deterministic theory and then modify it. So consider first a deterministic theory constructed in the following way. In some frame of reference, we can define surfaces of constant time. It is assumed that, at time zero, the values of all quantities that specify the dispositions of things at earlier times have already been generated by the mechanical process of nature; all of those earlier things are fixed and settled. Nature's process is asserted to be deterministic. This means that, if everything up to any given instant of time t is fixed and settled, then nature's process determines everything up to $t+1$. Nature's process advances into the future step by step, making more and more things fixed and settled. The locality requirement is taken to mean that the dispositions of things in any space-time region are determined by things in the backward light cone from that region.

To make the theory indeterministic, we modify the above deterministic theory in the following way. We suppose that scattered throughout the positive time region are special "break points." At each of these break points, there is a rupturing of space-time, and the interior of the future cone with apex at that point has two sheets. In these two sheets, the dispositions of things are different. The break points are rare enough so that only one of them is located in any unit interval of time. Nature's process is such that, if there is a break point in the region between t and $t+1$, then nature will select, at the step that fixes everything in that region, *one* of these two sheets. But there is absolutely nothing in the entire region lying earlier than this break point that determines which of the two sheets will be selected. The theory is therefore indeterministic, by definition.

Although nothing in the past of the break point determines which sheet is selected, a decision must somehow be made. In Dirac's words, "nature chooses." The demand of indeterminism says that nature's decision cannot be determined by what lies in the past of the break point. But then, what can fix nature's choice?

One possibility is the future. But that smacks of teleology. The more normal idea is that of a random-number generator that lies completely outside our space-time universe. At a step where a decision at a break point is needed, an appeal is made to this random-number generator, which then produces the needed decision. This is our model of how "nature chooses." To generate a model of an indeterministic universe of the kind imagined by quantum physicists, the output of the random-number generator is taken to be completely disconnected from everything in the universe lying earlier than the break point. And a decision associated with a break point is asserted to be made *only if* and *only when* that break point is encountered in nature's process of making things fixed and settled.

To make the picture more graphic, we may imagine that when nature's process reaches the stage where a break point decision must be made, the various alternative possibilities at that point are mapped into segments on a (Platonic) circle of unit circumference, with the length of each segment equated to the probability assigned to the corresponding possibility by some statistical theory—such as quantum theory. Then nature blindly throws a point down on the circle, and the segment in which it lands determines nature's decision. This model is essentially a picturesque version of the idea of nature's process harbored in the intuition of most quantum physicists.

In the GHZ experimental situation under consideration here, there are three choices that have been called "choices by experimenters." We can imagine that they are controlled by whether or not a certain atom in the experimenter's brain will undergo a radioactive decay in a certain time interval. Then the entire experimental setup is brought into the general framework just introduced. We can consider a simplified model of the universe in which the only break points are those associated with our GHZ experimental setup.

The break points corresponding to the three choices by experimenters differ from the other break in that they lie on the single-sheeted part of space and are therefore definitely encountered in nature's process of making things fixed and definite. The decisions associated with these three break points are supposed to be identifiable as independent free variables, and "elementary causes."

At the stage in nature's process where the decision pertaining to a certain break point must be made, the decisions associated with those break points that lie at later times have not yet been fixed. If an earlier decision were to depend causally upon what will come into existence only later, then a gridlock could ensue, and nature's process could be blocked. In order to achieve a model that is clearly contradiction-free, we specify that nature's process at each encountered break point will produce a definite decision, and that, in making this decision, nature proceeds as if the decisions to be made at future break points simply do not exist. When later break points are reached, decisions related to those later break points must be made. But the earlier decision will not be influenced by the later ones, because the earlier decision became fixed and settled already at the earlier stage, before the later decisions come into being. Each decision in our indeterministic model will therefore be one and the same decision, independently of what the later decisions turn out to be.

Let us suppose that the time differences between the experimenter's choice of a measurement and the subsequent performance of that measurement are small compared to the distances between the regions, and that the timings are such that the outcome of the first measurement appears *before* the experimenters in the other two regions make their choices. Then, in our indeterministic model, the outcome of the first measurement will be independent of the choices to be made by the two faraway experimenters. By virtue of the general structure of the model, this result must hold if the first measurement turns out to be the *first* of the two alternative possibilities for what it might be, and it must hold also if that measurement is the *second* of the two alternative possibilities for what it might be. Thus we have verified, within our indeterministic model, the first of the three conjunctive parts of assumption (4).

In Redhead's discussion, the question at issue is whether a certain atom will decay at a certain time t_2; this decay event is the analog, in his discussion, of one of the outcomes of the GHZ experiment. Redhead's analog of whether a faraway experimenter will choose to perform one measurement or another is whether he, Redhead, will choose to raise his hand or not. He supposes that he actually does raise his hand and that the atom decays at time t_2, and then asserts: "if I imagine running the course of events through again, with my hand not raised this time, the outcome at time t_2 might just as well have been that the atom did not decay."

Nature does not "run the course of events through again." What "would happen" under such an unrealizable condition is speculative; it can be debated endlessly. There is no absolutely correct and provable answer to that kind of question.

What scientists are interested in, however, is something quite different. They are interested in properties of well-defined classes of physical theories, particularly when the theories involved are of the kind they deal with, or might want to deal with. Questions of this kind can have absolutely correct and provable answers.

The indeterministic theory constructed above is an expression of normal ideas of physicists. It asserts that, if a certain measurement is performed in one experimental region, then nature's decision d about the outcome of that measurement must be functionally independent of the variable x that represents the choice that will later be made by a faraway experimenter: $d(x)=d(-x)$. This property of the theory is an expression of the fact that, *in this theory,* the effects of any elementary cause are confined to the region of space-time that lies in the future of the cause; cause *precedes* effects. In the relativistic generalization, the effects must be confined to the future cone, including interior, issuing from the region of the elementary cause. This is the requirement formalized in assumption (4).

Redhead's way of formulating the question leads him into the domain of modal logic. His conclusion that assumption (4) makes sense only in a deterministic setting is based on a *revision* that he makes of the standard modal logic of Lewis [12], to which he refers. In Lewis's framework itself, assumption (4) is the natural expression of the physical assumption of no faster-than-light influence. It works perfectly in an indeterministic setting. A detailed discussion of this point is given in Ref. [13].

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APPENDIX: THE QUANTUM PREDICTIONS

The three Pauli spin matrices

$$\sigma_x=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}, \tag{A1}$$

$$\sigma_y=\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}, \tag{A2}$$

and

$$\sigma_z=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}, \tag{A3}$$

have eigenvectors

$$\varphi_x^{\pm}=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ \pm 1\end{bmatrix}, \tag{A4}$$

$$\varphi_y^{\pm}=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ \pm i\end{bmatrix}, \tag{A5}$$

and

$$\varphi_z^{\pm}=\begin{bmatrix}\frac{1}{2} & \pm & \frac{1}{2}\\ \frac{1}{2} & -\pm & \frac{1}{2}\end{bmatrix}, \tag{A6}$$

which satisfy

$$\sigma_x\varphi_x^{\pm}=\pm\varphi_x^{\pm}, \tag{A7}$$

$$\sigma_y\varphi_y^{\pm}=\pm\varphi_y^{\pm}, \tag{A8}$$

and

$$\sigma_z\varphi_z^{\pm}=\pm\varphi_z^{\pm}. \tag{A9}$$

The following identities hold:

$$\varphi_z^{+}=\frac{1}{\sqrt{2}}(\varphi_x^{+}+\varphi_x^{-}), \tag{A10}$$

$$\varphi_z^{-}=\frac{1}{\sqrt{2}}(\varphi_x^{+}-\varphi_x^{-}), \tag{A11}$$

$$\varphi_z^{+}=\frac{1}{\sqrt{2}}(\varphi_y^{+}+\varphi_y^{-}), \tag{A12}$$

$$\varphi_z^{-}=\frac{-i}{\sqrt{2}}(\varphi_y^{+}-\varphi_y^{-}). \tag{A13}$$

The combined spin state ψ is

$$\psi=\frac{1}{\sqrt{2}}(\varphi_{z1}^{+}\varphi_{z2}^{+}\varphi_{z3}^{+}-\varphi_{z1}^{-}\varphi_{z2}^{-}\varphi_{z3}^{-}). \tag{A14}$$

It can be reexpressed by means of (A10)–(A13) in the four alternative forms

$$\psi=\psi_1=\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}(\varphi_{x1}^{+}+\varphi_{x1}^{-})\frac{1}{\sqrt{2}}(\varphi_{x2}^{+}+\varphi_{x2}^{-})\frac{1}{\sqrt{2}}(\varphi_{x3}^{+}+\varphi_{x3}^{-})-\frac{1}{\sqrt{2}}(\varphi_{x1}^{+}-\varphi_{x1}^{-})\frac{1}{\sqrt{2}}(\varphi_{x2}^{+}-\varphi_{x2}^{-})\frac{1}{\sqrt{2}}(\varphi_{x3}^{+}-\varphi_{x3}^{-})\right]$$

$$=\tfrac{1}{2}(\varphi_{x1}^{-}\varphi_{x2}^{+}\varphi_{x3}^{+}+\varphi_{x1}^{+}\varphi_{x2}^{-}\varphi_{x3}^{+}+\varphi_{x1}^{+}\varphi_{x2}^{+}\varphi_{x3}^{-}+\varphi_{x1}^{-}\varphi_{x2}^{-}\varphi_{x3}^{-}), \tag{A15}$$

$$\psi=\psi_2=\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}(\varphi_{x1}^{+}+\varphi_{x1}^{-})\frac{1}{\sqrt{2}}(\varphi_{y2}^{+}+\varphi_{y2}^{-})\frac{1}{\sqrt{2}}(\varphi_{y3}^{+}+\varphi_{y3}^{-})+\frac{1}{\sqrt{2}}(\varphi_{x1}^{+}-\varphi_{x1}^{-})\frac{1}{\sqrt{2}}(\varphi_{y2}^{+}-\varphi_{y2}^{-})\frac{1}{\sqrt{2}}(\varphi_{y3}^{+}-\varphi_{y3}^{-})\right]$$

$$=\tfrac{1}{2}(\varphi_{x1}^{+}\varphi_{y2}^{+}\varphi_{y3}^{+}+\varphi_{x1}^{+}\varphi_{y2}^{-}\varphi_{y3}^{-}+\varphi_{x1}^{-}\varphi_{y2}^{+}\varphi_{y3}^{-}+\varphi_{x1}^{-}\varphi_{y2}^{-}\varphi_{y3}^{+})\ , \quad \text{(A16)}$$

$$\psi=\psi_3=\tfrac{1}{2}(\varphi_{y1}^{+}\varphi_{x2}^{+}\varphi_{y3}^{+}+\varphi_{y1}^{+}\varphi_{x2}^{-}\varphi_{y3}^{-}+\varphi_{y1}^{-}\varphi_{x2}^{+}\varphi_{y3}^{-}+\varphi_{y1}^{-}\varphi_{x2}^{-}\varphi_{y3}^{+})\ , \quad \text{(A17)}$$

and

$$\psi=\psi_4=\tfrac{1}{2}(\varphi_{y1}^{+}\varphi_{y2}^{+}\varphi_{x3}^{+}+\varphi_{y1}^{+}\varphi_{y2}^{-}\varphi_{x3}^{-}+\varphi_{y1}^{-}\varphi_{y2}^{+}\varphi_{x3}^{-}+\varphi_{y1}^{-}\varphi_{y2}^{-}\varphi_{x3}^{+})\ . \quad \text{(A18)}$$

But then (A15) entails

$$\sigma_{x1}\sigma_{x2}\sigma_{x3}\psi=-\psi\ , \quad \text{(A19)}$$

(A16) entails

$$\sigma_{x1}\sigma_{y2}\sigma_{y3}\psi=\psi\ , \quad \text{(A20)}$$

(A17) entails

$$\sigma_{y1}\sigma_{x2}\sigma_{y3}\psi=\psi\ , \quad \text{(A21)}$$

and (A18) entails

$$\sigma_{y1}\sigma_{y2}\sigma_{x3}\psi=\psi\ . \quad \text{(A22)}$$

That is, the spin state ψ is a simultaneous eigenvector of the four spin operators $\sigma_{x1}\sigma_{x2}\sigma_{x3}$, $\sigma_{x1}\sigma_{y2}\sigma_{y3}$, $\sigma_{y1}\sigma_{x2}\sigma_{y3}$, and $\sigma_{y1}\sigma_{y2}\sigma_{x3}$, with eigenvalues -1, $+1$, $+1$, and $+1$, respectively. If the devices in region R_i measure σ_{xi} or σ_{yi} according to whether the orientation is "north" or "east," respectively, then the four predictions of quantum theory listed in the text follow directly from these eigenvector properties of the spin state ψ.

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