Bell theorem without inequalities for two spinless particles

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We use the Greenberger-Horne-Zeilinger [in *Bell's Theorem, Quantum Theory, and Conceptions of the Universe,* edited by M. Kafatos (Kluwer Academic, Dordrecht, 1989)] approach to present three demonstrations of the failure of Einstein-Podolsky-Rosen (EPR) [Phys. Rev. **47**, 777 (1935)] local realism for the case of two spinless particles in a two-particle interferometer. The original EPR assumptions of locality and reality do not suffice for this. First, we use the EPR assumptions of locality and reality to establish that in a two-particle interferometer, the path taken by each particle is an element of reality. Second, we supplement the EPR premises by the postulate that when the path taken by a particle is an element of reality, all paths not taken are empty. We emphasize that our approach is not applicable to a single-particle interferometer because there the path taken by the particle cannot be established as an element of reality. We point out that there are real conceptual differences between single-particle, two-particle, and multiparticle interferometry.

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INTRODUCTION

Einstein, Podolsky, and Rosen [1] (EPR), while considering various spacelike-separated measurements that may be made on two noninteracting particles, introduced a very specific viewpoint concerning local reality. "Since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system" (locality). "If, without in any way disturbing a system, we can predict with certainty (i. e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity" (reality).

It has long been known that these assumptions, (together with a third one, completeness, which we shall discuss later), reasonable as they appear to be, are inconsistent with quantum mechanics. This fact was discovered by Bell, who considered the system introduced by Bohm, of a spin-0 particle decaying into two spin-$\frac{1}{2}$ particles. In the simplest gedanken experiment, when the spin component of one of the decay particles is measured in a specific direction, quantum mechanics predicts that with 100% certainty its partner's spin will be opposite, if measured in the same direction. This behavior is sufficient to allow the use of the EPR locality and reality criteria to establish that this particular spin component of the second particle is an element of reality. By parallel arguments all spin components of both particles are such elements, and their reality must be established at the time of the original decay, even if they cannot all be simultaneously known experimentally. We call such a simple case, where the result of one measurement can be used to predict with 100% certainty the result of another measurement not yet performed, a case of perfect correlation. For two particles one may easily construct a deterministic, local, realistic model that can reproduce these perfect correlations, regardless of the direction in which the original spin measurement is made.

Bell [2] realized, however, that for more general measurements, where the spin components of the two particles are measured along different directions, the quantum-mechanical behavior cannot be modeled by any local, realistic, deterministic (or stochastic) model, thus upsetting the EPR program. He showed that starting from the EPR program one can derive an inequality, which must be obeyed by *any* such model, but which is violated by quantum theory. His proof refers to the imperfectly correlated cases, when the particles are measured along different directions. Bell's theorem says nothing about the case of perfect correlations, which indeed can be explained by such a model.

Recently it was noticed [3,4] by Greenberger, Horne, and Zeilinger (GHZ) that if three or more particles are produced in the decay, then one can show that even in the case of perfect correlations, which are at the core of the EPR viewpoint since they produce elements of reality, one cannot reproduce the quantum results by a local, realistic model. In fact, the perfect correlations by themselves contradict the other EPR assumptions.

In the present paper, we investigate the possibility of applying GHZ three-particle perfect-correlation-type arguments to two particles. We find that such an extension exists if one considers, instead of the familiar Bohm two-spin system, a two-particle interferometer. Whereas each particle in the Bohm system has only one available route from the source to the polarization analyzer, each particle in the two-particle interferometer has two available routes through the interferometer. As we shall see below, within the EPR viewpoint one and only one of these routes is actually taken by any given particle, and the route taken is an element of reality. However, in order to complete the two-particle argument, we find that we must augment the original EPR assumptions given in the first paragraph above with the following auxiliary assumption:

If the path taken by a particle is an element of reality, then there is no entity associated with this particle that in any way samples alternate paths (i.e., the paths not taken are truly *empty*).

We call this assumption "emptiness of paths not taken" (EPNT). This in turn implies that the particle can be affected only by manipulations along its path, and it cannot be affected by any manipulations that take place along alternate paths not taken.

One might initially suspect that this assumption is by itself tantamount to a denial of elementary quantum theory, since the prohibited entities include such subtleties as a de Broglie pilot wave, an Einstein "ghost" wave, any kind of information gathering Mermin fuzzy "cloud," a Wheeler "smoky dragon," or most emphatically, even a normal Schrödinger quantum-mechanical amplitude. (We are tempted to call our assumption "the law of the excluded muddle.") That is, the assumption seems to be incompatible with single-particle interference, the most basic wave-mechanical phenomenon.

This is not the case for two reasons. First, in a single-particle interferometer one cannot predict which path the particle will take through the interferometer. The path taken is therefore not an element of reality, and hence our assumption is inapplicable. Second, in the two-particle interferometer, where the path *can* be predicted and hence our assumption applies, there is in fact *no* single-particle interference. We believe that our assumption is one possible natural outgrowth of the EPR viewpoint, but in any case its consequences will be spelled out so that one may judge its plausibility.

We should also point out that the EPNT assumption is partially compatible with quantum mechanics, in the sense that if a particle is experimentally known to be in one beam, there is zero amplitude for it to be in any other beam. However, EPR assert the existence of such an element of reality, connecting the particle to a particular beam, even in cases where one has an experimental arrangement that cannot ascertain such information (see our first demonstration, below). Of course, quantum theory denies this possibility.

We will now proceed to give three demonstrations that the EPR program, if it is supplemented by the EPNT assumption, fails as profoundly at the two-particle level [5] as it has been previously demonstrated to do (GHZ) at the three-particle level. A central point of these demonstrations is the establishment of a particle's path as an element of reality and our consequent use of the EPNT assumption, which implies that only manipulations directly along a particle's path may have any influence on that particle.

In our discussion, we will focus principally on the two-particle interferometer arrangement [6] of Fig. 1, where a central source emits a pair of spinless particles, 1 and 2, in opposite directions. An experimental apparatus defines two directions a-a' and b-b'. Then the quantum state of the pair is

$$|\Psi_0\rangle=(|a\rangle_1|a'\rangle_2+|b\rangle_1|b'\rangle_2)/\sqrt{2}\ , \tag{1}$$

where ket $|a\rangle_1$ denotes particle 1 in beam a, etc.

THE FIRST DEMONSTRATION

For the *first demonstration,* we will consider a sequence of five different experiments with this arrangement. In our first experiment we establish that for each particle the path taken inside the interferometer is an EPR "element of reality" and, at the fifth experiment, we find that the "reality" of this internal path necessarily implies a total disagreement (i.e., a disagreement for each pair detected) with the quantum-mechanical predictions of the state (1).

In the first experiment, place four detectors directly in the source beams a, a', b, and b', and monitor for coincidences. It is assumed that the beam splitters at C and G have been removed, so that these coincidences can be monitored as far downstream in the beam as one wants. The state (1) predicts a-a' coincidences for half the pairs detected and b-b' coincidences for the other half of the pairs. These perfect correlations and the EPR local reality assumptions immediately imply that the path taken, (a' or b'), by particle 2 is an element of reality, for if particle 1 is found in path a (b) then particle 2 will, "with certainty," be found in path a' (b').

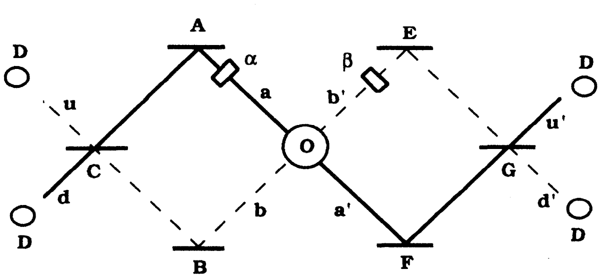

FIG. 1. *Two-particle interferometer.* A pair of particles, 1 and 2, emerges coherently at O, moving along the beams a-a' and b-b', in the state of Eq. (1). Beams a and b are deflected and pass through a beam splitter, so that particle 1 ends up in beam u or d. Similarly, particle 2 ends up in beam u' or d'. There is a phase shifter, α, along path a, and another, β, along path b'. Each beam terminates at a detector D. The outgoing state is given by Eq. (2). Note that this state implies that the two-particle coincident rates (u-u',u-d', etc.) depend on α and β. Nonetheless, there is no single-particle interference—the count rate at each detector D is independent of α and β.

A parallel argument establishes the reality of path (a or b) for particle 1. In short, according to the EPR view, half the pairs emitted by the source *really* take the a-a' paths and the other half *really* take the b-b' paths. Moreover, and we stress this point, in the EPR view these elements of reality must exist even when the detectors used to establish their existence are removed and one or both beam splitters are replaced [7], as in the next four experiments.

In the second experiment, place the beam splitter at position G in the path of particle 2, and place the detectors further downstream, beyond G. For particle 1, leave the beam splitter C out, and place the detectors directly in the paths a and b of particle 1. If these particle 1 detectors are sufficiently far from the source of decay, they can catch particle 1 after particle 2 has already passed through the beam splitter at G, either before or even after it strikes a detector [8]. Thus if the detector in beam a fires, one knows that particle 2 took path a', and this knowledge is available even after particle 2 passes the beam splitter at G. Therefore this element of reality, identifying the path that the particle 2 took within the interferometer, persists even after the particle has left the interferometer, to the time when particle 2 strikes a detector. (In fact, if we placed the detectors of particle 1 far enough downstream, we could determine which internal path particle 2 took, a week after it had been detected.) Similarly, an analogous argument establishes the reality of the path of particle 1 within the interferometer. Of course the experiment just performed cannot predict which beam u' or d' particle 2 takes after passing the beam splitter at G, so we have not yet established the reality of the path of each particle outside the interferometer, after the beam splitters at C and G. But we can know, even when a particle is beyond the interferometer, what path that particle took inside the interferometer.

For the next three experiments, place beam splitters at C and G in the paths of both particles, and move the detectors so that now all four detectors are in the beams u, d, u', and d'. Beam a encounters a device that produces a phase shift α, and beam b' encounters another one, which produces a phase shift β, as in Fig. 1. At the detectors the state (1) will evolve into (see, for example, GHSZ [4])

$$|\Psi\rangle=\frac{ie^{i(\alpha+\beta)/2}}{\sqrt{2}}[(|u\rangle_1|u'\rangle_2+|d\rangle_1|d'\rangle_2)\cos(\Delta/2)+(|u\rangle_1|d'\rangle_2-|d\rangle_1|u'\rangle_2)\sin(\Delta/2)]\ , \tag{2}$$

where $\Delta=\alpha-\beta$. (Here we have chosen specific beam-splitter phases. Other possible choices would not affect our conclusions. See GHSZ.)

In the third experiment, set the phase shifts $\alpha=\beta=0$, and again monitor for coincidences. With these phase values, the state (2) becomes

$$|\Psi\rangle=i(|u\rangle_1|u'\rangle_2+|d\rangle_1|d'\rangle_2)/\sqrt{2} \tag{3}$$

at the output. State (3) predicts u-u' coincidences for half the pairs detected, and d-d' coincidences for the other pairs. There are *no* u-d' or d-u' coincidences. As concerns the EPR program, simply assume that it successfully accounts for these coincidences. That is, when $\alpha=0$ ($\beta=0$) the pairs which take the a-a' (b-b') paths do not produce any u-d' or u'-d coincidences.

This third experiment has established the reality of the paths in the *outgoing* beams after the particles leave the interferometer, since a count at u (d) guarantees a count at $u'(d')$. The first experiment established the reality of the paths *inside* the interferometer. The second experiment forced us to accept the continuing reality of the path the particle took inside the interferometer even after the particle has left the interferometer [9]. So the EPR criterion leads us to the conclusion that the particle 2 took one and only one of the paths a',d' through the interferometer, and subsequently took one of the paths u',d' upon leaving the interferometer. We can establish any of these elements of reality for one particle by making an appropriate measurement on the other particle. (Even though *different* experiments are required on particle 1 to determine the reality of the path of particle 2 inside and beyond its interferometer, the particles no longer interact and the EPR locality assumption guarantees the existence of all these elements of reality [7].

Because the reality of the internal path can be established even after the particle has left the interferometer, we are now in a position to invoke the EPNT assumption. This assumption implies that if particle 2 took the path a', it cannot be affected by the particular value of the phase shifter β. Similarly, if particle 1 took path b, it cannot be affected by the value of the phase shifter α. We will use these results in what follows.

In the fourth experiment, keep the detectors in the outgoing beams, and adjust the phase shifters to $\alpha=\beta=\pi$, and again monitor for coincidences. Then the state at the detectors will be, from Eq. (2),

$$|\Psi\rangle=-i(|u\rangle_1|u'\rangle_2+|d\rangle_1|d'\rangle_2)/\sqrt{2}\ , \tag{4}$$

which also produces only u-u' and d-d' coincidences, i.e., *no* u-d' or d-u' coincidences will occur. For the EPR program, again assume it successfully reproduces these coincidences, namely, when $\alpha=\pi$ ($\beta=\pi$) the pairs taking the a-a' (b-b') paths produce no u-d' or u'-d coincidences.

Finally, in the fifth experiment, switch either of the phases, α or β, to 0 while leaving the other at π. We will consider for simplicity just the first case, $\alpha=0$ and $\beta=\pi$. In this case the wave function will be, according to Eq. (2),

$$|\Psi\rangle=+(|u\rangle_1|d'\rangle_2-|d\rangle_1|u'\rangle_2)/\sqrt{2}\ . \tag{5}$$

Hence quantum mechanics predicts that *only* u-d' and d-u' coincidences will occur. What does the EPR view predict here? Well, the a-a' pairs encounter $\alpha=0$ and they do not encounter the phase shifter in beam b'. Therefore, invoking the EPNT assumption, they cannot be affected by the value β of that phase shifter. We thus conclude that these pairs still produce only the same coincidences as in experiment 3 (i.e., u-u' and d-d' coincidences only). Similarly, consider now the b-b' pairs.

They encounter $\beta=\pi$ in beam b and, by the EPNT assumption, they cannot be influenced by the $\alpha=0$ phase setting in beam a. Therefore, by comparison with experiment 4, these b-b' pairs also can only produce u-u' and d-d' coincidences. So the EPR view predicts *no* u-d' or d-u' coincidences will occur, in complete disagreement with the quantum-mechanical prediction, [10].

COMMENTS

One might object that the EPNT assumption is already in obvious conflict with single-particle quantum mechanics. Consider, for example, the single-particle interferometer shown in Fig. (2). The essence of single-particle quantum mechanics [11] presents itself when one contemplates the following two experiments with this apparatus. In one experiment (call it a which-path experiment), place detectors in beams a and b and confirm that every particle is found in one of the beams only, approximately half in each beam. In the other experiment (call it an interference experiment), place detectors in beams u ad d and observe that the number of particles landing in each detector depends sensitively on the phase angle α. Quantum mechanics attributes these single-particle interference effects to the superposition of two probability amplitudes, one associated with each path.

Now one might think *(incorrectly)* that the first experiment establishes the path taken as an EPR element of reality for each particle, and one could then invoke the EPNT assumption, in the following way. Consider those particles that take route b during the second (interference) experiment. By the EPNT assumption they could not be affected by the setting α of the phase shifter in beam α. But this makes it impossible to reproduce the observed interference pattern, which has 100% contrast, i.e., for some settings of α, all the particles will be located in beam u, while for other settings, none will. The reasoning is as follows. If at one setting of α, all particles end up in beam u, then of course this includes the particles that took beam b. Similarly, for another setting of α, no particles will end up in beam u, which of course also means that no particles in beam b will have ended up in beam u. But this obviously contradicts our earlier conclusion (based on EPNT) that the particles in beam b are unaffected by the setting of α. However, as already indicated, the mistake here is to think that the above which-path experiment establishes path as an EPR element of reality. To do that one would have to be able to *predict*, without disturbing the particle, which particular path it *will take*. Instead, the which-path experiment, by simply counting particles in a beam, merely determines which path the particle *did take*, and in doing so it totally destroys the beam.

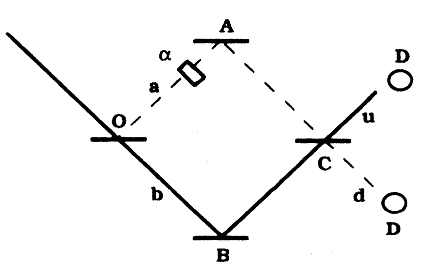

FIG. 2. *One-particle interferometer.* A single particle emerges from source O with a coherent amplitude for being in either beam, a or b. These beams recombine at C and produce interference at the detectors D. There is a phase shifter α along path a. The interference depends on α, but if we determine which path, a or b, the particle took, the interference will disappear.

One might contemplate making a more subtle experiment that does not disturb the particles in the beam, but this runs into a major problem of one-particle systems. As we saw in the previous paragraph, measuring the location (or more generally any property) of a single particle entails retrodiction, rather than prediction. For EPR, a measured property of a system is not necessarily an element of reality. For clearly, as Bohr often emphasized, the value of the property could be just an artifact of the measurement process, i.e., the observed value of the property could conceivably have only been created by the measurement. The original EPR definition of reality emphasizes and exploits prediction in formulating a criterion for reality. The objective properties of the particle are those that can be predicted, without actually touching the particle of interest. For only then can one be sure that the property exists, independently of observation.

This problem is pervasive in one-particle experiments, and will defeat even subtle attempts to impart an EPR element of reality to the path of a single particle. For example, we might have attempted in the spirit of EPR to determine the particle's path through the configuration of Fig. 2 by placing a detector in only one beam, say a, and concluding that the particle took the other beam, b, if the detector does not register. But this experiment does not predict that a given particle will be in beam b. It just eliminates the subset of particles that took beam a (certainly disturbing them), but gives no indication of when to expect a count in detectors placed in beam b, u, or d. Thus it fails to meet the predictability requirement of EPR.

In order to remedy this situation and be able to make a prediction, we must know when the particle is in fact inside the interferometer, which calls for some kind of coincidence measurement. But this transforms the entire experiment into a multiparticle one. (In fact, our two-particle interferometer is just a special case of this procedure.)

So the EPNT assumption does not apply in the single-particle case, and therefore can never conflict with one-particle interference. We also note that in the two-particle interferometer, where EPNT does apply, there is *no* one-particle interference. To see this, note from Eq. (1) that a path of particle 1 is correlated with a path of particle 2. Either the pair took paths a-a' or they took b-b'. The superposition of Eq. (1) involves the paths of the two particles, not the two possible paths of one particle. The probability of the detector in beam u counting is given from Eq. (2) as $P_u=|\langle u_1,u_2'|\Psi\rangle|^2+|\langle u_1,d_2'|\Psi\rangle|^2=\frac{1}{2}$, independent of α and β, and similarly $P_d=\frac{1}{2}$. So regardless of the settings of either phase shifter, these beams will each produce a count 50% of the time, and thus there is no single-particle interference. Clearly, the absence here of any single-particle interference indicates that two-particle interferometry places totally different demands on any theoretical explanation than does ordinary single-particle interferometry. We note that while our demonstration was given specifically in the context of the two-particle interferometer of Fig. 1, a parallel argument could be given in the context of other two particle interference arrangements, such as that of Franson [12], by utilizing other elements of reality.

Of course, one can deny EPNT and thereby imagine that *something* could travel down the empty beam, so as to provide information to the nonempty beam, when the two beams meet. And this something could be consistent with EPR locality, if the particles (and these somethings) on opposite sides of the origin do not communicate. However, one should be aware that there must be significant differences in the *nature* of this something, in the one- and the two-particle cases. In the one-particle interferometer of Fig. 2, quantum theory yields an amplitude for the particle to travel along each path through the interferometer. There is, as we have emphasized, no element of reality associated with either path. Nonetheless, one may try to assume a realist position (going beyond EPR) by asserting that the particle "really" took one path, and associating some sort of "empty," information-gathering wave with the other path. Then when they recombine, the empty wave and the particle can share the information acquired along their respective paths, in order to produce the correct behavior beyond the interferometer. But in the one-particle interferometer, once recombination occurs, there is no way to determine which path the particle took. In brief, the particle and the associated something must recombine in such a fashion as to prevent any future determination of which path the particle took inside the interferometer. The situation is quite different in the two-particle interferometer case of Fig. 1. Here, from the EPR point of view, separate elements of reality exist for the path of each particle both inside the interferometer and beyond it. Thus neither one of these particles and its associated something recombine in the same way as in the single-particle interferometer. For here, even after the particle and its associated something have recombined, the path which that particle took inside the interferometer can still be determined (by appropriate observation of the distant particle).

We emphasize that in the case of imperfect correlations, a something that samples the empty beam will still yield results in violation of the Bell inequality. [The quantum-mechanical correlation is given by Eq. (8), which is exactly the same as for two spin-1/2 particles, and leads by a parallel argument to Bell's theorem for this case.]

So something traveling along the empty path cannot explain imperfect correlations. But Bell's theorem gives no information for perfect correlations, because of the existence of an EPR element of path reality in the case. As we have stressed, it is precisely this existence of an element of reality that motivates the EPNT assumption, which goes beyond the EPR assumptions, in order to rule out these associated somethings in the perfect correlation case.

Our comments here do not apply to the theory of Bohm [13], a realistic theory that does not retain EPR locality. In this theory distant nonlocal effects can be carried instantaneously by the "quantum potential" which is built into the theory.

ALTERNATIVE DEMONSTRATIONS

We shall proceed to give the other two demonstrations. For our *second demonstration,* consider the joint probability P that detectors u and u' count coincidentally as a function of α and β. (Here, and in what follows, we shall denote the detectors by the beams they monitor, such as detector u, etc.) Quantum mechanically, this probability is a function, $P(\alpha-\beta)$, of the phase differences only, $P(\alpha-\beta)=(\frac{1}{2})[\cos^2(\alpha-\beta)/2]$, from the state (2). The EPR viewpoint, supplemented by the EPNT assumption, requires that this probability be expressible as

$$P(\alpha-\beta)=\tfrac{1}{2}f(\alpha)+\tfrac{1}{2}g(\beta) , \tag{6}$$

where $f(\alpha)$ $[g(\beta)]$ is the joint probability for those pairs that really take paths a-a'[b-b' and hence only encounter the $\alpha(\beta)$ phase shifter. The only solution [14] to Eq. (6) is $f(\alpha)=c\alpha+d$, c and d being constants, and similarly for $g(\beta)$. This simple linearity of P clearly contradicts quantum mechanics.

In order to present our third demonstration we must briefly review Bell's pioneering work, in the context of our apparatus. We assign to particle 1 the value $+1$ (-1) when it triggers detector u (d), and to particle 2 the value $+1$ (-1) when it triggers detector d' (u'). Let $P_{ij}(\alpha,\beta)$ denote the joint probability that particle 1 gives the result $i(=\pm1)$ and particle 2 the result $j(=\pm1)$. Then, following Bell, consider the expectation value of the product of the results,

$$E(\alpha,\beta)=P_{++}(\alpha,\beta)-P_{+-}(\alpha,\beta)-P_{-+}(\alpha,\beta) +P_{--}(\alpha,\beta) . \tag{7}$$

For the quantum state (2) this becomes

$$E_{\rm QM}(\alpha,\beta)=-\cos(\alpha-\beta) , \tag{8}$$

which is the same result as for the spin state Bell originally considered, except that phases replace Stern-Gerlach orientations.

Now EPR, in addition to their clearly stated views on locality and reality, also insisted on a third postulate (completeness): "Every element of the physical reality must have a counterpart in the [complete] physical theory." Bell's first crucial discovery was that *any* theory consistent with EPR's three postulates must admit the EPR-Bell form (see GHSZ)

$$E(\alpha,\beta)=\int d\lambda\,\rho(\lambda)A_\lambda(\alpha)\beta_\lambda(\beta) . \tag{9}$$

Here λ is simply Bell's notation for EPR's complete state [15] of a specific pair of particles, $A_\lambda(\alpha)$ denotes which detector particle 1 reaches and has values ±1 as given above, and $B_\lambda(\beta)$ is the same for particle 2, and the normalized density $\rho(\lambda)$ specifies the distribution of λ values within the ensemble of pairs.

Bell's second crucial discovery was that the EPR-Bell form of Eq. (9) implies an inequality not satisfied by the quantum-mechanical function of Eq. (8). Recent experimental confirmation [6] of Eq. (8) in the arrangement of Fig. 1 has added to the weight of previous polarization experiments violating Bell's inequality and thereby refuting the EPR viewpoint.

Our *third demonstration* uses the EPR-Bell form, Eq. (9). In view of the reality of the paths established above in the first demonstration, the EPR viewpoint, supplemented by the EPNT assumption, requires us to write this equation as

$$E(\alpha,\beta)=\int_{\lambda'\in\Lambda_a} d\lambda'\rho(\lambda')A_{\lambda'}(\alpha)B_{\lambda'} + \int_{\lambda''\in\Lambda_b} d\lambda''\rho(\lambda'')A_{\lambda''}B_{\lambda''}(\beta) , \quad (10)$$

where $\Lambda_a(\Lambda_b)$ is the subspace of λ states that describe pairs which take the a-a'(b-b') paths. In the first integral $B_{\lambda'}$ does not depend on β, since a particle on path a' cannot be affected by a phase shifter in path b', and similarly in the second integral $A_{\lambda''}$ does not depend on α.

Now reconsider the second experiment from our first demonstration (with detectors in the outgoing beams) but this time set the phase shifters so that $\alpha=\beta$ (but not necessarily $=0$). In this situation, the quantum-mechanical expectation value, Eq. (8), is an extremum, and $E_{\rm QM}=-1$. Consequently, if the EPR-Bell form of Eq. (9) and the EPR form (10) are to match this result, then we must have [16]

$$A_\lambda(\alpha)B_\lambda=-1 \text{ for } \lambda\in\Lambda_a \quad (11a)$$

(when $\alpha-\beta=0$:)

$$A_\lambda B_\lambda(\beta)=-1 \text{ for } \lambda\in\Lambda_b \quad (11b)$$

This is true for any individual value of λ, not merely for the integral as a whole. Similarly, if we redo this experiment again, this time with $\alpha-\beta=\pi$, we will again find that $E_{\rm QM}$ is an extremum, only now with $E_{\rm QM}=+1$. So we will now have

$$A_\lambda(\alpha)B_\lambda=+1 \text{ for } \lambda\in\Lambda_a \quad (12a)$$

(when $\alpha-\beta=\pi$:)

$$A_\lambda B_\lambda(\beta)=+1 \text{ for } \lambda\in\Lambda_b \quad (12b)$$

The angle β does not appear in Eq. (11a), so that by varying α and choosing the appropriate $\beta(=\alpha)$, we can make the equation hold for almost all α, and the same is true for all the Eqs. (11) and (12). However, it suffices to choose three appropriate pairs of phase angles α and β—for example, $(0,0)$, $(\pi,0)$, and (π,π)—one exhibits a contradiction between Eqs. (11) and (12), no matter how the space of λ states is partitioned into the mutually exclusive and exhaustive subspaces Λ_a and Λ_b. Hence it is impossible to define a consistent set of A_λ, and B_λ for this system. No EPR-Bell form (9) can agree with quantum mechanics.

CONCLUSIONS

We have shown that one can make a Bell theorem without inequalities in the case of two spinless particles. To do this we had to extend the EPR point of view to include the EPNT assumption. All three demonstrations given here depend crucially on the same feature: the paths taken inside the two-particle interferometer of Fig. 1 are EPR "elements of reality." Moreover, all three proceed without any inequalities. However, the different demonstrations have distinctive features. The first two do not employ the EPR-Bell form and hence do not use, at least explicitly, the EPR completeness postulate. The third demonstration does use the EPR-Bell form but only in the case of perfect correlation, i.e., where $E_{\rm QM}=\pm 1$, and where one can make 100% certain predictions, as demanded by EPR.

The extra assumption was chosen to exploit a real difference that exists between single-particle and two-particle interference. As already emphasized in the Comments section, it is not possible in a single-particle interferometer to establish the path taken by the particle as an EPR element of reality. Thus, to our knowledge, no one so far has exhibited any real connection between the EPR viewpoint and single-particle quantum mechanics [17]. This is not surprising, since the EPR viewpoint is couched critically in terms of two particles.

In this paper we focused on the two-particle interferometer of Fig. 1 where it is possible to establish paths as EPR elements of reality. We proposed that the paths not taken are truly empty, the EPNT assumption. This extra assumption allowed us to establish a contradiction with quantum mechanics even for the simple case of perfect correlations.

These considerations point out that there are real conceptual differences between single-particle, two-particle, and three- or multiparticle interferometry, which are not generally recognized. For single-particle interferometry, EPR considerations do not apply. For two-particle interferometry, they do apply, and one can obtain a Bell-type statistical contradiction between the EPR assumptions and quantum theory. With an auxiliary assumption (EPNT) one can obtain a contradiction using only perfect correlations. Finally, for three- or multiparticle interferometry one can obtain a contradiction between the EPR assumptions and quantum theory using only perfect correlations, without making any extra assumptions [18].

The two-particle experiments discussed here are more amenable to experimental realization with current technology than a three- experiment. Moreover, the argument presented here is also applicable to other two-particle arrangements [13] besides the interferometer of Fig. 1. One could also contemplate a realization exploiting the usual spin-entangled state, if suitable spin analyzers (Stern-Gerlach magnets for spin-$\frac{1}{2}$ particles, a Glans-Thomson prism for photons) are used to separate spin components into individual beams and later recombine them [19].

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[1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).

[2] The original demonstration was by J. S. Bell, Phys. **1**, 195 (1964). Reprinted in J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University, London, 1987).

[3] The basic technique we use for presenting Bell-type theorems without inequalities was developed by D. M. Greenberger, M. Horne, and A. Zeilinger, in *Bell's Theorem, Quantum Theory, and Conceptions of the Universe,* edited by M. Kafatos (Kluwer, Dordrecht, 1989). An especially cogent example of the three-particle case can be found in N. D. Mermin, Phys. Today **43**(6), 9 (1990).

[4] A critical review of the original EPR, Bell, and GHZ assumptions and results can be found in D. M. Greenberger, M. A. Horne, A. Shimony, and A. Zeilinger, Am. J. Phys. **58**, 1131 (1990). We refer to this paper as GHSZ.

[5] A. Peres, Phys. Lett. A **151**, 107 (1990) shows that perfect correlations for a two-particle system with spin-$\frac{1}{2}$ contradict certain plausible reality assumptions. However, he does not use the same assumptions as EPR. Similar comments apply to a complicated proof for a system of two particles with spin 1, given by S. Kochen (unpublished), and independently by P. Heywood and H. Redhead, Found. Phys. **13**, 481 (1983). This proof has been simplified by A. Stairs, Philos. Sci. **50**, 587 (1983), and even further by H. R. Brown and G. Svetlichny, Found. Phys. **20**, 1379 (1990), and A. Elby, *ibid.* **20**, 1389 (1990).

[6] The two-particle interferometer was first discussed by M. A. Horne and A. Zeilinger, in *Symposium on the Foundations of Modern Physics,* edited by P. Lahti and P. Mittelstaedt (World Scientific, Singapore, 1985), p. 435. See also M. A. Horne, A. Shimony, and A. Zeilinger, Phys. Rev. Lett. **62**, 2209 (1989). Recently, the technique of parametric down-conversion has transformed it into an experimental reality. The first Bell-type experiment not using polarization was performed by J. G. Rarity and P. R. Tapster, Phys. Rev. Lett. **64**, 2495 (1990).

[7] EPR anticipate the objection that only those elements of reality that can be simultaneously measured ought to exist for a system. They point out that such elements would depend for their reality on the order in which measurements are made on another system, not interacting with the first one. They conclude that "No reasonable definition of reality could be expected to permit this."

[8] By placing the detectors at various distances from the source, one can not only establish the path as an element of reality but also the position along that path as a function of time. This behavior is not included in our time-independent beam-entangled state (1) but it readily follows if one includes energy conservation in the description of the source. See M. Horne, A. Shimony, and A. Zeilinger, in *Quantum Coherence,* edited by J. Anandan (World Scientific, Singapore, 1990), p. 356.

[9] Note that the phase shifter used in experiment 3 could already have been in place during experiments 1 and 2 without affecting the argument.

[10] If one accepts the ENPT assumption, this demonstration can easily be turned into a Mermin-type red-green light thought experiment. See N. D. Mermin, Amer. J. Phys. **58**, 731 (1990).

[11] "[Double-slit interference is] a phenomenon which is impossible, *absolutely impossible,* to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the *only* mystery. We cannot make the mystery go away by explaining how it works." R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures in Physics* (Addison-Wesley, Reading, MA, 1965).

[12] J. D. Franson, Phys. Rev. Lett. **62**, 2205 (1989). This proposed experiment was performed by P. G. Kwiat, W. A. Vareka, C. K. Hong, H. Nathel, and R. Y. Chiao, Phys. Rev. A **41**, 2910 (1990); and also by Z. Y. Ou, X. Y. Zou, L. J. Wang, and L. Mandel, Phys. Rev. Lett. **65**, 321 (1990).

[13] D. Bohm, Phys. Rev. **85**, 166,180 (1952). This theory is described and analyzed in great detail in F. Selleri, *Quantum Paradoxes and Physical Reality* (Kluwer, Dordrecht, 1990).

[14] This is literally true only for rational α, and there are pathological solutions for general α, unless one imposes reasonable restrictions such as continuity, or weaker, monotonicity in a finite interval, or appropriate boundedness in a finite interval.

[15] The state λ is sometimes called a "hidden variable."

[16] Since these conditions are defined by extremum values of the integral in Eq. (10), they must be understood in the measure-theoretic sense of being true except on a set of measure zero; see GHSZ, and R. K. Clifton, M. L. G. Redhead, and J. Butterfield, Found. Phys. **21**, 149 (1990).

[17] There are theories such as Bohm's, Ref. [13], which contain hidden variables, but these variables cannot be established as EPR elements of reality in the one-particle case. S. M. Tan, D. F. Walls, and M. J. Collett [Phys. Rev. Lett. **66**, 252 (1991)] have discussed an interesting experiment which they interpret as a one-particle photon EPR example; however, we maintain that this is a multiphoton experiment.

[18] As this paper was going to press, another paper appeared, which offers a different Bell theorem without inequalities for the case of two particles: L. Hardy, Phys. Rev. Lett. **68**, 2981 (1992).

[19] Preliminary versions of this paper have been published in D. M. Greenberger *et al., Proceedings of the Cesena Conference on Problems in Quantum Theory,* edited by A. Van der Merwe (World Scientific, Singapore, in press); and in H. J. Bernstein, *Proceedings of the 1992 Moriand Conference,* edited by Tran Nguyen (Editions Frontières, Gif Sur Yvette, in press).