Dynamic Lie-algebra structure of a quantum system and the Aharonov-Anandan phase

Shi-Min Cui
Chinese Center for Advanced Science and Technology (World Laboratory), P.O. Box 8730, Beijing 100080, People's Republic of China
*and Institute of Condensed Matter Physics, Department of Applied Physics, Jiao Tong University, Shanghai 200030, People's Republic of China**

(Received 13 March 1992)

The dynamic Lie-algebra structure decomposition of the time-dependent Schrödinger equation for a quantum system is analyzed and the results are used to study the Aharonov-Anandan phase. The expression for Aharonov-Anandan phase is given, which is shown to be useful by the illustration of some examples and to be applicable to rather general systems.

PACS number(s): 03.65.−w

I. INTRODUCTION

In recent years there has been increased interest in the study of quantum systems that undergo a cyclic evolution. This is due to the original observation made by Berry [1] that, in the adiabatic case, the state of the system contains an additional nonintegrable phase which is observable [2–6]. Since then, many papers have been devoted to interpretation [7–12] and generalization [13–18] of this result. In particular, it turns out that a geometrical phase can be defined [13,19] for any path $[0,T]\rightarrow\mathcal{H}$ in the Hilbert space $\mathcal{H}$ of states such that $|\Psi(T)\rangle=\exp\{i\Phi\}|\Psi(0)\rangle$ for some real Φ (cyclic evolution), $|\Psi(t)\rangle$ being the state at time t according to the time-dependent Schrödinger equation. Moreover, such a geometrical phase is the same for all paths in $\mathcal{H}$ which project to a given closed curve in the projective Hilbert space. In the adiabatic limit, the phase so defined is a gauge-invariant generalization of Berry's phase. In the nonadiabatic case, however, such an important definition is too general and formal to provide a tractable calculation of the nonadiabatic Berry's phase, since it is related to the dynamical effect on the adiabatic Berry's phase as a whole physical problem [20]. The concrete study of the nonadiabatic Berry's phase depends on the specific structure of the Hamiltonian of the systems and some methods dealing with the special problems [21–24] are proposed for this reason. In this paper, we will study the Aharonov-Anandan (AA) phase [13] for a class of rather general cases that the Hamiltonian of the system $\hat{H}[R(t)]$ possesses a nonsymmetry dynamic Lie-algebra structure, i.e.,

$$\hat{H}(t)\equiv\hat{H}[R(t)]=\sum_{i=1}^{N}R_i(t)\hat{T}_i\ , \tag{1}$$

where R is a set of parameters on the parameter manifold $\mathcal{M}=\{R=(R_1(t),R_2(T),\ldots,R_N(t))\}$ with $R(0)=R(T)$, and $\hat{T}_i$ $(i=1,2,\ldots,N)$ are the generators of a semisimple Lie algebra $\mathcal{L}$. The AA phase for the system is obtained and calculated for several practical examples.

II. LIE-ALGEBRA STRUCTURE DECOMPOSITION OF THE SCHRÖDINGER EQUATION

According to the structure theory of Lie algebra [25], the Lie algebra $\mathcal{L}$ can be decomposed into a direct sum of some simple Lie algebras $\mathcal{L}_i$ $(i=1,2,\ldots,s)$, i.e., $\mathcal{L}=\mathcal{L}_1\oplus\mathcal{L}_2\oplus\cdots\oplus\mathcal{L}_s$. Correspondingly, the Hamiltonian $\hat{H}[R]$ is decomposed as $\hat{H}=\hat{H}_1\oplus\hat{H}_2\oplus\cdots\oplus\hat{H}_s$ with $\hat{H}_i\in\mathcal{L}_i$. Thus the time-dependent Schrödinger equation ($\hbar$ is taken to be 1 in this paper)

$$i\frac{\partial}{\partial t}|\Psi(t)\rangle=\hat{H}|\Psi(t)\rangle\ , \tag{2}$$

can be decomposed as

$$\begin{aligned}&i\frac{\partial}{\partial t}|\Psi_i(t)\rangle=\hat{H}_i|\Psi_i(t)\rangle,\quad i=1,2,\ldots,s\ ,\\&|\Psi(t)\rangle=|\Psi_1(t)\rangle|\Psi_2(t)\rangle\cdots|\Psi_s(t)\rangle\ .\end{aligned} \tag{3}$$

Therefore, the problem with $\mathcal{L}$ as a semisimple Lie algebra can be reduced to the several cases with simple Lie algebras. Without losing the generality, in the following discussions we assume that $\mathcal{L}$ is a simple Lie algebra. We rewrite the Hamiltonian (1) on the Cartan-Weyl basis $\{\mathfrak{H}_i,\mathfrak{E}_\alpha\}$ of $\mathcal{L}$ as

$$\hat{H}(t)\equiv\hat{H}[R(t)]=\sum_{i=1}^{l}\gamma_i[R(t)]\mathfrak{H}_i+\sum_{\alpha=1}^{d=N-l}\gamma_\alpha[R(t)]\mathfrak{E}_\alpha\ , \tag{4}$$

where $\gamma_i[R(t)]$ and $\gamma_\alpha[R(t)]$ are the linear combinations of $R_i(t)$, $\mathfrak{H}_i$ are the generators of the Cartan subalgebra $\mathcal{C}$ or $\mathcal{L}$, and $\mathfrak{E}_\alpha$ ($\mathfrak{E}_{-\alpha}$) are the raising (lowering) operators. $\mathfrak{H}_i$ and $\mathfrak{E}_\alpha$ obey standard commutation relations [25]

$$\begin{aligned}&[\mathfrak{H}_i,\mathfrak{H}_j]=0,\quad i,j=1,2,\ldots,l\ ,\\&[\mathfrak{H}_i,\mathfrak{E}_\alpha]=\alpha_i\mathfrak{E}_\alpha,\quad \alpha=1,2,\ldots,d\ ,\\&[\mathfrak{E}_\alpha,\mathfrak{E}_{-\alpha}]=\alpha^i\mathfrak{H}_i\ ,\\&[\mathfrak{E}_\alpha,\mathfrak{E}_\beta]=N_{\alpha\beta}\mathfrak{E}_{\alpha+\beta},\quad\text{for }\alpha+\beta\neq0\ ,\end{aligned} \tag{5}$$

with α_i (α^i) the covariant (contravariant) component of a

 ©1993 The American Physical Society

root α. In order to find the solution of the time-dependent Schrödinger equation we introduce a nonunitary operator

$$\hat{S}(t)=\prod_{\alpha=1}^{d}\hat{S}_{\alpha}(t)=\prod_{\alpha=1}^{d}\exp[-i\beta_{\alpha}(t)\mathfrak{E}_{\alpha}] . \tag{6}$$

Let

$$|\Psi(t)\rangle=\hat{S}(t)|\psi(t)\rangle , \tag{7}$$

then the equation of motion for $|\psi(t)\rangle$ is

$$i\frac{\partial}{\partial t}|\psi(t)\rangle=\hat{h}(t)|\psi(t)\rangle , \tag{8}$$

where the operator $\hat{h}(t)$ is

$$\hat{h}(t)=\hat{S}^{-1}\hat{H}\hat{S}-i\hat{S}^{-1}\frac{\partial\hat{S}}{\partial t} . \tag{9}$$

Obviously, $\hat{h}$ is not Hermitian due to the nonunitarity of $\hat{S}$. By using Eq. (5), Eq. (9) can be written as

$$\hat{h}(t)=\sum_{i=1}^{l}\Gamma_i\left[\left\{\beta_{\alpha'}(t),\frac{\partial\beta_{\alpha'}(t)}{\partial t}\right\}\right]\mathfrak{H}_i+\sum_{\alpha=1}^{d}\Gamma_{\alpha}\left[\left\{\beta_{\alpha'}(t),\frac{\partial\beta_{\alpha'}(t)}{\partial t}\right\}\right]\mathfrak{E}_{\alpha} , \tag{10}$$

where Γ_i and Γ_α are functions related to the structure of $\mathcal{L}$. Now we assume that the d unknown parameters $\beta_\alpha(t)$ is determined by the equations

$$\Gamma_{\alpha}\left[\left\{\beta_{\alpha'}(t),\frac{\partial\beta_{\alpha'}(t)}{\partial t}\right\}\right]=0,\quad \alpha=1,2,\ldots,d , \tag{11}$$

with the boundary condition

$$\beta_\alpha(0)=\beta_\alpha(T),\quad \alpha=1,2,\ldots,d . \tag{12}$$

Then Eq. (10) becomes

$$\hat{h}(t)=\sum_{i=1}^{l}\Gamma_i\left[\left\{\beta_{\alpha'}(t),\frac{\partial\beta_{\alpha'}(t)}{\partial t}\right\}\right]\mathfrak{H}_i . \tag{13}$$

The solutions of Eqs. (11) and (12) determine a submanifold $\mathcal{N}=\{\beta\equiv(\beta_1,\beta_2,\ldots,\beta_d)\}$. The closed curve C_R: $\{R(t)|R(0)=R(T)\}$ on $\mathcal{M}$ corresponds to a closed curve C_β: $\{\beta(t)|\beta(0)=\beta(T)\}$ on $\mathcal{N}$. Obviously, in the adiabatic limit, the submanifold $\mathcal{N}=\{\beta\equiv(\beta_1,\beta_2,\ldots,\beta_d)\}$ is determined by the equation

$$\Gamma_\alpha(\{\beta_{\alpha'}(t),0\})=0,\quad \alpha=1,2,\ldots,d . \tag{14}$$

III. AHARONOV-ANANDAN GEOMETRIC PHASE

Suppose $|LM\rangle$ are eigenvectors of $\mathfrak{H}_i$,

$$\mathfrak{H}_i|LM\rangle=M_i|LM\rangle,\quad i=1,2,\ldots,l , \tag{15}$$

where L is the greatest weight and M_i is a component of a weight M. The solutions of (2) and (8) are

$$|\psi(t)\rangle=\prod_{i=1}^{l}\exp\left[-i\int_0^t dt\,\Gamma_i\left[\left\{\beta_\alpha,\frac{\partial\beta_\alpha}{\partial t}\right\}\right]\mathfrak{H}_i\right]|\psi(0)\rangle , \tag{16}$$

$$|\Psi(t)\rangle=\hat{U}(t)|\psi(0)\rangle , \tag{17}$$

with

$$\hat{U}(t)=\prod_{\alpha=1}^{d}\exp[-i\beta_\alpha\mathfrak{E}_\alpha]\prod_{i=1}^{l}\exp\left[-i\int_0^t dt\,\Gamma_i\left[\left\{\beta_\alpha,\frac{\partial\beta_\alpha}{\partial t}\right\}\right]\mathfrak{H}_i\right] . \tag{18}$$

We now consider a cyclic solution which is related to Berry's topological phase. If the initial state is chosen as

$$|\psi_{LM}(0)\rangle=|LM\rangle , \tag{19}$$

then

$$|\Psi_{LM}(t)\rangle=\prod_{i=1}^{l}\exp\left[-i\int_0^t dt\,\Gamma_i\left[\left\{\beta_\alpha,\frac{\partial\beta_\alpha}{\partial t}\right\}\right]M_i\right]\prod_{\alpha=1}^{d}\exp[-\beta_\alpha\mathfrak{E}_\alpha]|LM\rangle . \tag{20}$$

After one period, a nonadiabatic topological phase shift will be induced. Using (12) in (20) we have

$$|\Psi_{LM}(T)\rangle=\exp[-i\Phi_{LM}]|\Psi_{LM}(0)\rangle , \tag{21}$$

where the total phase shift is

$$\Phi_{LM}=\int_0^T dt\sum_{i=1}^{l}\Gamma_i\left[\left\{\beta_\alpha,\frac{\partial\beta_\alpha}{\partial t}\right\}\right]M_i . \tag{22}$$

Following the definition given by Aharonov and Anandan [13], the dynamical phase is readily obtained by using Eq. (20):

$$\int_0^T dt\langle\Psi_{LM}(t)|\hat{H}(t)|\Psi_{LM}(t)\rangle=\int_0^T dt\sum_{i=1}^{l}\Gamma_i\left|\beta_\alpha,\frac{\partial\beta_\alpha}{\partial t}\right|M_i+i\int_0^T dt\left\langle LM\left|\hat{S}^{-1}\frac{\partial\hat{S}}{\partial t}\right|LM\right\rangle , \tag{23}$$

and the AA phase

$$\gamma_{LM}=i\int_0^T dt\left\langle LM\left|\hat{S}^{-1}\frac{\partial\hat{S}}{\partial t}\right|LM\right\rangle$$
$$=\oint_{C_\beta}d\beta_\alpha\left\langle LM\left|\sum_{\alpha-1}^{d}\left\{\prod_{\alpha'=\alpha+1}^{d}\exp[-i\beta_{\alpha'}\mathfrak{E}_{\alpha'}]\right\}^{-1}\mathfrak{E}_\alpha\left\{\prod_{\alpha'=\alpha+1}^{d}\exp[-i\beta_{\alpha'}\mathfrak{E}_{\alpha'}]\right\}\right|LM\right\rangle . \quad (24)$$

Here γ_{LM} is expressed as an element of holonomy group on the submanifold $\mathcal{N}$. This means that the AA phase is only related to the connection and holonomy group on the d-dimensional submanifold $\mathcal{N}$ but not on the N-dimensional manifold $\mathcal{M}$. In other words, the AA phase for all the parameter manifolds $\mathcal{M}$ with the same submanifold $\mathcal{N}$ is completely determined by the geometrical properties of $\mathcal{N}$ and the embedding ways of $\mathcal{N}$ in $\mathcal{M}$.

IV. THE APPLICATION OF THE SU(1,1) AND SU(2) CASES

We now turn to the discussion of the AA phase for two practical examples to illustrate the general formalism above. Let us first consider the SU(1,1) system [22]:

$$\hat{H}(t)=A(t)K_0+f(t)K_+ +f^*(t)K_- , \quad (25)$$

and A is real, and $K_\pm$ and K_0 satisfy the su(1,1) algebra

$$[K_+,K_-]=-2K_0,\quad [K_0,K_\pm]=\pm K_\pm . \quad (26)$$

Let us introduce

$$\hat{S}(t)=\exp[-i\beta_+K_+]\exp[-i\beta_-K_-] . \quad (27)$$

By using (26) in (9)–(12), we obtain

$$\hat{h}(t)=(A-2if^*\beta_+)K_0 , \quad (28)$$

whose eigenvector $|mk\rangle$ obeys $K_0|mk\rangle=(m+k)|mk\rangle$ with k the Bargmann index, and

$$\frac{d\beta_+}{dt}=f-iA\beta_+-\beta_+^2f^* ,$$
$$\frac{d\beta_-}{dt}=f^*+iA\beta_-+2\beta_+\beta_-f^* . \quad (29)$$

The AA phase is given by

$$\gamma_{mk}=i(m+k)\oint_{C_\beta}(\beta_-d\beta_+-\beta_+d\beta_-)$$
$$=2i(m+k)\oint_{C_\beta}\beta_i d\beta_+ , \quad (30)$$

Thus, the AA phase on $\mathcal{M}=\{A,f,f^*\}$ depends only on the geometry of the two-dimensional submanifold $\mathcal{N}=\{\beta_+,\beta_-\}$. In the adiabatic limit, Eq. (29) reduces to

$$f-iA\beta_+-\beta_+^2f^*=0 ,$$
$$f^*+iA\beta_-+2\beta_+\beta_-f^*=0 , \quad (31)$$

and Berry's phase can be obtained from (30) and (31),

$$\gamma_{mk}=-i(m+k)\oint_C\frac{A-f-f^*}{(A^2-4f^*f)^{1/2}}d\left[\frac{f-f^*}{A-f-f^*}\right] , \quad (32)$$

where C is a closed curve in the parameter space.

Now consider the SU(2) dynamical group which is of practical and theoretical interest. The Hamiltonian is assumed to be [24]

$$\hat{H}=\boldsymbol{\omega}\cdot\mathbf{L}=\tfrac{1}{2}(\omega_x-i\omega_y)L_+ +\tfrac{1}{2}(\omega_x+i\omega_y)L_- +\omega_zL_z , \quad (33)$$

where L_z and $L_\pm$ are the generators of the su(2) Lie algebra. Following the procedure described above, we obtain the expression on the two-dimensional submanifold $\mathcal{N}=\{\beta_+,\beta_-\}$ of the AA phase on the manifold $\mathcal{M}=\{\omega_x,\omega_y,\omega_z\}$:

$$\gamma_{lm}=im\oint_{C_\beta}(\beta_+d\beta_--\beta_-d\beta_+) , \quad (34)$$

corresponding to the eigenvector $|lm\rangle$ of the operator L_z with the eigenvalue m. The closed curve C_ω: $\{\boldsymbol{\omega}(t)|\boldsymbol{\omega}(0)=\boldsymbol{\omega}(T)\}$ on $\mathcal{M}$ corresponds to a closed curve C_β: $\{\beta(t)|\beta_\pm(0)=\beta_\pm(T)\}$ on $\mathcal{N}$:

$$\frac{d\beta_+}{dt}=\frac{1}{2}\omega_x(1+\beta_+^2)-\frac{i}{2}\omega_y(1-\beta_+^2)-i\omega_z\beta_+ ,$$
$$\frac{d\beta_-}{dt}=\frac{1}{2}\omega_x(1-2\beta_+\beta_-)+\frac{i}{2}\omega_y(1-2\beta_+\beta_-)+i\omega_z\beta_- . \quad (35)$$

Equations (34) and (35) show that the AA phase is completely determined by the geometrical properties of $\mathcal{N}$ (not $\mathcal{M}$) and the embedding ways of $\mathcal{N}$ in $\mathcal{M}$. In general, the AA phase should be calculated numerically, although it is straightforward to obtain it from Eqs. (34) and (35), since the solutions of Eq. (35) should be obtained by numerical calculation. In order to illustrate analytically the above results, we consider a simple case: $\omega_x-i\omega_y=\omega_{xy}\exp(-i\omega_0t)$, ω_{xy} and ω_z are time-independent real parameters. From Eqs. (34) and (35) we can easily obtain the AA phase [24],

$$\gamma_{lm}=-2m\pi\left[1-\frac{\omega_z-\omega_0}{(\omega_{xy}^2+\omega_z^2+\omega_0^2-2\omega_z\omega_0)^{1/2}}\right] . \quad (36)$$

V. CONCLUDING REMARKS

In conclusion, we have investigated the dynamic Lie-algebra structure of the Schrödinger equation and the AA phase for a quantum system with a dynamical Lie group. An expression for the AA phase is given, which is useful indicated by the illustration of the SU(1,1) and SU(2) examples.

Finally, we want to extend the approach adopted in this paper to rather general systems as follows.

(1) The above discussion can be extended to any case with the Hamiltonian $\hat{H}$ taking the form of Eq. (9):

$$\hat{H}=\hat{S}\hat{h}\hat{S}^{-1}+i\frac{\partial\hat{S}}{\partial t}=\hat{S}^{-1} , \quad (37)$$

where $\hat{S}$ is representation of a Lie group and $\hat{h}$ can be solved explicitly. For example, the Hamiltonian that produces the coherent state is

$$\hat{H}=\omega(t)[\hat{a}^{\dagger}-\alpha^*(t)][\hat{a}-\alpha(t)] , \quad (38)$$

where $\hat{a}^{\dagger}$ and $\hat{a}$ are creation and annihilation operators, respectively, that satisfy

$$[\hat{a},\hat{a}^{\dagger}]=1,\quad [\hat{a},\hat{a}]=[\hat{a}^{\dagger},\hat{a}^{\dagger}]=0 . \quad (39)$$

Let us introduce

$$\begin{aligned}\hat{S}&=\exp(\beta\hat{a}^{\dagger}-\beta^*\hat{a})\\ &\equiv\exp(\beta\hat{a}^{\dagger})\exp(-\beta^*\hat{a})\exp(\tfrac{1}{2}\beta^*\beta) .\end{aligned} \quad (40)$$

Using (40) in (9)–12), we have

$$\hat{h}(t)=\omega(t)\hat{a}^{\dagger}\hat{a}+\frac{\omega(t)}{2}(\alpha^*\alpha+\beta^*\beta-\alpha^*\beta-\alpha\beta^*) , \quad (41)$$

whose eigenvector $|n\rangle$ obeys $\hat{a}^{\dagger}|\hat{a}|n\rangle=n|n\rangle$, and

$$\begin{aligned}\frac{d\beta}{dt}&=-i\omega\beta+i\omega\alpha ,\\ \frac{d\beta^*}{dt}&=i\omega\beta^*+i\omega\alpha^* .\end{aligned} \quad (42)$$

The AA phase is given by

$$\gamma_n=\frac{i}{2}\oint_{C_\beta}(\beta^*\,d\beta-\beta\,d\beta^*)=i\oint_{C_\beta}\beta^*\,d\beta . \quad (43)$$

When $|d\alpha/dt|\ll\omega$, we have $\beta\simeq\alpha$ and recover the results obtained in the adiabatic limit [26] under a more general condition. This example further indicates that the approach adopted in this paper is useful.

(2) The expression (24) of the AA phase is also applicable to a rather general case with the Hamiltonian as a nonlinear function of the Lie group generators: $\hat{H}[R]=H(\sum_{i=1}^{N}R_iT_i)$. For example, we have $\hat{H}[J]=[\beta_zJ_0+\frac{1}{2}(B_x-iB_y)J_+ +\frac{1}{2}(B_x+iB_y)J_-]^2$ for the nuclear quadrupole resonance [11,17,27].

(3) It may be of interest to point out that a noncyclic and nonunitary generalization [14] is being studied.

ACKNOWLEDGMENT

The work is supported in part by the Chinese National Science Foundation.

*Address for correspondence.

[1] M. V. Berry, Proc. R. Soc. London, Ser. A **392**, 45 (1984).
[2] R. Y. Chiao and Y. S. Wu, Phys. Rev. Lett. **57**, 933 (1986); A. Tomita and R. Y. Chiao, *ibid.* **57**, 937 (1986); R. Simon, H. J. Kimble, and E. G. C. Sudarshan, *ibid.* **61**, 19 (1988).
[3] T. Bitter and D. Dubbers, Phys. Rev. Lett. **59**, 251 (1987); D. J. Richardson, A. L. Kilvington, T. Green, and S. K. Lamoreaux, *ibid.* **61**, 2030 (1988).
[4] D. M. Bird and A. R. Preston, Phys. Rev. Lett. **61**, 2863 (1988).
[5] R. Tycko, Phys. Rev. Lett. **58**, 2281 (1987).
[6] G. Delacretaz, E. R. Grant, R. L. Whetten, L. Wöste, and J. W. Zwanziger, Phys. Rev. Lett. **56**, 2598 (1986); F. S. Ham, *ibid.* **58**, 725 (1987).
[7] B. Simon, Phys. Rev. Lett. **51**, 2167 (1983); A. J. Niemi and G. W. Semenoff, *ibid.* **55**, 927 (1985).
[8] Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485 (1959).
[9] F. Wilczek and A. Zee, Phys. Rev. Lett. **52**, 2111 (1984); J. Moody, A. Shapere, and F. Wilczek, *ibid.* **56**, 893 (1986); E. Gozzi and W. Thacker, Phys. Rev. D **35**, 398 (1987).
[10] P. Nelson and L. Alvarez-Gaume, Commun. Math. Phys. **99**, 103 (1985); H. Sonoda, Nucl. Phys. **B266**, 410 (1986); A. J. Niemi, G. W. Semenoff, and Y. S. Wu, *ibid.* **B276**, 173 (1986).
[11] D. Arovas, J. R. Schrieffer, and F. Wilczek, Phys. Rev. Lett. **53**, 722 (1984); F. D. M. Haldane and Y. S. Wu, *ibid.* **55**, 2887 (1985).
[12] D. Thouless, M. Kohmoto, M. Nightingale, and M. den Nijs, Phys. Rev. Lett. **49**, 405 (1982); G. W. Semenoff and P. Sodano, *ibid.* **57**, 1195 (1986).
[13] Y. Aharonov and J. Anandan, Phys. Rev. Lett. **58**, 1593 (1987).
[14] J. Samuel and R. Bhandari, Phys. Rev. Lett. **60**, 2339 (1988); T. F. Jordan, Phys. Rev. A **38**, 1590 (1988); Y. S. Wu and H. Z. Li, Phys. Rev. B **38**, 11 907 (1988).
[15] R. Y. Chiao, A. Antaramian, K. M. Ganga, H. Jiao, S. R. Wilkinson, and H. Nathel, Phys. Rev. Lett. **60**, 1214 (1988); H. Jiao, S. R. Wilkinson, R. Y. Chiao, and H. Nathel, Phys. Rev. A **39**, 3475 (1989).
[16] R. Bhandari and J. Samuel, Phys. Rev. Lett. **60**, 1211 (1988).
[17] D. Suter, K. T. Mueller, and A. Pines, Phys. Rev. Lett. **60**, 1218 (1988).
[18] H. Weinfurter and G. Badurek, Phys. Rev. Lett. **64**, 1318 (1990).
[19] M. V. Berry, Proc. R. Soc. London, Ser. A **414**, 31 (1987); Nature (London) **326**, 277 (1987).
[20] J. Anandan and Y. Aharonov, Phys. Rev. D **38**, 1863 (1988); J. Anandan and A. Pines, Phys. Lett. A **141**, 335 (1989).
[21] E. Layton, Y. Huang, and S. I. Chu, Phys. Rev. A **41**, 42 (1990).
[22] J. Xu, T. Qian, and X. Gao, Europhys. Lett. **15**, 119 (1991).
[23] S. J. Wang, Phys. Rev. A **42**, 5103 (1990); **42**, 5107 (1990).
[24] S.-M. Cui, Phys. Rev. A **45**, 5525 (1992).
[25] J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory* (Springer-Verlag, Berlin, 1972).
[26] S. Chaturvedi, M. S. Sriram, and V. Srinivasan, J. Phys. A **20**, L1071 (1987).
[27] A. Zee, Phys. Rev. A **38**, 1 (1988).