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Quantum field theory on curved low-dimensional space embedded in three-dimensional space

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Recently, the quantum mechanics on a curved low-dimensional space was studied. There is an embedded effect when the space embedded in three-dimensional Cartesian space has some curvature. In this paper, we will consider second quantization of the spinless Schrödinger field there at finite temperature and show that there is also an embedded effect even though the low-dimensional space has no curvature as a manifold. This effect appears as an effective chemical potential.

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I. INTRODUCTION

Recently the quantum mechanics on a curved low-dimensional space embedded in three-dimensional (3D) space was studied [1–7]. It is known that if the low-dimensional space has curvature, the curvature sometimes makes an attractive potential appear in the Schrödinger equation even though there is no interaction. This effective potential comes from a geometrical correction at the quantum level. It is proportional to the sum of the squares of the curvatures and also to the square of the Planck constant $\hbar$. This effect was studied by da Costa and others in terms of the operator formalism [1–4] and by us in terms of the path-integral method [5].

Due to this effect, the particles in the curved low-dimensional space cannot move around freely even if there are no impurities nor other interactions. Some of them are recoiled by the effective potential. Taking account of this effect, we proposed a typical shape for a reflectionelss quantum wire so that its effective potential becomes the Landau potential [6].

In Ref. [7], we found that the Dirac equation along a thin elastic rod can be regarded as the Lax operator of the modified Korteweg–de Vries equation. Since the dynamics of the elastic rod is governed by the modified Korteweg–de Vries equation, it implies that the fictitious quantum mechanics, which appears when we solve the soliton equation, is a real quantum-mechanical effect on the soliton as a base space.

In this paper, we will quantize the spinless Schrödinger field on the low-dimensional curved space (submanifold) in 3D space at finite temperature. We will start from the field in 3D space and confine it in the submanifold through the harmonic oscillator along the submanifold. Here we will show that there appears an effective chemical potential as an embedded effect. We will embody this problem on two-dimensional (2D) space in Sec. II and on one-dimensional (1D) space in Sec. III. In the conclusion and discussion (Sec. IV), we will also comment on the phase transition in such a system.

II. QUANTUM FIELD THEORY ON A 2D CURVED SURFACE

In this section, we will quantize the spinless Schrödinger field on 2D submanifold in 3D space at a finite temperature using a path-integral method. Here we formally include the four-point vertex interaction but it does not play an important role in this section.

First of all, let us define the 3D ordinary system by the Cartesian coordinates, x^i, $i=1-3$. We will use the Latin indices for the flat 3D components and the Einstein convention. The original partition function at finite temperature in 3D space is given by [8]

$$Z[\psi^*,\psi]=\int D\psi^* D\psi \exp\left[\frac{1}{\hbar}\int d^4x \mathcal{L}[\psi^*,\psi]\right], \quad (1)$$

where $d^4x \equiv dt\, d^3x$ and t ($\equiv x^0$) is an imaginary time; $t\in[0,\beta]$, $\beta\equiv 1/k_B T$, T is the absolute temperature of the system and k_B is the Boltzmann constant. t is imposed to the periodic boundary condition and its conjugate space is Matsubara frequency space [8]. The Lagrangian is given by

$$\mathcal{L}=\hbar\psi^*\partial_t\psi-\frac{\hbar^2}{2m}\delta^{ij}\partial_i\psi^*\partial_j\psi+\psi^*[\mu-V(x)]\psi - \tfrac{1}{2}\int d^3y\, u(x^i-y^i)\psi^*(x)\psi^*(y)\psi(y)\psi(x)\big|_{y^0\equiv x^0}, \quad (2)$$

where $\partial_i \equiv \partial/\partial x^i$, m is mass of a particle, μ is a chemical potential, $u(x-y)$ is an interaction function for a four-point vertex and $V(x)$ is a confinement potential, which constrains the particle to a submanifold as we see later. δ_{ij} is the Kronecker symbol. We note that we used the notation of Popov [8], in which the symmetry of time $t \to -t$ is applied. Therefore, the sign in front of the Lagrangian appears the opposite of the ordinary definition [9]. Hereafter we assume that ψ vanishes on the boundary of the region we consider; we will use the symbol "$\cong$" and will neglect a total-derivative term.

Let us define the general coordinates, in terms of which a curved system will be expressed after some of its degrees of freedom are suppressed. Let the middle parts of the Greek indices $(q^\mu, q^\nu, \ldots)$ indicate the curved system; $\mu = 1,2,3$. The relation between the Cartesian and general coordinates is given through the dreibein [10],

$$e^i{}_\mu \equiv \partial_\mu x^i , \tag{3}$$

where $\partial_\mu \equiv \partial/\partial q^\mu$. The metric is written as

$$g_{\mu\nu} \equiv \delta_{ij} e^i{}_\mu e^j{}_\nu . \tag{4}$$

We will embody the problem that we constrain the field on a 2D curved surface S. Let the first and second coordinates indicate the position attached on S. The normal unit vector of S is denoted by $\mathbf{e}_3$. The confinement potential V is given along S and constraints the particle to be on S. Let us assume that V has the form, $V^{2D}_{\rm conf}(q^3) \equiv \frac{1}{2} m\omega^2 (q^3)^2$ for a large ω, where q^3 is the normal coordinate of the surface. After confinement, we can realize the 2D submanifold in the 3D space and then express this system using the 2D parameters.

Since we wish that the 3D metric $g_{\mu\nu}$ (4) around S is expressed by the variables of S, we will define the geometry in the vicinity of S. Let the position on S be written by $\mathbf{r}(q^1, q^2)$. We can express a point $\mathbf{x} \equiv (x^1, x^2, x^3)$ in the vicinity of S in terms of the curved system,

$$\mathbf{x}(q^\mu) = \mathbf{r}(q^\alpha) + q^3 \mathbf{e}_3 . \tag{5}$$

The beginning parts of Greek indices $(q^\alpha, q^\beta, \ldots)$ span from one to two. We define the zweibein along S as $b^i{}_\alpha \equiv \partial r^i/\partial q^\alpha$. We divide the ordinary derivative along S into the horizontal and vertical parts; the horizontal part is written by D_α defined as $D_\alpha \mathbf{X} \equiv \partial_\alpha \mathbf{X} - \langle \partial_\alpha \mathbf{X}, \mathbf{e}_3 \rangle \mathbf{e}_3$ for a vector $\mathbf{X}$. Here $\langle\ ,\ \rangle$ denotes the canonical inner product. The 2D Christoffel symbol is thus defined as $D_\alpha \mathbf{b}_\beta = \Gamma^\gamma{}_{\beta\alpha} \mathbf{b}_\gamma$. The second fundamental form [11] is defined by

$$\Gamma^3_{\beta\alpha} \equiv \langle \mathbf{e}_3, \partial_\alpha \mathbf{b}_\beta \rangle . \tag{6}$$

The Weingarten map, defined by $-\Gamma^\gamma{}_{\beta 3} \equiv \langle \mathbf{b}_\gamma, \partial_3 \mathbf{b}_\beta \rangle$, is associated with the second fundamental form through the relation

$$\Gamma^3{}_{\beta\alpha} = -\Gamma^\gamma{}_{3\alpha} \eta_{\gamma\beta} , \tag{7}$$

where $\eta_{\alpha\beta} \equiv \delta_{ij} b^i{}_\alpha b^j{}_\beta$ is the surface metric. We can therefore express $e^i{}_\mu (=\partial x^i/\partial q^\mu)$ in the vicinity of S in terms of $b^i{}_\alpha$,

$$e^i{}_\alpha = b^i{}_\alpha + q^3 \Gamma^\beta{}_{3\alpha} b^i{}_\beta . \tag{8}$$

The 3d metric around S (4) becomes

$$\begin{aligned} g_{\alpha\beta} &= \eta_{\alpha\beta} + [\Gamma^\gamma{}_{3\alpha}\eta_{\gamma\beta} + \eta_{\alpha\gamma}\Gamma^\gamma{}_{3\beta}] q^3 \\ &\quad + [\Gamma^\gamma{}_{3\alpha}\eta_{\gamma\delta}\Gamma^\delta{}_{3\beta}](q^3)^2 , \\ g_{3\alpha} &= g_{\alpha 3} = 0 , \\ g_{33} &= 1 , \end{aligned} \tag{9}$$

and $g \equiv \det(g_{\mu\nu})$

$$\begin{aligned} g &= \eta\zeta , \\ \zeta^{1/2} &\equiv [1 + \mathrm{tr}_2(\Gamma^\alpha{}_{3\beta}) q^3 + \det{}_2(\Gamma^\alpha{}_{3\beta})(q^3)^2] . \end{aligned} \tag{10}$$

Here tr_2 and $\det_2$ are the 2D trace and determinant over α and β, respectively. These values are invariant for the coordinate transformation if we fix the submanifold, and they are known as the mean and Gaussian curvatures on S [11].

As we finished the geometrical preliminary, we will consider the field around S. It is known that the coordinate transformation in the field theory needs some subtle treatment [12]. Since the scalar product, which is the intensity of the field, is expressed by $(\psi_1|\psi_2) \equiv \int d^3x\, \psi_1^*(x)\psi_2(x)$ in the Cartesian coordinate, it becomes $(\psi_1|\psi_2) = \int d^3q\, g^{1/2} \psi_1^*(q)\psi_2(q)$ in the curved coordinate system. In general, the Jacobian impedes the Hermiticity of the natural differential operator. In order to avoid the problem after confinement we expect to get the intensity distribution along S as $(\phi_T^* \cdot \phi_T)(q^1,q^2) \equiv \int d(q^3)(\phi^* \cdot \phi)(q^1,q^2,q^3)$. Here we eliminate the normal part of the Jacobian in the measure. In order to get the distribution, we define a new field, $\phi \equiv \zeta^{1/4}\psi$. Then the action changes its form,

$$S = \int d^4q\, \eta^{1/2} \tilde{\mathcal{L}}[\phi^*, \phi] , \tag{11}$$

with a new Lagrangian including ζ expressed as

$$\begin{aligned} \tilde{\mathcal{L}}[\phi^*,\phi] &\cong \phi^*\hbar\partial_t\phi + \frac{\hbar^2}{2m}\phi^*\zeta^{1/4} g^{-1/2}\partial_\mu g^{\mu\nu} g^{1/2}\partial_\nu \zeta^{-1/4}\phi + \phi^*[\mu - V(q)]\phi \\ &\quad - \tfrac{1}{2}\int d^3q'\eta'^{1/2} u(q^i - q'^i)\phi^*(q)\phi^*(q')\phi(q')\phi(q)\big|_{q'^0 \equiv q^0} \\ &= \phi^*\hbar\partial_t\phi + \frac{\hbar^2}{2m}\phi^*\zeta^{1/4} g^{-1/2}\partial_\alpha g^{\alpha\beta} g^{1/2}\partial_\beta \zeta^{-1/4}\phi + \frac{\hbar^2}{2m}\phi^*\partial_3^2\phi + \frac{\hbar^2}{2m}\phi^*\left[\frac{3}{8}\frac{1}{\zeta^2}(\partial_3\zeta)^2 - \frac{1}{4}\frac{1}{\zeta}\partial_3^2\zeta\right]\phi \\ &\quad + \phi^*[\mu - V(q)]\phi - \tfrac{1}{2}\int d^3q'\eta'^{1/2} u(q^i - q'^i)\phi^*(q)\phi^*(q')\phi(q')\phi(q)\big|_{q'^0 \equiv q^0} . \end{aligned} \tag{12}$$

Since the Feynman measure in 3D space is defined by

$$D\psi \equiv \prod_x d\psi(x) , \tag{13}$$

it can be rewritten in the following form:

$$D\psi = \zeta^{1/4} D\phi \equiv \prod_q \zeta^{1/4}(q) d\phi(q) . \tag{14}$$

The partition function in this case is

$$Z[\phi^*,\phi] = \int \zeta^{1/2} D\phi^* D\phi \exp\left[\frac{1}{\hbar}\int d^3q\, \eta^{1/2}\tilde{\mathcal{L}}[\phi^*,\phi]\right] . \tag{15}$$

Let us now consider the confinement effect from the given potential $V^{2D}_{\rm conf}$. Suppose ω is sufficiently large. We express the Lagrangian as

$$\tilde{\mathcal{L}} \cong \hbar\phi^*\partial_t\phi + \frac{\hbar^2}{2m}\phi^*\partial_3^2\phi - \phi^* V^{2D}_{\rm conf}\phi + \cdots . \tag{16}$$

Though there are terms which depend on q^3 in the omitted terms, we can neglect them, since $V^{2D}_{\rm conf}$ is a rapidly increasing potential and dominant. We decompose $\phi(q^1,q^2,q^3)$ into $\phi_T(q^1,q^2)\phi_N(q^3)$,

$$\tilde{\mathcal{L}} \cong \phi_T^*\phi_T\left[\hbar\phi_N^*\partial_t\phi_N + \frac{\hbar^2}{2m}\phi_N^*\partial_3^2\phi_N - \phi_N^* V^{2D}_{\rm conf}\phi_N\right] + \cdots . \tag{17}$$

We expand ϕ_N by the eigenfunctions which satisfy the equation

$$\left[-\hbar\partial_t - \frac{\hbar^2}{2m}\partial_3^2 + \tfrac{1}{2}m\omega^2(q^3)^2\right]\phi_n = \Lambda_n\phi_n , \tag{18}$$

with eigenvalues Λ_n as

$$\phi_N = \sum_n a_n\phi_n . \tag{19}$$

If we express $\phi(t,q^3) = \sum_{n,m} a_{n,m}\exp(E_m t)\phi_{n,m}(q^3)$, which E_m is an imaginary energy, this equation becomes

$$\left[-\frac{\hbar^2}{2m}\partial_3^2 + \tfrac{1}{2}m\omega^2(q^3)^2\right]\phi_{n,m} = (\Lambda_n + E_m)\phi_{n,m} . \tag{20}$$

Due to the harmonic potential, the eigenvalue of (20) is written as $\Lambda_n + E_m = \frac{1}{2}\hbar\omega[2(n+m)+1]$ for integer m,n. When we take one-body-limit corresponding to $\Lambda_{n_0}=0$, we obtain $E_m = \frac{1}{2}\hbar\omega(2m+1)$, $m \ge 0$. Hence $\Lambda_n = \hbar\omega n$. Since $\Delta\Lambda_{n,m} \equiv \Lambda_n - \Lambda_{n-1} + E_m - E_{m-1}$ is proportional to ω, only zero mode, $\phi_{0,0}$ with $\Lambda_{n_0}=0$ and $E_0 = \hbar\omega/2$, is permitted. We make ϕ_N shrink to a point and it becomes the delta function. $\phi \sim a_0\phi_T(m\omega/\pi\hbar)^{1/4}\exp[-m\omega(q^3)^2/2]$. Accordingly we can safely integrate over q^3 in (11) or make it vanish. Then we have the relation $g_{\alpha\beta} \sim \eta_{\alpha\beta}$ in (9) and $\zeta = 1$.

We have a new Lagrangian, which is functional of only $a_0\phi_T$. We rewrite the remainder field as $\phi_{2D} \equiv a_0\phi_T$ and the coupling function as $u_{2D} \equiv u|_{q^3 \equiv q'^3 \equiv 0}$. Due to above relations, we obtain the action

$$S^{2D}[\phi^*_{2D},\phi_{2D}] = \int dt\, d^2q\, \mathcal{L}^{2D} , \tag{21}$$

with

$$\begin{aligned}\mathcal{L}^{2D} \cong{}& \hbar\phi^*_{2D}\partial_t\phi_{2D} + \frac{\hbar^2}{2m}\eta^{-1/2}\phi^*_{2D}\partial_\mu\eta^{\mu\nu}\eta^{1/2}\partial_\nu\phi_{2D} \\ &+ \phi^*_{2D}(\mu + \mu^{2D}_{\rm eff})\phi_{2D} \\ &- \tfrac{1}{2}\int d^2q' u_{2D}(q^i - q'^i)\phi^*_{2D}(q)\phi^*_{2D}(q') \\ &\times \phi_{2D}(q)\phi_{2D}(q)|_{q'^0 \equiv q^0} ,\end{aligned} \tag{22}$$

where $\mu^{2D}_{\rm eff}$ is an effective chemical potential,

$$\mu^{2D}_{\rm eff} = \frac{\hbar^2}{2m}\{[\tfrac{1}{2}{\rm tr}_2(\Gamma^\alpha{}_{3\beta})]^2 - \det{}_2(\Gamma^\alpha{}_{3\beta})\} . \tag{23}$$

Equation (23) is invariant, as we described below (10). We notice that the kinetic term in (22) has a geometrical form; the kinetic term has the relations

$$\begin{aligned}\eta^{\alpha\beta}\partial_\alpha\phi^*_{2D}\partial_\beta\phi_{2D} &\cong -\eta^{-1/2}\phi^*_{2D}\partial_\alpha\eta^{1/2}\eta^{\alpha\beta}\partial_\beta\phi_{2D} \\ &= -\eta^{\alpha\beta}\phi^*_{2D}D_\alpha D_\beta\phi_{2D}\end{aligned} \tag{24}$$

where $D_\alpha X_\beta \equiv \partial_\alpha X_\beta + \Gamma^\gamma_{\beta\alpha}X_\gamma$ for a vector X_α and $D_\alpha\phi \equiv \partial_\alpha\phi$ for a scalar field ϕ. In other words, the kinetic term has the properties as a 2D manifold while $\mu^{2D}_{\rm eff}$ expresses the properties of the submanifold. We should, thus, interpret $\mu^{2D}_{\rm eff}$ as an embedded effect. For example, though the book-cover-type 2D surface is flat as a manifold, $\mu^{2D}_{\rm eff}$ does not vanish [1]. However, for a 2D sphere, which is a symmetrical case, it vanishes. We also note that (23) agrees with the surface potential of da Costa [1,2].

Let us consider the measure of the path integral. We note that the action on the exponent is independent of $\phi(q^1,q^2,q^3\neq 0)$. Since the Feynman measure on the partition function is written as

$$\int\int\prod_{q^1,q^2}\left[\int\prod_{q^3}\zeta^{1/2}d\phi^* d\phi\, e^{S^{2D}/\hbar}\right] , \tag{25}$$

we can integrate over $\prod_{q^3\neq 0} d\phi$. The partition functional with (22) and (23) can be expressed by

$$\begin{aligned}Z^{2D}[\phi^*_{2D},\phi_{2D}] ={}& \int D\phi^*_{2D}D\phi_{2D}\exp \\ &\times\left[\frac{1}{\hbar}\int dt\, d^2q\, \eta^{1/2}\mathcal{L}^{2D}[\phi^*_{2D},\phi_{2D}]\right] ,\end{aligned} \tag{26}$$

where we use $=$ as the equivalent class in the meaning of the path integral.

III. QUANTUM FIELD THEORY ON A 1D CURVE

In this section, we shall consider a 1D problem. Similar to the previous section, we shall define a system in terms of the 1D geometry. Let **r** be the position along a curve C. For the sake of simplicity, we assume that C exits on a flat surface F. We introduce the orthonormal coordinate system along C; the tangent unit vector $\mathbf{n}_3$, the normal unit vector $\mathbf{n}_1$ of C attached to F, and th other normal unit vector $\mathbf{n}_2$ of both C and F. We indicate the position around C,

$$\mathbf{x}=\mathbf{r}+\mathbf{n}_1 q^1+\mathbf{n}_2 q^2 . \tag{27}$$

We assume that the confinement potential is given by $V_{\text{conf}}^{1\text{D}}=\frac{1}{2}m\omega^2[(q^1)^2+(q^2)^2]$. We express the 1D Christoffel symbol along C as [11]

$$\partial_3\mathbf{n}_3=\Gamma^1{}_{33}\mathbf{n}_1 , \quad \partial_2\mathbf{n}_1=\Gamma^3{}_{13}\mathbf{n}_3 . \tag{28}$$

We can rewrite $\Gamma^1{}_{33}$ by the ordinary curvature $k(q^3)$ as $-\Gamma^3{}_{13}=\Gamma^1{}_{33}\equiv k$. The dreibein is written as

$$e^i{}_\mu=[1-k(q^3)q^1\delta_{\mu 3}]n^i{}_\mu \quad \text{not summed over } \mu . \tag{29}$$

As we can argue the confinement similar to the previous section, $\omega\to\infty$, the field becomes $\phi(q^1,q^2,q^3)=\phi_{1\text{D}}(q^3)\phi_N(q^1,q^2)\to\phi_{1\text{D}}$. Appropriately redefining the variables, the partition functional on C, corresponding to the 2D case (26), becomes

$$Z^{1\text{D}}[\phi_{1\text{D}}^*,\phi_{1\text{D}}]=\int D\phi_{1\text{D}}^* D\phi_{1\text{D}} \times\exp\left[\frac{1}{\hbar}\int dt\, dq^3 \mathcal{L}^{1\text{D}}[\phi_{1\text{D}}^*,\phi_{1\text{D}}]\right] ,$$

$$\mathcal{L}^{1\text{D}}\simeq\phi_{1D}^*\hbar\partial_t\phi_{1\text{D}}+\frac{\hbar^2}{2m}\phi_{1\text{D}}^*\partial_3{}^2\phi_{1\text{D}}+\phi_{1\text{D}}^*(\mu+\mu_{\text{eff}}^{1\text{D}})\phi_{1\text{D}} -\frac{1}{2}\int dq'^3 u_{1\text{D}}(q^3-q'^3)\phi_{1\text{D}}^*(q')\phi_{1\text{D}}(q') \times\phi_{1\text{D}}^*(q)\phi_{1\text{D}}(q)\big|_{q'^0\equiv q^0} . \tag{30}$$

The effective potential $\mu_{\text{eff}}^{1\text{D}}$ has the simple form

$$\mu_{\text{eff}}^{1\text{D}}=\frac{\hbar^2}{8m}k^2 . \tag{31}$$

Note that the square of k is an invariant value in the system. Our argument of the 1D space is also performed under the sense similar to that discussed in the last section. We notice that the 1D space is flat as a manifold but there is also an embedded effect and a bent effect. If we take a classical limit in the meaning of the second quantization, (32) also agrees with da Costa's result [1,2].

IV. CONCLUSION AND DISCUSSION

We find that there appears an effective chemical potential in quantum-field theory when the confined low-dimensional space is embedded in 3D space and has some curvature. This result is in agreement with that of quantum mechanics [1,2].

Finally, we will comment on the phase transition on a submanifold. Let us assume that our system has some phase transition and the field we discuss expresses some Bose condensation [8,9]. In other words, the chemical potential μ is a monotonically decreasing function in the temperature and $\mu(T=0)>0$ after some corrections. The four-point function behaves like $u(q)\cong u_0\delta(q)$, $u_0>0$. The Landau-Ginzburg theory can be applied. According to the theory, at the point that the chemical potential vanishes, the phase transition occurs. On the other hand, in our results, the magnitude of the effective chemical potential μ_{eff} depends on the geometry of the submanifold and thus makes the total chemical potential increase compared to one in the flat low-dimensional space. If the 2D system is a cylinder with the radius ~ 1 nm, $\mu_{\text{eff}}^{2\text{D}}\sim 100$ K for an electron. (For the same radius, it is ~ 50 K for a particle with a two-electron mass.) Due to the effective chemical potential $\mu_{\text{eff}}^{2\text{D}}$, a phase transition should shift to the high-temperature side there. We may consider the field we discussed to be a phenomenological Cooper pair. If the Cooper pair on a C_{60} superconductor occurs along the equator of C_{60}, this chemical potential seems to have an effect on the critical temperature.

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