Separation and superposition of atomic wave packets by reflection and transmission by an optical ripple mirror

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In this paper, we investigate the reflection and transmission of an atomic wave by an optical ripple mirror which may be realized by a standing-wave laser field. We show that the ripple mirror can operate as a separator of the incident atomic waves corresponding to different internal states by reflection and transmission. For different laser parameters, the atomic ripple mirror can reflect one linear combination of two ground states and transmit the orthogonal linear combination. The transmitted superposition state corresponds to the well-known "dark" state. The atomic wave function is a coherent superposition of reflected and transmitted waves entangled with different internal states.

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I. INTRODUCTION

The motion of an atom in a laser field has been the subject of much research over the last few years. The reflection and diffraction of an atomic beam by a laser field have been theoretically predicted [1–8] and experimentally observed. However, the investigations have mainly concentrated on the two-level atom. The adoption of a two-level model can greatly simplify the theoretical treatment. For example, if the photon recoil frequency is smaller than the spontaneous emission decay rate of the internal excited state, the light field is completely equivalent to a mechanical potential for the center-of-mass motion of a two-level atom in the adiabatic approximation. Some authors [5,8,9] have applied the effective-potential model to study the interference, reflection and channeling of atoms in both a travelling- and standing-wave laser field. Recently, these studies have been extended to multilevel atoms [10–12].

In this paper, we consider a scheme for the reflection and diffraction of a three-level atom with an excited state and two ground states by a standing-wave laser. We assume that the laser has a nonuniform transverse distribution with beam width w_0. The atomic beam is assumed to pass through the laser beam with a glancing angle of incidence along the laser propagation direction. Hence our scheme differs from the atomic beam splitter which has a large angle of incidence of the atomic beam as studied in [6–8]. With the arrangement of glancing incidence of the atomic beam, we find that the standing-wave laser field can reflect the atomic beam in a perpendicular direction to the laser propagation by light-induced gradient potentials. Due to the periodic structure of the standing-wave laser in the propagation direction, the reflection of the atomic beam by such a laser beam is very similar to that of a light beam by a ripple mirror. Hence in our scheme, the standing-wave laser with nonuniform transverse distribution acts as an atomic ripple mirror which is a position-dependent periodic potential for the atomic center-of-mass motion. Theoretically, we show that for different internal states of the three-level atom, the light-induced potentials have different forms which depend on the detunings of the laser from the atomic levels. For some regions of the laser detuning, the atomic ripple mirror can operate as a state "separator" which can reflect the atomic waves corresponding to one of the ground-state sublevels and transmit the other. For other regions of laser detuning, we find that the light field not only induces a gradient potential for every atomic sublevel but also induces a coupling between the atomic waves for each sublevel. We show that due to this coupling, the atomic ripple mirror can be used as a state-preparation device which can reflect one linear combination of two ground states and transmit the orthogonal linear combination. The transmitted combination corresponds to the well-known "dark" state.

This paper is organized as follows: In Sec. II, we present a general theory of a three-level atom interacting with a standing-wave laser. A vector Schrödinger equation describing the atomic center-of-mass motion corresponding to three internal states is derived. With the assumption of glancing incidence of the atomic beam, we reduce the vector Schrödinger equation into coupled equations for the two ground-state atomic waves. A general discussion of the solutions for the coupled equations is given. Section III discusses the conditions under which the standing-wave laser may separate the atomic waves corresponding to different internal states, or prepare them in a coherent superposition state. Finally, the conclusions are given in Sec. IV.

II. EQUATIONS OF MOTION FOR ATOMIC WAVES

In this section, we will derive the equations of motion for the atomic waves in a standing-wave laser field. We consider a three-level atom with an excited state and two ground-state sublevels. The Hamiltonian has the following form in the dipole and rotating-wave approximation:

$$
\begin{aligned}
H &= \frac{\mathbf{p}^2}{2m} + H_A + H_F + H_C + H_R \,, \\
H_A &= \sum_{j=1}^{3} \hbar\varepsilon_j |j\rangle\langle j| \,, \\
H_F &= \sum_{\lambda} \int d^3k \, \hbar\omega_k b^{\dagger}_{\mathbf{k}\lambda} b_{\mathbf{k}\lambda} \,, \\
H_C &= -\int d^3r \, \mathbf{D}^{(-)}(\mathbf{r}) \cdot \mathbf{E}_S^{(+)}(\mathbf{r},t) + \text{c.c.} \,, \\
H_R &= -\int d^3r \, \mathbf{D}^{(-)}(\mathbf{r}) \cdot \mathbf{E}_R^{(+)}(\mathbf{r},t) + \text{c.c.} \,,
\end{aligned} \tag{1}
$$

where $\mathbf{p}$ is the momentum operator describing the center-of-mass motion of the atom. H_A and H_F are, respectively, the free Hamiltonians for the atomic internal degrees of freedom and the vacuum electromagnetic field. H_C is the interaction Hamiltonian of the atom with the standing-wave laser field which is treated as a classical field with $\mathbf{E}_S^{(+)}(\mathbf{r},t) = 2\mathbf{E}_0 F(x,y)\cos(k_L z) e^{-i\omega_L t}$. $F(x,y)$ is the transverse distribution of the laser field. ω_L and k_L are its frequency and wave vector. H_R is the interaction Hamiltonian of the atom with the vacuum electromagnetic field with $\mathbf{E}_R^{(+)}(\mathbf{r},t) = i\sum_\lambda \int d^3k (\hbar\omega_k/4\pi^2)^{1/2} e^{i\mathbf{k}\cdot\mathbf{r}} \boldsymbol{\varepsilon}_{\mathbf{k}\lambda} b_{\mathbf{k}\lambda}$. $\mathbf{D}^{(-)}(\mathbf{r}) = (\boldsymbol{\mu}_{31}|3\rangle\langle 1| + \boldsymbol{\mu}_{32}|3\rangle\langle 2|) \otimes |\mathbf{r}\rangle\langle \mathbf{r}|$ denotes the negative-frequency part of the dipole-operator density. $|\mathbf{r}\rangle$ is the atomic center-of-mass position eigenstate and $\mathbf{u}_{3j}$ is the dipole matrix element corresponding to the transitions between levels $|3\rangle$ and $|j\rangle$ $(j=1,2)$. The transition between levels $|2\rangle$ and $|1\rangle$ is electric-dipole forbidden. In terms of the Hamiltonian (1), one has the following Schrödinger equation for the atom-field state $|\Psi\rangle$:

$$
i\hbar \frac{\partial |\Psi\rangle}{\partial t} = H|\Psi\rangle \,. \tag{2}
$$

In order to investigate the time evolution of the state $|\Psi\rangle$, we follow the Weisskoff-Wigner radiation theory and expand $|\Psi\rangle$ in the space of Fock states of the vacuum electromagnetic field as in Ref. [10,11]:

$$
|\Psi\rangle = \sum_{\{n_{\mathbf{k}\lambda}\}} |\psi_{\{n_{\mathbf{k}\lambda}\}}(t)\rangle \otimes |\{n_{\mathbf{k}\lambda}\}\rangle \,. \tag{3}
$$

Then the coupled equations for the state vectors $|\psi_{\{0\}}\rangle$ and $|\psi_{\{n_{\mathbf{k}\lambda}\}}(t)\rangle$ are

$$
\begin{aligned}
i\hbar \frac{\partial |\psi_{\{0\}}\rangle}{\partial t} &= H_0 |\psi_{\{0\}}\rangle + \sum_{\{n_{\mathbf{k}\lambda} > 0\}} \langle \{0\}| H_R^I | \{n_{\mathbf{k}\lambda}\}\rangle |\psi_{\{n_{\mathbf{k}\lambda}\}}(t)\rangle \,, \\
i\hbar \frac{\partial |\psi_{\{n_{\mathbf{k}\lambda}\}}(t)\rangle}{\partial t} &= H_0 |\psi_{\{n_{\mathbf{k}\lambda}\}}(t)\rangle + \langle \{n_{\mathbf{k}\lambda}\}| H_R^I |\{0\}\rangle |\psi_{\{0\}}\rangle \\
&\quad + \sum_{\{n'_{\mathbf{k}\lambda} > 0\}} \langle \{n_{\mathbf{k}\lambda}\}| H_R^I | \{n'_{\mathbf{k}\lambda}\}\rangle \times |\psi_{\{n'_{\mathbf{k}\lambda}\}}(t)\rangle \,,
\end{aligned} \tag{4a}
$$

where

$$
H_0 = \frac{\mathbf{p}^2}{2m} + H_A + \hbar\omega_L |1\rangle\langle 1| + \hbar\omega_L |2\rangle\langle 2| + H_C^I \tag{4b}
$$

and we have transformed to the interaction picture $H_\alpha^I = U^\dagger H_\alpha U$ $(\alpha = C, R)$ with the unitary operator $U = \exp[-iH_F t/\hbar + i\omega_L t(|1\rangle\langle 1| + |2\rangle\langle 2|)]$. We can solve Eqs. (4) by second-order perturbation theory where we limit the state vectors to the zero-photon space. As a result, we may reduce (4) into a simple form for the state vector $|\psi_A\rangle \equiv |\psi_{\{0\}}\rangle$:

$$
i\hbar \frac{\partial |\psi_A\rangle}{\partial t} = [H_0 - i\hbar(\gamma/2)|3\rangle\langle 3|]|\psi_A\rangle \,, \tag{5}
$$

where $\gamma = \gamma_1 + \gamma_2$ is the total decay rate of level $|3\rangle$. γ_1 and γ_2, respectively, give the radiative decay rates of level $|3\rangle$ into $|1\rangle$ and $|2\rangle$. Now, we use Eq. (5) to study the scattering of atomic waves by a standing-wave laser field. We can make the ansatz of a stationary oscillation with the incident energy E

$$
|\psi_A(E,t)\rangle = \int d^3r \, e^{-i(E/\hbar + \varepsilon_3)t} |\mathbf{r}\rangle \otimes [\phi_1(\mathbf{r})|1\rangle + \phi_2(\mathbf{r})|2\rangle + \phi_3(\mathbf{r})|3\rangle] \,. \tag{6}
$$

Inserting (6) into (5), one has the equations for the atomic center-of-mass wave:

$$
\begin{aligned}
E\phi_1 &= \frac{-\hbar^2\nabla^2}{2m}\phi_1 + \hbar\Delta_1\phi_1 - \hbar\Omega_1^* F(x,y)\cos(k_L z)\phi_3 \,, \\
E\phi_2 &= \frac{-\hbar^2\nabla^2}{2m}\phi_2 + \hbar\Delta_2\phi_2 - \hbar\Omega_2^* F(x,y)\cos(k_L z)\phi_3 \,, \\
E\phi_3 &= \frac{-\hbar^2\nabla^2}{2m}\phi_3 - i\hbar(\gamma/2)\phi_3 \\
&\quad - \hbar(\Omega_1\phi_1 + \Omega_2\phi_2)F(x,y)\cos(k_L z) \,,
\end{aligned} \tag{7}
$$

where $\Delta_j = \omega_L - (\varepsilon_3 - \varepsilon_j)$ $(j=1,2)$ denote the detuning of the atomic transition frequencies $\varepsilon_3 - \varepsilon_j$ from the laser frequency ω_L. $\Omega_j = 2\boldsymbol{\mu}_{3j}\cdot\mathbf{E}_0/\hbar$ is the Rabi frequency corresponding to the transition between levels $|3\rangle$ and $|j\rangle$. In this paper, we assume the angle of incidence of the atomic beam to the z axis Θ is very small so that $v_z \gg v_x, v_y$ and $E \approx E_z = \frac{1}{2}mv_z^2$. The incident energy E is assumed to be larger than $\hbar\Delta_j, \hbar\gamma$ and $\hbar^2 k_L^2/2m$. After these considerations, the atomic wave packet may be assumed to be slowly varying in the z direction on the scale of the de Broglie wavelength $\lambda_d = 2\pi/K_z$ with $K_z = mv_z/\hbar$, and then we separate out the fast-varying part by the transformation

$$
\begin{aligned}
\phi_j(\mathbf{r}) &= e^{iK_z z - i\Delta_j z/v_z}\varphi_j(\mathbf{r}) \quad (j=1,2) \,, \\
\phi_3(\mathbf{r}) &= e^{iK_z z}\varphi_3(\mathbf{r}) \,.
\end{aligned} \tag{8}
$$

After neglecting the second derivatives $\partial^2\varphi_j/\partial z^2$ and the terms $\hbar^2\Delta_j^2\varphi_j/mv_z^2$ compared with $K_z(\partial\varphi_j/\partial z)$, we have the equations for the slowly varying envelopes:

$$
i\hbar \frac{\partial\varphi_1}{\partial\tau} = \frac{-\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right]\varphi_1 - \hbar\Omega_1^* F(x,y)\cos(\Delta\omega\tau)e^{i\Delta_1\tau}\varphi_3 \,, \tag{9a}
$$

$$i\hbar\frac{\partial\varphi_2}{\partial\tau}=\frac{-\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\varphi_2 -\hbar\Omega_2^*F(x,y)\cos(\Delta\omega\tau)e^{i\Delta_2\tau}\varphi_3\ , \quad (9b)$$

$$i\hbar\frac{\partial\varphi_3}{\partial\tau}=\frac{-\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\varphi_3-i\hbar(\gamma/2)\varphi_3 -\hbar(\Omega_1e^{-i\Delta_1\tau}\varphi_1+\Omega_2e^{-i\Delta_2\tau}\varphi_2)F(x,y)\cos(\Delta\omega\tau)\ , \quad (9c)$$

where the effective time variable $\tau=z/v_z$ and $\Delta\omega=k_Lv_z$ is the Doppler shift. For a glancing angle of incidence of the atomic wave, one can expect a long interaction time $\tau_0\gg\gamma^{-1}$. As a result, we can follow the same treatment as used in Ref. [8] and make the semiclassical adiabatic approximation for the excited-state wave packet. By integrating Eq. (9c) and taking out the τ slowly varying wave packets $\varphi_{1,2}$, we obtain the adiabatic solution φ_3:

$$\varphi_3(\mathbf{r})=\frac{i\Omega_1F(x,y)}{2}(\beta_1^-e^{-i\delta_{1-}\tau}+\beta_1^+e^{-i\delta_{1+}\tau})\varphi_1(\mathbf{r}) +\frac{i\Omega_2F(x,y)}{2}(\beta_2^-e^{-i\delta_{2-}\tau}+\beta_2^+e^{-i\delta_{2+}\tau})\varphi_2(\mathbf{r})\ , \quad (10)$$

where $\beta_j^\pm\equiv1/(\gamma/2-i\delta_{j\pm})$ and $\delta_{j\pm}\equiv\Delta_j\pm k_Lv_z$ $(j=1,2)$. It is evident that the spatial distribution of the excited-state wave packet depends on that of the light field. In the field-free space, the atoms cannot be excited and one cannot obtain the excited-state atomic wave. Therefore, we deduce that the excited-state wave is located in the region of the laser field in the semiclassical adiabatic approximation. We substitute Eq. (10) into Eqs. (9a) and (9b), which gives the following coupled equations for φ_1 and φ_2:

$$i\hbar\frac{\partial\varphi_1}{\partial\tau}=\frac{-\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\varphi_1+V_{11}\varphi_1+V_{12}\varphi_2\ , \quad (11a)$$

$$i\hbar\frac{\partial\varphi_2}{\partial\tau}=\frac{-\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\varphi_2+V_{22}\varphi_2+V_{21}\varphi_1\ . \quad (11b)$$

V_{ij} $(j=1,2)$ give the effective-potential functions and coupling coefficients. They have the following definitions:

$$V_{ij}=\frac{-i\hbar\Omega_i^*\Omega_j}{4}F(x,y)^2(\beta_j^-+\beta_j^++\beta_j^-e^{2i\Delta\omega\tau} +\beta_j^+e^{-2i\Delta\omega\tau})e^{i(\Delta_i-\Delta_j)\tau}\ . \quad (12)$$

Equations (11) are difficult to solve. However, in some simple cases, we can give approximation solutions. The first case corresponds to the decoupling of two ground-state wave packets. From Eqs. (12), we see that the coupling coefficients V_{ij} $(i\neq j)$ depend on the oscillation factor $e^{i(\Delta_i-\Delta_j)\tau}$. For nondegenerate ground-state sublevels, if the separation between two sublevels $|\Delta_i-\Delta_j|$ has a large value, the oscillation factor will rapidly wash out the coupling terms in Eqs. (11) with increasing τ. As a result, Eqs. (11) are further reduced into two independent Schrödinger equations as follows:

$$i\hbar\frac{\partial\varphi_j}{\partial\tau}=\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial y^2}\varphi_j+V_{jj}\varphi_j\quad(j=1,2)\ , \quad (13)$$

where we have assumed that the atomic wave packet has a smaller width than the laser beam width w_0 along the x axis and a zero kinetic energy E_x so that the second derivatives with respect to x can be dropped. Equations (13) are time-dependent Schrödinger equations with effective time periodic potentials. The scattering of particles by a time periodic potential has been well studied in quantum mechanics [13,14]. According to Ref. [13], we rewrite the potentials into two parts $V_{jj}=V_{jj}^0+V_{jj}^1(\tau)$ with the static potentials $V_{jj}^0=(-i\hbar|\Omega_j|^2/4)F(x,y)^2(\beta_j^-+\beta_j^+)$ and the time-dependent parts $V_{jj}^1(\tau)=(-i\hbar|\Omega_j|^2/4)F(x,y)^2(\beta_j^-e^{2i\Delta\omega\tau}+\beta_j^+e^{-2i\Delta\omega\tau})$. For convenience, we assume that the laser beam has a uniform distribution in its width w_0, i.e., $F(x,y)^2=1$ for $-a<y<a=w_0/\sqrt{2}$ and $F(x,y)=0$ in elsewhere. If the time-independent problem $H_j^0=-(\hbar^2/2m)\partial^2/\partial y^2+V_{jj}^0$ has $H_j^0\chi_j^\mp(q)=\epsilon(q)\chi_j^\mp(q)$, then we have the analytic solutions for the time-dependent equations (13) in the region $-a<y<a$ [13]:

$$\varphi_j^\mp(q)=\exp[-i\epsilon(q)\tau/\hbar]\chi_j^\mp(q)\sum_{n=-\infty}^{\infty}J_ne^{-2in\Delta\omega\tau}\ , \quad (14)$$

where $J_n=\sum_{l=0}^{\infty}\{(-i)^n/[l!(1+n)!]\}[|\Omega_j|^2\beta_j^-/(8\Delta\omega)]^l[|\Omega_j|^2\beta_j^+/(8\Delta\omega)]^{l+n}$ and the time-independent solutions $\chi_j^\mp(q)=\exp(\pm iq'y)$ with $q'=\{2m[\epsilon(q)-V_{jj}^0]/\hbar^2\}^{1/2}$. For simplicity, we omit the subscript j for $\epsilon(q)=\hbar^2q^2/2m$, q' and J_n. Equation (14) shows that the Doppler shift results in energy sidebands. To find the solutions for the reflected wave and the transmitted wave in the regions $y>a$ and $y<-a$, we match a linear superposition of incident and reflected waves, and also transmitted waves on the boundary $y=a$ and $-a$ with the energies $\epsilon(q)+2n\hbar\Delta\omega$ $(n=-\infty,\ldots,\infty)$, to the linear superposition of solutions $\varphi_j^-(q)$ and $\varphi_j^+(q)$ in (14). In this paper, we will mainly concentrate on zero-order sideband scattering corresponding to a very large Doppler shift so that higher-order sideband coefficients $\{J_n\}$ may be neglected. This is quite reasonable for a glancing incidence of the atomic beam with a large velocity v_z. In this case, the atomic wave only "sees" a static average potential in the scattering process. Now, we assume that a thermal atomic source is located at the position $z=z_{\rm in}=0$. It provides a random-phase thermal atomic beam which is statistically described by the density operator

$$\rho_{\rm in}(t)=\int \mathcal{P}(E)dE\langle z_{\rm in}=0|\overline{|\psi_A(E,t)\rangle\langle\psi_A(E,t)|}|z_{\rm in}=0\rangle$$
$$=\int \mathcal{P}(E)dE\left[\int\int dy\,dy'|y\rangle\langle y'|\otimes(P_1^2|1\rangle\langle 1|+P_2^2|2\rangle\langle 2|)g(y)g(y')\exp[-iK_y(y-y')]\right], \quad (15)$$

where $\mathcal{P}(E)$ gives the energy distribution of the thermal atomic beam. The bar denotes the average over all possible phases. P_1^2 and P_2^2 are the statistical weights of the two ground-state sublevels. $g(y)$ is the envelope function of the incident wave with the center velocity $v_y=\hbar K_y/m$ in the y direction. According to (15), we can express the state vector for a thermal atomic beam with energy E as

$$\langle z_{\rm in}=0|\psi_A(E,t)\rangle=e^{-i(E/\hbar+\varepsilon_3)t}\times\int dy\,|y\rangle\otimes\exp(-iK_yy)\times[P_1e^{-i\theta_1}g(y)|1\rangle+P_2e^{-i\theta_2}g(y)|2\rangle]\,. \quad (16)$$

$\theta_{1,2}$ are the random phases of the atomic beam. The average over θ_1 and θ_2 destroys the coherent terms $|2\rangle\langle 1|$ and $|1\rangle\langle 2|$ in the density operator $\rho_{\rm in}(t)$. Hence, the state vector (16) is not a coherent superposition state. In terms of (16), we have the incident atomic wave packet $\varphi_j(\mathbf{r})=P_je^{-i\theta_j}g(y)\exp(-iK_yy)$ at $\tau=z_{\rm in}/v_z=0$. As a result, we have the expressions for the wave packets in region $y>a$ after reflection:

$$\varphi_j(\mathbf{r})=P_je^{-i\theta_j}w(\tau,y)+P_je^{-i\theta_j}f_j(\tau,y)\,. \quad (17a)$$

$w(\tau,y)\equiv\int_{-\infty}^{\infty}dq\,G(q+K_y)\exp[iqy-ie(q)\tau/\hbar]$ gives the evolution of the incident atomic wave packets. $f_j(\tau,y)\equiv\int_0^{\infty}dq\,R_j(q)\exp[iqy-ie(q)\tau/\hbar]$ are the reflected wave packets. Similarly, we have transmitted wave packets in the region $y<-a$

$$\varphi_j(\mathbf{r})=P_je^{-i\theta_j}\int_0^{\infty}dq\,S_j(q)\exp[-iqy-ie(q)\tau/\hbar]\,. \quad (17b)$$

The reflection coefficient $R_j(q)$ and the transmission coefficient $S_j(q)$ have the definitions [15]

$$R_j(q)=\frac{(q^2-q'^2)e^{-2iqa}\sin 2q'a}{(q^2+q'^2)\sin 2q'a+2iqq'\cos 2q'a}G(K_y-q)\,, \quad (18a)$$

$$S_j(q)=\frac{2iqq'e^{-2iqa}}{(q^2+q'^2)\sin 2q'a+2iqq'\cos 2q'a}G(K_y-q)\,, \quad (18b)$$

where $G(q)=(1/2\pi)\int_{-\infty}^{\infty}dy\,g(y)\exp(-iqy)$ is the Fourier transform of $g(y)$. Generally, the higher-order reflection and transmission coefficients have a more complicated form. However, from the solutions (14) and the discussions of Ref. [13], one can conclude that the ratios of the probabilities of the higher-order reflected and transmitted waves with wave vector q to those of the zero-order waves are of the order of $|J_n|^2$. The higher-order reflection and transmission correspond to the diffraction of atomic waves due to the ripple structure of the atomic mirror. We point out that the expressions (18) for the reflection and transmission coefficient are the results obtained with the assumption of a rectangular transverse distribution of the laser beam. The transverse distribution of a real laser field is a Gaussian function. In this case, more exact expressions for the reflection and transmission coefficient can be given by employing the WKB method. However, the expressions (18) can be considered as a valid approximation for the reflection and transmission of a glancing incident atomic wave or a slow atomic wave with very small wave vector K_y compared with the inverse width of a real Gaussian laser beam. In this paper, we have assumed a glancing incident condition.

III. SEPARATION AND SUPERPOSITION OF ATOMIC WAVES

We have given the reflection and transmission coefficient in the previous section. In this section, we will discuss their dependence on the energy $e(q)$ and the laser parameters. For convenience, we define the reflectivity $\mathcal{R}_j=|R_j(q)|^2/G(K_y-q)^2$ and the transmittivity $\mathcal{T}_j=|S_j(q)|^2/G(K_y-q)^2$. According to Eq. (18) we have the relation $\mathcal{R}_j+\mathcal{T}_j=1$. Therefore, we only need to work out the reflectivity. The reflectivity and transmittivity determine the probabilities of atomic waves in the region $y>a$ and $y<-a$. For a real q' which corresponds to an effective potential well ($V_{jj}^0<0$) or a potential barrier with height less than the incident energy in the y direction [$V_{jj}^0<e(q)$], we can maximize the reflectivity if $2q'a=(2n+1)\pi/2$. This is the resonant condition for the reflection of particles in quantum mechanics. This gives the following relation between the incident energy $e(q)$ in the y direction and the laser parameters for the maximized reflectivity:

$$e(q)=V_{jj}^0+\frac{\pi^2\hbar^2}{8ma^2}(n+1/2)^2\quad(n=0,1,2,\ldots) \quad (19a)$$

Similarly, one can find the condition to maximize the transmittivity:

$$e(q)=V_{jj}^0+\frac{\pi^2\hbar^2}{8ma^2}n^2\quad(n=1,2,3,\ldots)\,. \quad (19b)$$

In Fig. 1, we show the reflectivity as a function of $\xi=2q'a$ and $\eta=V_{jj}^0/e(q)$ for a potential well and a low potential barrier. We find that when $V_{jj}^0=0$, the reflectivity is zero for any value $q'a$ and also $e(q)$. In addition, the resonant conditions (19) and Fig. 1 also imply that the reflection and transmission of atomic waves have the property of energy or velocity selection in the y direc-

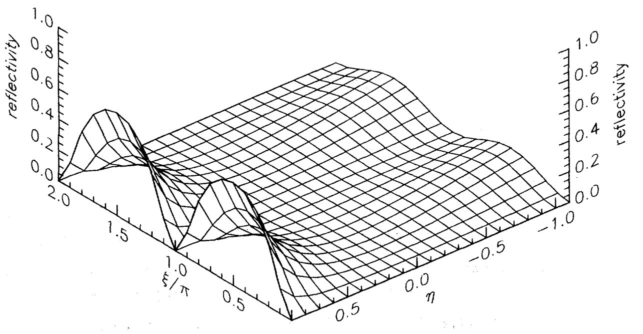

FIG. 1. The reflectivity dependence on the parameters $\xi=2q'a$ and $\eta=V_{jj}^0/e(q)$. The peak values of the reflectivity correspond to the resonant condition (19a).

tion. However, for a potential well or a low potential barrier, the reflectivity is very small even when the resonant condition (19a) is satisfied. Therefore, in later discussions, we will concentrate on the case where one has a high potential barrier by selecting the laser frequency above the atomic resonance. For a high potential barrier $V_{jj}^0>e(q)$, formula (18) and the relation $\mathcal{R}_j+\mathcal{T}_j=1$ is still applicable but q' should be replaced by $i\{2m[V_{jj}^0-e(q)]/\hbar^2\}^{1/2}$. In this case, the transmission of the atomic wave through the laser beam is just an example of quantum-mechanical tunneling through a potential barrier. The reflectivity is plotted in Fig. 2 for this case. We find the reflectivity rapidly approaches one when $\xi=2|q'|a$ increases. This means that the wider the laser beam and the higher the effective potential barrier, the larger the reflectivity. On the other hand, in terms of formula (18), we have

$$R_j(q)=\sqrt{\mathcal{R}_j}\,G(K_y-q)\exp(-i\Theta_R)\ ,$$
$$S_j(q)=\sqrt{\mathcal{T}_j}\,G(K_y-q)\exp(-i\Theta_T)\ .$$

This means that the process of reflection and transmission induces a phase shift for both reflected waves and transmitted waves. The phase shift has the following expression for the reflected wave:

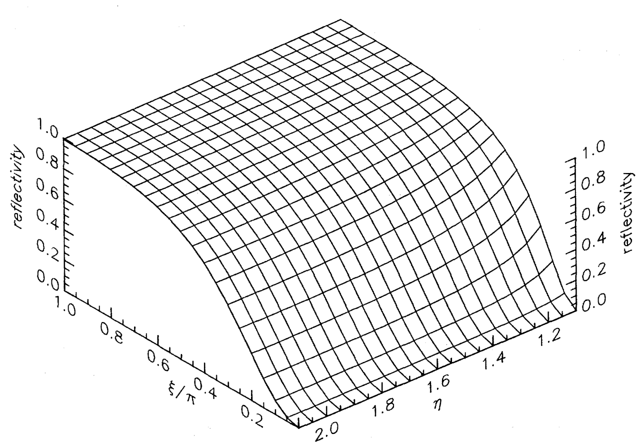

FIG. 2. The reflectivity dependence on the parameters $\xi=2|q'|a$ and $\eta=V_{jj}^0/e(q)>1$.

$$\Theta_R=\begin{cases}2qa+\arctan\left[\dfrac{2qq'}{q^2+q'^2}\cot(2q'a)\right] \\ \qquad [\text{for } e(q)>V_{jj}^0] \\ 2qa+\arctan\left[\dfrac{2q|q'|}{q^2-|q'|^2}\coth(2|q'|a)\right] \\ \qquad [\text{for } e(q)<V_{jj}^0]\end{cases} \tag{20a}$$

and for the transmitted wave

$$\Theta_T=\Theta_R-\frac{\pi}{2}\ . \tag{20b}$$

This shows that there is a fixed phase difference of $\pi/2$ between the reflected wave and the transmitted wave. What is interesting is the fact that for an incident plane monochromatic atomic wave, one can select the laser parameters to achieve a reflectivity $\mathcal{R}_j=$ the transmittivity $\mathcal{T}_j=\frac{1}{2}$. In this case the incident wave is split into two parts with the phase relation $\Theta_R-\Theta_T=\pi/2$. In this sense, the laser field is equivalent to a "semimirror" for a plane monochromatic atomic wave.

We shall now demonstrate how we may use these results to obtain state-dependent reflection and transmission. For the separation of states, we need to have a reflectivity close to 1 for the wave packet corresponding to one internal state and a transmittivity close to 1 for the wave packet corresponding to the other internal state. This may be achieved by selecting different detunings for the ground-state sublevels. For a large value of detuning, we can neglect the dissipative term induced by spontaneous emission in the potential V_{jj}^0 and approximately have $V_{jj}^0=\hbar|\Omega_j|^2\Delta_j/[2(\Delta_j^2-\Delta\omega^2)](j=1,2)$. If the detuning Δ_j for some sublevel $|j\rangle$ is adjusted near zero, the wave corresponding to the internal state $|j\rangle$ transmitted into the region $y<-a$ will have transmittivity close to 1. For a three-level atom interacting with a single-frequency laser, the detuning Δ_i for another sublevel $|i\rangle$ is limited by the relation $\Delta_i-\Delta_j=\Delta$. $|\Delta|$ is the separation between two sublevels. As a result we have $\Delta_i=\Delta$. If sublevel $|i\rangle$ is a higher level than $|j\rangle$ and $\Delta>\Delta\omega$, the wave corresponding to level $|i\rangle$ is reflected into the region $y>a$. One can adjust the other laser parameters to maximize the reflectivity to 1. If $|i\rangle$ is a lower level than $|j\rangle$, we have $\Delta_i=-|\Delta|$. In this case, the reflection occurs for the wave with level $|i\rangle$ when $|\Delta|<\Delta\omega$. Generally, we can always obtain reflection for the higher sublevel with $\Delta>\Delta\omega$. This illustrates how we may separate atomic wave packets by reflection and transmission. A schematic diagram is given in Fig. 3.

If the two ground-state sublevels are nearly degenerate or the standing-wave laser field has two components with different frequencies matching the large separation between two nondegenerate sublevels, we have $\Delta_1=\Delta_2\equiv\delta$. In this case, one cannot neglect the coupling terms in Eqs. (11). Fortunately, we can solve Eqs. (11) by noting $\beta_1^{\pm}=\beta_2^{\pm}\equiv\beta^{\pm}$ and introducing the following transformations:

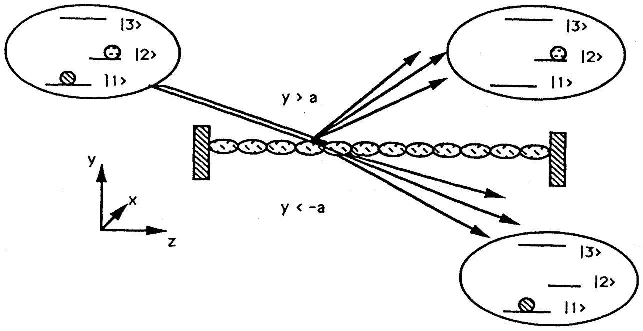

FIG. 3. A schematic diagram for the separation of atomic wave packets by reflection and transmission by an optical ripple mirror. The lines with arrows in the reflected and transmitted wave represent various diffraction orders from the periodic structure of the standing-wave laser.

$$\begin{aligned}\Psi^{(+)}&=\Omega_1\varphi_1+\Omega_2\varphi_2 ,\\ \Psi^{(-)}&=\Omega_2^*\varphi_1-\Omega_1^*\varphi_2 .\end{aligned} \tag{21}$$

Applying (21), we transform Eqs. (11) into two independent Schrödinger equations:

$$i\hbar\frac{\partial\Psi^{(+)}}{\partial\tau}=\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial y^2}\Psi^{(+)}+V\Psi^{(+)} , \tag{22a}$$

$$i\hbar\frac{\partial\Psi^{(-)}}{\partial\tau}=\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial y^2}\Psi^{(-)} . \tag{22b}$$

The potential

$$V=\left[\frac{-i\hbar(|\Omega_1|^2+|\Omega_2|^2)}{4}\right]F(x,y)^2 \times(\beta^-+\beta^++\beta^-e^{2i\Delta\omega\tau}+\beta^+e^{-2i\Delta\omega\tau}) .$$

Equation (22a) has the same form as Eq. (13). Equation (22b) is the Schrödinger equation in free space. This means that when the incident atomic wave passes through the laser field, the combination $\Psi^{(-)}=\Omega_2^*\varphi_1-\Omega_1^*\varphi_2$ will not be affected and directly transmit into the region $y<-a$. $\Psi^{(-)}$ corresponds to the probability amplitude of a "dark" state [16]. To show this, using the transformations (21), we express the atomic waves $\varphi_{1,2}$ in terms of $\Psi^{(\pm)}$ and then have

$$\begin{aligned}\varphi_1(\mathbf{r})&=\frac{\Omega_1^*\Psi^{(+)}+\Omega_2\Psi^{(-)}}{|\Omega_1|^2+|\Omega_2|^2} ,\\ \varphi_2(\mathbf{r})&=\frac{\Omega_2^*\Psi^{(+)}-\Omega_1\Psi^{(-)}}{|\Omega_1|^2+|\Omega_2|^2} .\end{aligned} \tag{23}$$

Substituting the expressions (23) into the transformations (8) and the expression (6), we have the atomic state vector in the regions $y>a$ and $y<-a$:

$$|\psi_A(E,t)\rangle=\int d^3r\, e^{-i(E/\hbar+\varepsilon_3)t}|\mathbf{r}\rangle \otimes[e^{-iK_z z-i\delta z/v_z}(\Psi^{(+)}|+\rangle+\Psi^{(-)}|-\rangle)] . \tag{24}$$

The new orthogonal state vectors $|+\rangle$ and $|-\rangle$ have the definitions

$$|+\rangle=\frac{\Omega_1^*|1\rangle+\Omega_2^*|2\rangle}{|\Omega_1|^2+|\Omega_2|^2},\quad |-\rangle=\frac{\Omega_2|1\rangle-\Omega_1|2\rangle}{|\Omega_1|^2+|\Omega_2|^2} . \tag{25}$$

It is evident that the state vector $|-\rangle$ is a "dark" state [16] with the probability amplitude $\Psi^{(-)}$. Schrödinger equation (22b) implicits that the "dark" state does not "see" the light field effectively. If one selects the detuning $\delta>\Delta\omega$ and the other laser parameters to maximize the reflectivity close to 1, the other combination $\Psi^{(+)}$ will be fully reflected into the region $y>a$. Thus we have an entangled state with the reflected and transmitted waves corresponding to different internal states.

To investigate the coherence of the atomic waves, we still consider the thermal atomic beam described by state vector (16) as the initial incident beam. Equation (22b) gives the freely propagating wave packet in all space $-\infty<y<\infty$:

$$\Psi^{(-)}=(\Omega_2^*P_1e^{-i\theta_1}-\Omega_1^*P_2e^{-i\theta_2})w(\tau,y) . \tag{26}$$

The wave packet $\Psi^{(+)}$ has the following form for perfect reflection ($\mathcal{R}_j=1$):

$$\Psi^{(+)}=\begin{cases}(\Omega_1P_1e^{-i\theta_1}+\Omega_2P_2e^{-i\theta_2})w(\tau,y)\\ \quad+(\Omega_1P_1e^{-i\theta_1}+\Omega_2P_2e^{-i\theta_2})f(\tau,y)\ \ (y>a) , & (27a)\\ 0\ \ (y<-a) . & (27b)\end{cases}$$

$f(\tau,y)$ is the spatial dependence of the reflected atomic wave packet. It has the same form as $f_j(\tau,y)$ but the reflection coefficient should be calculated in terms of the potential V. The expressions (23) and the solutions (26) and (27) give the atomic wave packets corresponding to

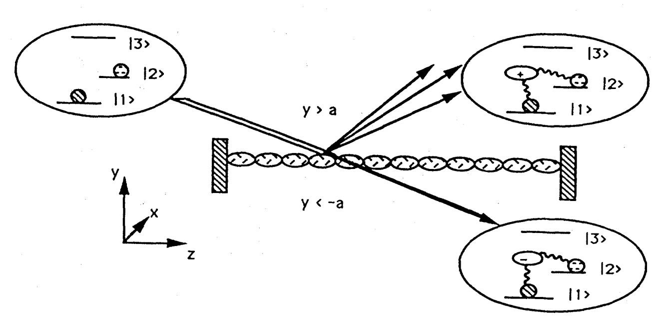

FIG. 4. A schematic diagram for the coherent superposition of atomic wave packets by reflection and transmission by an optical ripple mirror. The wave lines linking the two sublevels in the reflected and transmitted atomic wave represent the coherence between these atomic levels.

different sublevels $|j\rangle$ $(j=1,2)$:

$$\varphi_j=P_je^{-i\theta_j}w(\tau,y)+\frac{\Omega_j^*}{|\Omega_1|^2+|\Omega_2|^2}(\Omega_1P_1e^{-i\theta_1}+\Omega_2P_2e^{-i\theta_2})f(\tau,y) \quad \text{(for the region } y>a) \tag{28a}$$

and

$$\varphi_j=\frac{(-1)^{j+1}\Omega_k}{|\Omega_1|^2+|\Omega_2|^2}(\Omega_2^*P_1e^{-i\theta_1}-\Omega_1^*P_2e^{-i\theta_2})w(\tau,y) \quad (k\neq j=1,2) \quad \text{(for the region } y<-a) . \tag{28b}$$

According to the expressions (28) and (6), we can give the statistically observable density operator $\rho_{\text{out}}(E,t)$ for the atomic wave with energy E after reflection and transmission:

$$\rho_{\text{out}}(E,t)=\langle z=z_{\text{out}},y|\overline{|\psi_A(E,t)\rangle\langle\psi_A(E,t)|}|z=z_{\text{out}},y\rangle$$
$$=\overline{\phi_1^*\phi_1}|1\rangle\langle 1|+\overline{\phi_1^*\phi_2}|2\rangle\langle 1|+\overline{\phi_2^*\phi_1}|1\rangle\langle 2|+\overline{\phi_2^*\phi_2}|2\rangle\langle 2| , \tag{29a}$$

$$\overline{\phi_i^*\phi_j}=P_iP_j\delta_{ij}|w(\tau_{\text{out}},y)|^2+\frac{\Omega_i}{|\Omega_1|^2+|\Omega_2|^2}P_j(\Omega_1^*P_1\delta_{j1}+\Omega_2^*P_2\delta_{j2})w(\tau_{\text{out}},y)f(\tau_{\text{out}},y)^*$$
$$+\frac{\Omega_j^*}{|\Omega_1|^2+|\Omega_2|^2}P_i(\Omega_1P_1\delta_{i1}+\Omega_2P_2\delta_{i2})w(\tau_{\text{out}},y)^*f(\tau_{\text{out}},y)$$
$$+\frac{\Omega_i\Omega_j^*}{(|\Omega_1|^2+|\Omega_2|^2)^2}(|\Omega_1|^2P_1^2+|\Omega_2|^2P_2^2)|f(\tau_{\text{out}},y)|^2 \quad (i,j=1,2;\ \text{for } y>a) \tag{29b}$$

and

$$\overline{\phi_i^*\phi_j}=\frac{(-1)^{i+j}\Omega_n^*\Omega_k}{(|\Omega_1|^2+|\Omega_2|^2)^2}(|\Omega_1|^2P_2^2+|\Omega_2|^2P_1^2)|w(\tau_{\text{out}},y)|^2 \quad (n\neq i=1,2;\ k\neq j=1,2,\ \text{for } y<-a) \tag{29c}$$

where $\tau_{\text{out}}=z_{\text{out}}/v_z$. The density operator $\rho_{\text{out}}(E,t)$ excludes the contribution from the excited-state wave packet, since it is located in the region $-a<y<a$. The second and third terms, in (29b) are determined by the interference of incident atomic wave packets with the reflected atomic wave packets. The interference region depends on the overlap between incident wave packets and reflected wave packets. The interference terms vanish with the reflected wave packets propagating into the region far from the incident wave. The fourth term in (29b) is caused by the self-interference term of the reflected wave packet which does not vanish anywhere for $y>a$. For the transmitted wave, in the expression (29c) one only finds the self-interference term. The self-interference terms in both the reflected wave and the transmitted wave play an important role in preserving the coherence of the atomic waves as they propagates into a far region from the mirror. We can see this by putting $i\neq j=1,2$ in (29). This gives the dependence of the coherent terms $\overline{\phi_1^*\phi_2}|2\rangle\langle 1|+\overline{\phi_2^*\phi_1}|1\rangle\langle 2|$ on the self-interference terms. We always have nonzero values for $\overline{\phi_1^*\phi_2}$ and $\overline{\phi_2^*\phi_1}$ in the regions after reflection and transmission. Comparing (29a) with (15), we deduce that the standing-wave laser field creates a coherent superposition state of reflected and transmitted atomic waves entangled with different internal states. We give a schematic diagram in Fig. 4.

IV. CONCLUSIONS

In this paper, we derive the coupled wave equations describing the scattering of atomic waves with a very small angle of incidence from a standing-wave laser field. The reflection coefficient and transmission coefficient are obtained for the zero-order scattering. We demonstrate the possibility to separate the atomic waves corresponding to different internal states by reflection and transmission. We can make a "semimirror" with one-half of the atomic beam being reflected and the other half being transmitted. The phase difference between the transmitted and reflected wave is $\pi/2$. We also show the standing-wave laser field can reflect and transmit the atomic waves in a superposition of internal states entangled with different center-of-mass motion.

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