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Quantum-optical version of Cramer's theorem

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Cramer's theorem is formulated in the context of quantum optics. A physical meaning for the theorem is given and is illustrated by the generation of thermal noise from a pure quantum state.

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I. INTRODUCTION

Consider a beam (designated by 1) that is split by an ideal splitter into two beams (3 and 4). We now inquire: can we have, within quantum mechanics, these beams (3 and 4) independent? That is, can we have their combined density matrix factorized: $\rho(3,4)=\rho_3(3)\otimes\rho_4(4)$? If so, we could, in principle, emulate them by two independent sources. This problem was raised and solved about 25 years ago [1]. It was shown that only if beam 1 is in a pure Glauber coherent state [2], then does this factorization obtain. This was [1] contrasted with classical physics, where it is always possible to generate two independent beams that will emulate the splitting of a single source (1, in our example).

A natural generalization of this is as follows: consider two independent beams 1 and 2 that are (nontrivially) split to form two new beams 3 and 4. What could 1 and 2 be if we require, as before, that the resultant beams be independent [i.e., $\rho(3,4)=\rho_s(3)\otimes\rho_4(4)$]? Thus, if, in the previous analysis, we had port 2 nonempty, with $\tilde{\rho}(1,2)=\rho_1(1)\otimes\rho_2(2)$, yet we still required the "final" beams 3 and 4 to be independent—what would be the allowed $\tilde{\rho}(1,2)$? Density matrices possessing this property were termed bifactorizable [3]. It was shown [3] that they are necessarily Gaussian (i.e., their density matrices are exponential in, at most, a quadratic form of the appropriate creation or annihilation operators). It is always possible to associate (uniquely) a temperaturelike parameter to a Gaussian density matrix (e.g., cf. Appendix A). In this parametrization, a general property of a bifactorizable density matrix is that all of its constituent single-beam density matrices (ρ_i, $i=1$, 2, 3, and 4) are at the same temperature. [Thus, e.g., the first case that was discussed above, port 2, was in its vacuum state ($\rho_2=|0\rangle_2\otimes{}_2\langle 0|$); this forced all the other density matrices (ρ_i, $i=1,3,4$) to be at zero temperature (the temperature of the vacuum), and with the added requirement of being Gaussian, we got a pure coherent state.] Further generalization can be studied with the aid of Cramer's theorem, which we now introduce.

In classical probability theory, Cramer's theorem is considered with special interest [4]. The theorem exposes a peculiar property of Gaussian distributions that pertains to any number of random variables. Thus it asserts: given two independent random variables X_1 and X_2, and given that the distribution of their sum X_1+X_2 is Gaussian, then all the distributions (i.e., for both X_1 and X_2) are Gaussian. The transcription of the theorem to quantum mechanics and field theory was given by Hegerfeldt [5], and he and Emch [6] used it in their study of thermal coherent states [7]. Interrelation of the quantum and the classical versions of the theorem was considered recently [8] in a study of Gaussian distributions. In the present paper, we use the quantum version of Cramer's theorem [5] to generalize the studies discussed above to the case where the "final" beams (3 and 4 in our example) are not independent. Indeed we will, after defining means of ordering correlations [9], consider maximally correlated beams. This will allow us to accommodate the case of "temperature in a pure state" [10,11] within the quantum version of Cramer's theorem.

The paper is organized as follows. In Sec. II, we give the basic formalism and definitions. In Sec. III, the quantum version of Cramer's theorem is presented and some of its physical contents is discussed. Section IV is devoted to conclusions and remarks. Appendix A includes a proof that the most general Gaussian density matrix can be parametrized as a thermal squeezed state. A proof for the quantum-optical version of Cramer's theorem is given in Appendix B, while Appendix C contains a derivation of the thermal distribution for one mode when the two-mode density matrix is maximally correlated.

II. COHERENT-STATE FORMALISM

Our analysis will be given in terms of Glauber's coherent-state representation [2]. This representation allows visualization of mode interchange, which we now proceed to elucidate.

Consider an idealized splitter, as depicted in Fig. 1. The effect of the splitter is given by

$$a_1^\dagger=\mu a_3^\dagger+\nu a_4^\dagger\ , \tag{1}$$

$$a_2^\dagger=-\nu^* a_3^\dagger+\mu^* a_4^\dagger\ , \tag{2}$$

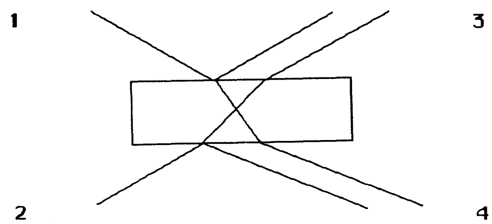

FIG. 1. Idealized splitter: two uncorrelated beams 1 and 2, resulting in beams 3 and 4.

$$|\mu|^2+|\nu|^2=1 \ . \tag{3}$$

In the case of pure states, the state in terms of modes 1 and 2 (to the left of the splitter) is

$$f_1(a_1^\dagger)f_2(a_2^\dagger)|0\rangle \ , \tag{4}$$

while in terms of the modes 3 and 4, the state is

$$f_1(\mu a_3^\dagger+\nu a_4^\dagger)f_2(-\nu^* a_3^\dagger+\mu^* a_4^\dagger)|0\rangle \equiv F(a_3^\dagger,a_4^\dagger)|0\rangle \ . \tag{5}$$

In modes 1 and 2, the density matrix factorizes,

$$\langle \alpha_1\alpha_2|\rho|\alpha_1'\alpha_2'\rangle=\langle\alpha_1|\rho_1|\alpha_1'\rangle\langle\alpha_2|\rho_2|\alpha_2'\rangle \tag{6}$$

with [2]

$$\langle\alpha_i|\rho_i|\alpha_i'\rangle=e^{-|\alpha_i|^2/2-|\alpha_i'|^2/2}f_i(\alpha_i^*)f_i^*(\alpha_i') \ , \tag{7}$$

$i=1,2$. Here $|\alpha_i\rangle$ is the eigenfunction of the annihilation operator a_i, of mode i. In terms of the modes 3 and 4, the density matrix is

$$\langle\alpha_3\alpha_4|F(a_3^\dagger,a_4^\dagger)|0\rangle\langle 0|F^*(a_3,a_4)|\alpha_3'\alpha_4'\rangle \ . \tag{8}$$

Here

$$\alpha_3=\mu\alpha_1-\nu^*\alpha_2 \ , \tag{9}$$

$$\alpha_4=\nu\alpha_1+\mu^*\alpha_2 \ , \tag{10}$$

with similar expressions for α_3' and α_4'. The above formulas were given to illustrate our approach. In general, we assume throughout this paper that, in terms of modes 1 and 2, the density matrix factorizes:

$$\rho(1,2)=\rho_1(1)\otimes\rho_2(2) \ . \tag{11}$$

Furthermore, we shall take all beams to be of equal frequency (we shall return to this point later).

As was stated in the Introduction, an additional requirement that

$$\rho(3,4)=\rho_3(3)\otimes\rho_4(4) \tag{12}$$

would imply that all density matrices are Gaussian [3], i.e., their characteristic functions [12] are given by

$$\begin{aligned}C_j(\lambda)&\equiv \mathrm{Tr}[\rho_j\exp(\lambda a_j^\dagger-\lambda^* a_j)]\\ &=\exp[\tfrac{1}{2}\langle(\lambda a_j^\dagger-\lambda^* a_j)^2\rangle] \ .\end{aligned} \tag{13}$$

It can be shown (and is outlined in Appendix A) that we may always parametrize a general Gaussian density matrix (GGDM) as a thermal squeezed state [7,13]. The latter is defined by

$$\rho_{\mathrm{TSS}}=S(\zeta)e^{-\beta\omega a^\dagger a}S^\dagger(\zeta) \ , \tag{14}$$

$$S(\zeta)=\exp[(\zeta a^{\dagger 2}-\zeta^* a^2)/2] \ , \quad \zeta=re^{i\varphi}, \quad r,\varphi \text{ real} \ . \tag{15}$$

(Here we take $\langle a\rangle=\langle a^\dagger\rangle=0$, as we can always consider $a'=a-\langle a\rangle$.) Now, in the above, the parameter $\beta\omega$ is non-negative, β is referred to as inverse temperature, and ω is the frequency. Returning to the case of bifactorization [(11) and (12)], in this case it was shown that $\beta\omega$ must be common to all (i.e., $\beta_i\omega_i=\beta_j\omega_j$, $i=1,2,3,4$). In our case, where we consider modes of the electromagnetic-radiation field all of the same frequency, bifactorization implies common temperature to all modes involved.

It is convenient at this point to define the index of correlation [9] $I_c^{3,4}$ of modes 3 and 4:

$$I_c^{3,4}=S_3+S_4-S \ , \tag{16}$$

$$S=-\mathrm{Tr}\rho\ln\rho \ , \tag{17}$$

$$S_i=-\mathrm{Tr}\rho_i\ln\rho_i \ . \tag{18}$$

Here ρ is the two-mode density matrix, and ρ_i is the density matrix obtained upon tracing out the coordinates of mode j ($\neq i$). Clearly,

$$I_c^{ij}=0 \quad \text{whenever } \rho=\rho_i\otimes\rho_j \ . \tag{19}$$

III. QUANTUM VERSION OF CRAMER'S THEOREM AND ITS PHYSICAL CONTENT

The quantum-optics version of Cramer's theorem is the following theorem (proven originally by Hegerfeldt [5], cf. also Appendix B): Given (i) two independent beams 1 and 2 (Fig. 1), which are intermixed via a splitter to form modes 3 and 4, and, (ii) upon tracing out the coordinates of mode 3 (say), the resultant density matrix (for mode 4) is a general Gaussian density matrix; then, states the theorem, both mode 1 (constituent of beam 1) and mode 2 are given by Gaussian density matrices. A more detailed connection between the classical and quantum versions is given in Appendix B. Our point is that concurrence with Cramer's theorem allows, with modes intermixed via a splitter, the inclusion of correlated density matrices for the 3 and 4 modes with striking Gaussian properties. We now illustrate this in a particularly interesting case: consider the distribution of a pure state ψ for modes 3 and 4,

$$\psi(3,4)=\exp[\gamma a_4^\dagger a_3^\dagger-\gamma^* a_3 a_4]|0\rangle \ . \tag{20}$$

This state was shown [9] to be maximally correlated, i.e., to have a maximal index of correlation (for fixed average energy), for modes 3 and 4. Since it is a pure state, it is also a pure state in modes 1 and 2. We show in Appendix C that tracing out mode 3 leads to a thermal (hence Gaussian) density matrix for mode 4,

$$\rho_4 = \sum_{n=0}^{\infty} e^{-\beta n\omega}|n\rangle\langle n| \Big/ (1-e^{-\beta\omega}) , \tag{21}$$

with

$$\tanh|\gamma| = e^{-\beta\omega/2} . \tag{22}$$

In terms of modes 1 and 2, the states must be Gaussian because of Cramer's theorem. The distributions are (i.e., the pure state wave functions)

$$\psi(i) = \exp\left[-i\gamma\frac{a_i^{\dagger 2}}{2} + i\gamma^*\frac{a_i^2}{2} \right]|0\rangle , \quad i=1,2 , \tag{23}$$

i.e., the splitter parameters $\mu=|\mu|\exp(i\varphi_\mu)$ and $\nu=|\nu|\exp(i\varphi_\nu)$ in Eqs. (1)–(3) are such that

$$|\mu| = \nu| = \frac{1}{\sqrt{2}} , \tag{24}$$

and

$$\varphi_\nu + \varphi_\mu = \frac{\pi}{2} . \tag{25}$$

This example illustrates how Cramer's theorem accommodates the possibility of combining two independent states (1 and 2) to form a state such that expectation values involving only the coordinates of one mode (4, in our example) cannot be distinguished from a thermal state. The example we considered is particularly simple, but it underscores a physical implication of Cramer's theorem: the generation of thermal noise from a pure state [10]. Here it is a limiting special case, e.g., a splitter whose parameters are not those given by (24) and (25), but for which $\varphi_\nu+\varphi_\mu \neq n\pi$, n an integer will give rise to, upon tracing out of one variable, a nonzero temperature distribution for the other variable, i.e., $\varphi_\nu+\varphi_\mu \neq n\pi$ leads to correlations between modes 3 and 4. The example considered gives rise to maximal correlation and hence to maximal temperature difference.

The underlying mathematical reason is that, whenever we start (i.e., the independent modes 1 and 2) with nontrivial Gaussian states (e.g., squeezed states), intermixtures of modes via unitary transformation (splitter's parameters μ and ν complex) are allowed inasmuch as Cramer's theorem is applicable to the resultant state. This is to be contrasted with the requirement for bifactorization, which, when we start with a nontrivial Gaussian density matrix, allows only orthogonal transformations (μ and ν real) to retain the independence of the modes.

IV. SUMMARY AND CONCLUSIONS

The quantum-mechanical formulation of Cramer's theorem that was given by Hegerfeldt and Emch [5,6] is, in this paper, formulated in quantum-optics language. This formulation of the theorem revealed new physical interpretations for it. One such interpretation showed the relation of the theorem to the general problem of generating thermal noise from pure states. As such, this interpretation is realizable in radiation from a black hole (Hawking's radiation [14]). For example, consider two independent (such as are emanated from sources that are a spacelike distance apart) pure states that combine and involve both coordinates of interest and coordinates of a mode that is a part of a black hole that are to be traced out. This leaves the modes of interest in a thermal state. An analogous situation is present in thermofield dynamics [15], where one traces out the unobserved ("tilde") degrees of freedom, leaving the physical degrees of freedom at a finite temperature. Cramer's theorem assures us that in these cases the sources (if independent) are Gaussian. The quantum-optics version of Cramer's theorem that was studied in this paper allows a fresh interpretation of the result of Ref. [6]. Here the authors took the modes 1 and 2 as two independent particles in harmonic potentials. Modes 3 and 4 were the center of mass and relative coordinates of these particles. In this case, if different frequencies are allowed for the motion of the center of mass and that of the particles, we cannot get a simple linear relation between the field operators of modes 1 and 2 and those of modes 3 and 4. It is possible, in this case, to study the problem in terms of position and momentum operators [6]. Here, too, Cramer's theorem allows different temperatures for the various modes.

Our conclusion is that the field-theoretic Cramer's theorem expresses the possibility, within quantum field theory, of bringing together two systems of temperature T_1 and T_2, and obtaining a system whose (observational) temperature is higher than either, and all of whose distributions are Gaussian. As such, the theorem deals with situations that are generalizations of the interesting studies (involving Gaussian distributions, too) where two independent beams led to two (other) independent beams.

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APPENDIX A: PARAMETRIZATION OF A GENERAL GAUSSIAN DENSITY MATRIX AS A TSS

A thermal-squeezed-state (TSS) density matrix is defined [cf. Eq. (14)] by [7]

$$\rho_{\mathrm{TSS}} = S^\dagger(\zeta)e^{-Ka^\dagger a}S(\zeta)/Z , \tag{A1}$$
$$Z = \mathrm{Tr}\rho_{\mathrm{TSS}} .$$

Here $K=\beta\omega$ [Eq. (14)] and

$$S(\zeta) = \exp[(\zeta a^{\dagger 2} - \zeta^* a^2)/2] , \quad \zeta = re^{i\varphi} . \tag{A2}$$

Our definition (A1) assumes (this incurs no loss of generality) that $\langle a^\dagger\rangle = \langle a\rangle = 0$.

One readily derives

$$S(\zeta)a^\dagger S^\dagger(\zeta) = a^\dagger\cosh r - ae^{-i\varphi}\sinh r . \tag{A3}$$

We can now evaluate the characteristic function,

$$C_{\rm TSS}(\lambda)=\mathrm{Tr}\rho_{\rm TSS}e^{\lambda a^{\dagger}-\lambda^{*}a}$$
$$=\mathrm{Tr}e^{-Ka^{\dagger}a}S(\zeta)e^{\lambda a^{\dagger}-\lambda^{*}a}S^{\dagger}(\zeta)/Z \ . \qquad \text{(A4)}$$

Using (A3), we get, in terms of thermal [i.e., $\rho=Z^{-1}\exp(-Ka^{\dagger}a)$] expectation values,

$$C_{\rm TSS}(\lambda)=\exp\langle \tfrac{1}{2}(\alpha a^{\dagger}-\alpha^{*}a)^{2}\rangle \ , \qquad \text{(A5)}$$

with

$$\alpha=\lambda\cosh r+\lambda^{*}e^{i\varphi}\sinh r \ . \qquad \text{(A6)}$$

Now

$$\langle(\alpha a^{\dagger}-\alpha^{*}a)^{2}\rangle=-\alpha\alpha^{*}\coth(K/2) \ . \qquad \text{(A7)}$$

Using (A6), this becomes

$$\tfrac{1}{2}\langle(\alpha a^{\dagger}-\alpha^{*}a)^{2}\rangle$$
$$=\frac{\coth(K/2)}{2}[\,|\lambda|^{2}(\cosh^{2}r+\sinh^{2}r)$$
$$+(\lambda^{2}e^{-i\varphi}+\mathrm{c.c.})\cosh r\sinh r\,] \ . \qquad \text{(A8)}$$

Taking the general Gaussian density matrix as

$$\langle g|\rho|g'\rangle=N_{\rho}\exp\left[-\frac{A}{2}g^{2}-\frac{A^{*}}{2}g'^{2}+Cgg'\right] \ , \qquad \text{(A9)}$$

the requirements for Hermiticity, normalizability, and positive definiteness lead to Re $A>0$, $C\geq 0$, $C=C^{*}$. Defining

$$\hat{a}=\left[\frac{\alpha}{2}\right]^{1/2}\hat{Q}+\frac{i}{\sqrt{2\alpha}}\hat{P} \ , \qquad \text{(A10)}$$

$$A=A_{1}+iA_{2} \ , \qquad \text{(A11)}$$

$$D=\alpha^{2}-(|A|^{2}-C^{2})+2i\alpha A_{2} \ , \qquad \text{(A12)}$$

we can make the following identification after evaluating the characteristic function corresponding to (A9):

$$e^{2i\varphi}=\frac{D}{D^{*}} \ , \qquad \text{(A13)}$$

$$\cosh r=\frac{\alpha}{2(A_{1}^{2}-C^{2})^{1/2}}\left[1+\frac{|A|^{2}-C^{2}}{\alpha^{2}}\right] \ , \qquad \text{(A14)}$$

$$\coth(K/2)=\frac{A_{1}+C}{A_{1}-C} \ . \qquad \text{(A15)}$$

The essential result is (A15) which, with $A_{1}>0$ and $C\geq 0$, allows the parameter K to be non-negative and thus be interpreted as the temperature of a TSS ($C=0$ corresponds to a pure state—$\beta\rightarrow\infty$).

APPENDIX B: QUANTUM FIELD VERSION OF CRAMER'S THEOREM

Given (1) that the two-mode density matrix factorizes in modes 1 and 2 [Eq. (6)], and (2) that mode 4 is Gaussian, i.e., Eq. (13) holds for $j=4$; we wish to show that ρ_{j} for $j=1,2$ are Gaussian. Define [8] a Hermitian operator ($i=\sqrt{-1}$),

$$G_{4}=i(\lambda a_{4}^{\dagger}-\lambda^{*}a_{4})=G_{4}^{\dagger} \ . \qquad \text{(B1)}$$

Upon introducing a scaling (real) parameter u [8], Eq. (13) reads

$$C_{4}(u)=\exp\left[-\frac{u^{2}}{2}\langle(G_{4})^{2}\rangle\right] \ . \qquad \text{(B2)}$$

Utilizing Eqs. (1) and (2), we have

$$\hat{G}_{4}=\hat{G}_{1}+\hat{G}_{2} \ , \qquad \text{(B3)}$$

with

$$\hat{G}_{1}=i(\lambda\nu^{*}a_{1}^{\dagger}-\lambda^{*}\nu a_{1})=\hat{G}_{1}^{\dagger} \ , \qquad \text{(B4)}$$

$$\hat{G}_{2}=i(\lambda\mu a_{2}^{\dagger}-\lambda^{*}\mu^{*}a_{2})=\hat{G}_{2}^{\dagger} \ . \qquad \text{(B5)}$$

Let us express the trace [left-hand side Eq. (13)]

$$\mathrm{Tr}\rho\exp(-iuG_{4}) \qquad \text{(B6)}$$

in terms of complete orthonormal sets of eigenfunctions and eigenvalues of G_{1} and G_{2}. In this case, (B6) reads

$$\int dg_{1}\langle g_{1}|\rho_{1}|g_{1}\rangle e^{-iug_{1}}\int dg_{2}\langle g_{2}|\rho_{2}|g_{2}\rangle e^{-iug_{2}} \ . \qquad \text{(B7)}$$

Here we used

$$\hat{G}_{i}|g_{i}\rangle=g_{i}|g_{i}\rangle \ , \quad i=1,2 \ . \qquad \text{(B8)}$$

We have that $\langle g_{i}|\rho_{i}|g_{i}\rangle\geq 0$ and $\int dg_{i}\langle g_{i}|\rho_{i}|g_{i}\rangle=1$, i.e., we may consider it as a classical distribution. Equating (B7) to (B2), we have, by the classical version of Cramer's theorem, that each of these factors is Gaussian in λ. The unique relation between density matrix and characteristic functions ensures that ρ_{i} is Gaussian.

APPENDIX C: THE DERIVATION OF THE THERMAL DENSITY MATRIX, EQ. (21)

The operators

$$K_{+}=a_{3}^{\dagger}a_{4}^{\dagger} \ , \qquad \text{(C1)}$$

$$K_{-}=a_{4}a_{3} \ , \qquad \text{(C2)}$$

$$K_{0}=\tfrac{1}{2}(a_{3}^{\dagger}a_{3}+a_{4}a_{4}^{\dagger}) \qquad \text{(C3)}$$

close the su(1,1) algebra, hence it is readily shown that

$$\mathrm{sech}|\gamma|\exp\left[\left[\frac{\gamma}{|\gamma|}\tanh|\gamma|\right]a_{3}^{\dagger}a_{4}^{\dagger}\right]|0\rangle$$
$$=\exp(\gamma a_{3}^{\dagger}a_{4}^{\dagger}-\gamma^{*}a_{4}a_{3})|0\rangle \ . \qquad \text{(C4)}$$

Thus our pure-state density matrix is

$$\rho(3,4)=(\mathrm{sech}|\gamma|)^{2}\exp\left[\left[\frac{\gamma}{|\gamma|}\tanh|\gamma|\right]a_{3}^{\dagger}a_{4}^{\dagger}\right]|0\rangle$$
$$\times\langle 0|\exp\left[\left[\frac{\gamma^{*}}{|\gamma|}\tanh|\gamma|\right]a_{4}a_{3}\right] \ ,$$

$$\mathrm{Tr}_{3}\rho(3,4)=\sum_{n_{3}}\langle n_{3}|\rho(3,4)|n_{3}\rangle$$
$$=(\mathrm{sech}|\gamma|)^{2}\sum_{n}(\tanh|\gamma|)^{2n}|n\rangle\langle n| \ ,$$

with $\tanh|\gamma|=e^{-\beta\omega/2}$, and we get Eq. (36).

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