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Vacuum-Rabi-splitting-induced transparency

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The vacuum Rabi splitting may be observed with population-trapping techniques. The trapped population has a lifetime much greater than the natural lifetime of the atomic transition; as a result, vacuum Rabi splittings that are much smaller than the natural lifetime may be observed. A condition for lasing without inversion in this system without any externally injected coherent field or coherence is stated.

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A two-level atom placed in a resonant cavity tuned to the transition frequency will exhibit a two-peaked absorption spectrum that is called a vacuum Rabi splitting. This two-peaked structure arises from the resonant interaction of the electromagnetic-field mode in the cavity with the atomic transition. At microwave frequencies, because of the very high Q factor of available resonators and the very slow relaxation of microwave atomic transitions, this effect has been observed [1,2]. At optical frequencies, typical spontaneous emission linewidths are much larger and cavity finesses are much smaller than in the microwave regime. Doppler widths are also much larger. To avoid these problems very high finesse short optical cavities have been constructed, and the effect has been observed with an atomic beam of sodium atoms [3–5]. This paper describes a scheme to exploit population trapping to reduce the cavity and atom temperature requirements and still resolve the effect.

The cancellation of optical loss with a quantum interference has been considered by many workers [4,5]. Stroud, Mollow, Knight, and others have considered a discrete three-level system based on population trapping with a strong coherent field [6–9]. The system described in this paper is very similar to the one considered by these workers except that there is no external laser field present to create the interference.

The interference is created by the vacuum field inside a resonant cavity. The cavity vacuum fluctuations couple two states of a three-level atom and cancel the absorption. The total solid angle subtended by the cavity is small and so the total spontaneous emission rates for the atoms are essentially unchanged. The atom behaves as if a laser field sufficient to cause the vacuum Rabi splitting had been applied. It has been found both theoretically and experimentally that the presence of small amplitude or phase fluctuations can dramatically affect the optical loss in these systems [10]. Consequently, on the basis of previous work, one might expect that the large fractional amplitude and phase fluctuations of the vacuum field would destroy the transparency. This is not the case. Since the Rabi splitting is a function of the cavity photon number operator, the vacuum field which is an eigenstate of the number operator does not cause fluctuations in the Rabi splitting. Because of this fact, a vacuum Rabi frequency that is much smaller than the reciprocal lifetime of the transition may be resolved. This does not lead to a reduction in the required cavity finesse; it is only a reduction in the required coupling of the cavity mode to the atoms. A possible system in sodium is presented. A threshold condition for lasing without inversion [11] will also be stated.

The prototypical system is shown in Fig. 1. The atoms couple to a single mode cavity which is resonant at the frequency of the $|2\rangle$ to $|3\rangle$ atomic transition. The probe laser that couples from $|1\rangle$ to $|3\rangle$ is very weakly coupled to the atom and may be assumed to be in a coherent state. Finally, the atom-cavity system is coupled to reservoirs of vacuum modes at zero or finite temperature to allow for relaxation. The Hamiltonian for this system is

$$\begin{aligned}\hat{H}=&E_{21}|2\rangle\langle 2|+E_{31}|3\rangle\langle 3|+\hbar\omega_c\hat{a}^\dagger\hat{a}\\&+\{-\alpha\exp(-i\omega_p t)|1\rangle\langle 3|+ig\hat{a}^\dagger|2\rangle\langle 3|+\text{c.c.}\}\\&+\sum_k\hbar\omega_k\hat{b}_k^\dagger\hat{b}_k+\{i\sum_{i,j}g_{ij}(k)\hat{b}_k^\dagger|i\rangle\langle j|+\text{c.c.}\}\\&+\sum_k\hbar\omega_k\hat{c}_k^\dagger\hat{c}_k+\{q_k\hat{c}_k^\dagger\hat{a}+\text{c.c.}\}\ ,\end{aligned}\qquad(1)$$

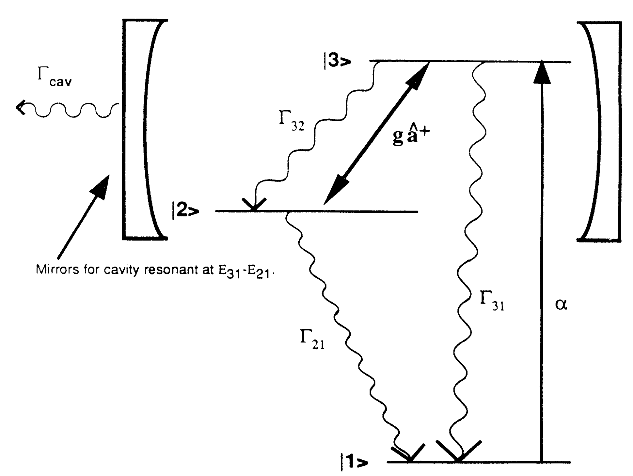

FIG. 1. Prototype energy-level diagram for vacuum Rabi transparency. Γ_{31}, Γ_{32}, and Γ_{cav} represent couplings to photon reservoirs. Γ_{21} is the decay rate of level $|2\rangle$ and $g\hat{a}^\dagger$ represents the mode of the $|2\rangle$-$|3\rangle$ resonant cavity.

where α represents the coherent coupling from $|1\rangle$ to $|3\rangle$, g is the coupling constant between the atoms and the resonant cavity mode described by the $\hat{a}$ operator, and $g_{ij}(k)$ is the coupling between the atoms and the normal vacuum modes described by the $\hat{b}$ operators. ω_{21} and ω_{31} are the frequencies of the $|2\rangle$ and $|1\rangle$ and the $|3\rangle$ and $|1\rangle$ transitions, respectively. The reservoir couplings are included in the first summation term. The cavity loss is accounted for by coupling the cavity mode to a reservoir of modes c_k and the coupling constants q_k are chosen to achieve the correct cavity decay rate. After making a rotating-wave approximation and tracing over the effects of the reservoirs, the atom-cavity evolution equations are

$$\begin{aligned}
&\hat{\sigma}_{ij}=|i\rangle\langle j| ,\\
&\hat{\sigma}_{11}+\hat{\sigma}_{22}+\hat{\sigma}_{33}=1 ,\\
&\frac{d\hat{\sigma}_{11}}{dt}=-\frac{i}{\hbar}(\alpha\hat{\sigma}_{13}-\alpha^*\hat{\sigma}_{31})+\Gamma_{31}(1+n_{31})\hat{\sigma}_{33}\\
&\qquad+\Gamma_{21}\hat{\sigma}_{22}-\Gamma_{31}n_{31}\hat{\sigma}_{11} ,\\
&\frac{d\hat{a}^\dagger\hat{\sigma}_{22}\hat{a}}{dt}=\frac{g}{\hbar}\hat{a}^\dagger\hat{\sigma}_{23}\hat{a}\hat{a}^\dagger+\frac{g^*}{\hbar}\hat{\sigma}_{32}\hat{a}\hat{a}^\dagger\hat{a}\\
&\qquad-(\Gamma_{21}+\Gamma_{32}n_{32}+\Gamma_{\text{cav}})\hat{a}^\dagger\hat{\sigma}_{22}\hat{a}\\
&\qquad+\Gamma_{\text{cav}}\hat{c}^\dagger\hat{c}(t)\hat{\sigma}_{22}+\Gamma_{32}(1+n_{32})\hat{a}^\dagger\hat{\sigma}_{33}\hat{a} ,\\
&\frac{d\hat{\sigma}_{33}}{dt}=\frac{i}{\hbar}(\alpha\hat{\sigma}_{13}-\alpha^*\hat{\sigma}_{31})-\frac{g}{\hbar}\hat{a}^\dagger\hat{\sigma}_{23}-\frac{g^*}{\hbar}\hat{\sigma}_{32}\hat{a}\\
&\qquad+\Gamma_{32}n_{32}\hat{\sigma}_{22}+\Gamma_{31}n_{31}\hat{\sigma}_{11}\\
&\qquad-[\Gamma_{31}(1+n_{31})+\Gamma_{32}(1+n_{32})]\hat{\sigma}_{33},\\
&\frac{d\hat{\sigma}_{13}\hat{a}}{dt}=\frac{i\alpha^*}{\hbar}(\hat{\sigma}_{33}-\hat{\sigma}_{11})-\frac{g^*}{\hbar}\hat{\sigma}_{12}\hat{a}\\
&\qquad-[\Gamma_{31}(1+2n_{31})+\Gamma_{32}(1+n_{32})+2i\Delta\omega_{31}]\frac{\hat{\sigma}_{13}}{2} ,\\
&\frac{d\hat{\sigma}_{32}\hat{a}}{dt}=\frac{i\alpha}{\hbar}\hat{\sigma}_{12}\hat{a}+\frac{g}{\hbar}\hat{\sigma}_{33}\hat{a}\hat{a}^\dagger-\frac{g}{\hbar}\hat{a}^\dagger\hat{\sigma}_{22}\hat{a}\\
&\qquad+[\Gamma_{31}(1+n_{31})+\Gamma_{32}(1+2n_{32})+\Gamma_{\text{cav}}+\Gamma_{21}\\
&\qquad+2i(\Delta\omega_{21}-\Delta\omega_{31})]\frac{\hat{\sigma}_{32}\hat{a}}{2} ,\\
&\frac{d\hat{\sigma}_{12}\hat{a}}{dt}=\frac{i\alpha^*}{\hbar}\hat{\sigma}_{32}\hat{a}+\frac{g}{\hbar}\hat{\sigma}_{13}\hat{a}\hat{a}^\dagger\\
&\qquad-[\Gamma_{31}n_{31}+\Gamma_{21}+\Gamma_{\text{cav}}+\Gamma_{32}n_{32}\\
&\qquad+2i\Delta\omega_{21}]\frac{\hat{\sigma}_{12}\hat{a}}{2} ,\\
&\frac{d\hat{a}^\dagger\hat{a}}{dt}=\Gamma_{\text{cav}}(\hat{c}^\dagger\hat{c}(t)-\hat{a}^\dagger\hat{a})+\frac{g^*}{\hbar}\hat{\sigma}_{32}\hat{a}+\frac{g}{\hbar}\hat{a}^\dagger\hat{\sigma}_{23} .
\end{aligned} \tag{2}$$

In these equations, n_{31}, n_{32}, and n_{21} represent the average thermal occupation numbers of the corresponding thermal reservoirs. It is important to note that the cavity reservoir occupation number $\hat{c}^\dagger\hat{c}(t)$ has not been ensemble averaged. The reason for this is that in the examples presented in this paper the absorption depends nonlinearly on this value. Ensemble averaging neglects the quantum statistics of this field. The first case to consider is that of weak probe-laser excitation of the system with all reservoirs at zero temperature. Initially, assume that the system is entirely in the ground state $|1\rangle$. The probe-laser coupling α is a perturbation to the eigenstate $|1\rangle$ and the loss can be found in powers of α. Since the probe laser is weak, all terms of α^3 and higher order may be neglected. The loss rate from the ground state due to the probe-laser beam is

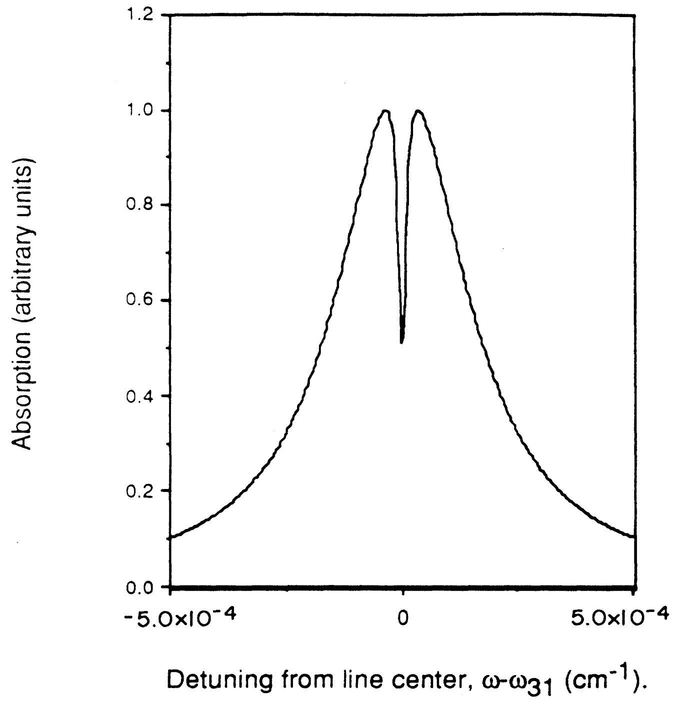

FIG. 2. Resonance absorption profile for atoms in single mode cavity. In this graph, $\Gamma_{\text{cav}}+\Gamma_{21}=10^{-5}$ cm^{-1}, $\Gamma_3=3.33\times10^{-4}$ cm^{-1}, $g=5.9\times10^{-5}$ cm^{-1}. Notice the very sharp reduction in the absorption at the line center.

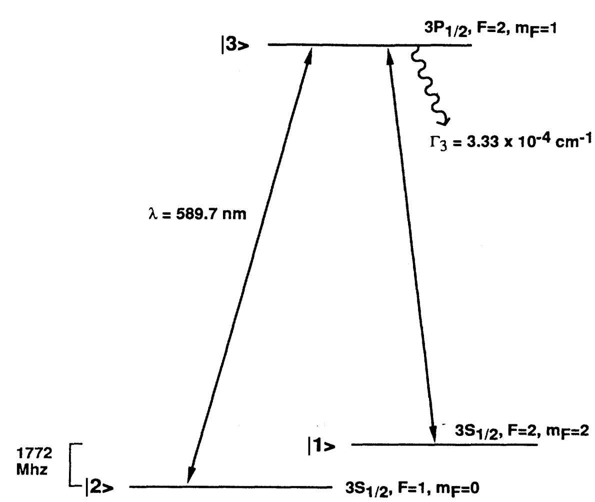

FIG. 3. Possible system in sodium to observe the vacuum-Rabi-splitting-induced transparency.

$$\frac{d\langle\hat{\sigma}_{11}\rangle}{dt}=-\frac{2|\alpha|^2}{\hbar^2}\mathrm{Re}\left\{\frac{i\Delta\omega_{21}+\dfrac{\Gamma_{\mathrm{cav}}+\Gamma_{21}}{2}}{\left[i\Delta\omega_{31}+\dfrac{\Gamma_{31}+\Gamma_{32}}{2}\right]\left[i\Delta\omega_{21}+\dfrac{\Gamma_{\mathrm{cav}}+\Gamma_{21}}{2}\right]+\dfrac{g^2}{\hbar^2}}\right\}\langle\hat{\sigma}_{11}\rangle\ . \tag{3}$$

For simplicity's sake, assume that the detunings from states $|2\rangle$ and $|3\rangle$ are zero; $\Delta\omega_{21}=\Delta\omega_{31}=0$. The total loss rate is then proportional to the sum of the cavity decay rate Γ_{cav} and the lifetime decay rate of state $|2\rangle$, Γ_{21}. If Γ_{cav} and Γ_{21} are small, the decay rates from state $|3\rangle$ do not affect the loss observed by the probe laser. The above expression is exactly the loss in a three-level atom with a coupling laser Rabi frequency of $g/\hbar$ and a laser bandwidth of Γ_{cav} [12]. Figure 2 shows the loss as a function of detuning of the probe laser from the $|1\rangle$-$|3\rangle$ resonance transition frequency. Note the sharp reduction of the loss at the center. The width of this transmission hole is *much narrower* than the Lorentzian width of the upper state. This is the principal result of this paper.

One might expect that since a smaller vacuum Rabi splitting can be observed with this interference, the cavity finesse requirements are lower. This is not the case. For an optical cavity, the mode coupling constant, and therefore the vacuum Rabi splitting, are inversely proportional to the square root of the cavity length. Γ_{cav} is inversely proportional to the cavity length. As a result, the absorption, which usually is approximately $\Gamma_{\mathrm{cav}}/g^2$, is invariant with cavity length. It is clear, however, that by increasing the cavity finesse, Γ_{cav} decreases without any change in the mode coupling constant thereby increasing the ratio of the peak absorption to the absorption at the center frequency. This makes the effect more observable.

Doppler widths much larger than the splitting do not destroy the observability of the effect. Provided that the inhomogeneous width of state $|3\rangle$ is less than the lifetime width $\Gamma_3=\Gamma_{31}+\Gamma_{32}$, Eq. (3) is approximately correct for the absorption of the atoms. Note that since the splitting is much smaller than Γ_3, this permits the Doppler widths to be much larger than the observed splitting. It is rather like two-photon Doppler-free spectroscopy. On the other hand, the inhomogeneous broadening of state $|2\rangle$ will destroy the narrow transparency. A system in which states $|1\rangle$ and $|2\rangle$ are chosen to be hyperfine structure split is a good candidate for this experiment. The close spacing of the $|1\rangle$ and $|2\rangle$ states essentially eliminates the Doppler broadening of the $|1\rangle$ to $|2\rangle$ transition.

With an ensemble of many atoms, cooperative effects between the atoms may lead to a cavity average photon number greater than zero. These effects are not considered in this single-atom formulation of the problem. More atoms will lead to a larger Rabi splitting, and this is an effect that has already been studied in the limit of a coherent field in the cavity. In order to guarantee that the mean photon number in the cavity is essentially zero and that our solutions are accurate, the decay rate from the cavity must be much greater than the emission rate into the cavity. By making $\Gamma_{\mathrm{cav}}\gg N_{\mathrm{atoms}}(d\langle\hat{\sigma}_{11}\rangle/dt)$, the cavity photon number is guaranteed to be much less than 1. This places a constraint on the maximum probe-laser intensity at a given atom density.

This system is very similar to a laser without inversion that was recently proposed by Imamoglu *et al.* [5,10]. One might then expect that this system would also lase without inversion. It turns out that lasing requires a nonzero photon number circulating in the cavity. Warming the cavity decay reservoir achieves this condition. Physically, this corresponds to applying thermal light to the back surface of one of the cavity mirrors. Lasing without inversion can be achieved in a three-level closed system. A lasing threshold condition for this system is

$$\Gamma_{\mathrm{cav}}\langle\hat{c}_k^\dagger\hat{c}_k\rangle>\Gamma_{\mathrm{cav}}+\Gamma_{21}+\Gamma_{31}n_{31}\ , \tag{4}$$

when $\Gamma_{32}\gg\Gamma_{31},\Gamma_{21},\Gamma_{\mathrm{cav}}$; $n_{32}=n_{21}=0$; $0<n_{31}\ll 1$. n_{31}, n_{32}, and n_{21} are the mean thermal occupation numbers for the reservoirs coupled to the atoms. $\langle\hat{c}_k^\dagger\hat{c}_k\rangle$ is the mean occupation number of the cavity thermal reservoir. $\Gamma_{31}n_{31}<\Gamma_{21}$ is sufficient to ensure that the atomic populations are not inverted and this is not inconsistent with the above threshold condition. This is essentially a requirement that thermal photons are entering the cavity and being scattered out by the atoms faster than laser photons are being lost. The gain is driven by the thermal light entering the cavity. There is no applied coherent field aside from the generated laser field. It is apparent that in this system free energy in the form of coherent photons is being extracted from the thermal reservoirs despite the fact that there is no atomic population inversion and no external input of energy except for thermal photons. The system is acting as a heat engine.

Figure 3 shows an energy-level diagram for a possible system in sodium. The atoms are prepared with optical pumping into state $|1\rangle$, which is the $3S_{1/2}$ $F=2$, $m_F=2$ hyperfine level. The relevant atomic decays are $\Gamma_3=\Gamma_{31}+\Gamma_{32}=6.29\times10^7$ sec^{-1}, Γ_{21} is equal to the atomic transit time through the cavity since state $|2\rangle$ is stable. In order to observe the interference effect, the cavity decay rate and the atom transit time must be smaller than Γ_3. The cavity is a single mode circularly polarized ring and approximately 8 cm long with a beam waist $1/e$ diameter of 10 μm. The total solid angle occupied by the mirrors is only 2×10^{-3} sr, and therefore the spontaneous emission rates for the atoms remain unchanged. For a cavity decay rate of 2×10^6 sec^{-1}, the finesse is 12 500. For sodium atoms at the beam waist, this cavity provides a vacuum Rabi frequency of approximately 5.6×10^6 sec^{-1}. Figure 2 shows the absorption spectrum as seen by a probe laser tuned and polarized to the $|1\rangle$-$|3\rangle$ transition.

The optical loss can be destroyed by creating an interference with a vacuum mode. The vacuum mode takes the place of the coupling laser in the normal population-trapping scheme. The loss in the presence of the cavity is determined by the sum of the cavity decay rate and the lifetime of state $|2\rangle$. Lasing without inversion can occur when thermal light is injected into the single mode cavity.

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