# Reply to "Phase measurements"

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In response to the Comment by Barnett and Pegg [preceding paper, Phys. Rev. A **47**, 4537 (1993)] it is pointed out that their objection is really based on a different measurement scheme. If our measurement procedure is repeated many times with different phase shifters inserted at one input port, the method yields an almost continuous probability density.

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The article by Barnett and Pegg [1] on phase measurements criticizes our operational approach to the problem of identifying phase operators, on the grounds that in one simple example a different choice of operators yields different answers, whereas a classical field does not present this problem.

In response, we first reemphasize what is stated repeatedly in our papers on the phase problem [2–4] that different measurement schemes lead to different phase operators, and therefore also to possibly different answers. What Barnett and Pegg [1] have done is to introduce a new measurement scheme that yields different answers, even though it is based on the same classical equations from which we started. This is possible because there is no unique procedure for constructing quantum operators from classical variables. However, that does not imply that all possible operator constructions are equally good, and one needs to use other criteria for making a choice. Our measured phase operators $\hat{C}_M,\hat{S}_M$ were adopted because they yield results that agree with classical optics in certain limits, whereas Barnett and Pegg's choice does not.

Consider the eight-port measurement scheme we have labeled scheme 2, in which four photodetectors labeled $D_3$, $D_4$, $D_5$, and $D_6$ are used to count photons, and the differences between the numbers $n_3$, $n_4$, $n_5$, and $n_6$ are used to define the cosine and sine of the measured phase difference $(\phi_2-\phi_1)$ according to

$$\begin{aligned}\hat{C}_M&=(\hat{n}_4-\hat{n}_3)/[(\hat{n}_4-\hat{n}_3)^2+(\hat{n}_6-\hat{n}_5)^2]^{1/2}\ ,\\ \hat{S}_M&=(\hat{n}_6-\hat{n}_5)/[(\hat{n}_4-\hat{n}_3)^2+(\hat{n}_6-\hat{n}_5)^2]^{1/2}\ .\end{aligned}\tag{1}$$

These two dynamical variables make use of all the information registered by the four detectors. Barnett and Pegg introduce new operators, such as [1]

$$\begin{aligned}&\hat{C}_{M_1}(\phi_2-\phi_1-\pi/4)\\ &\qquad\equiv(\hat{n}_6-\hat{n}_3)/[(\hat{n}_6-\hat{n}_3)^2+(\hat{n}_6-\hat{n}_4)^2]^{1/2}\ ,\\ &\hat{S}_{M_1}(\phi_2-\phi_1-\pi/4)\\ &\qquad\equiv(\hat{n}_6-\hat{n}_4)/[(\hat{n}_6-\hat{n}_3)^2+(\hat{n}_6-\hat{n}_4)^2]^{1/2}\ ,\end{aligned}\tag{2}$$

which satisfy $\hat{C}_{M_1}^2+\hat{S}_{M_1}^2=1$, but deliberately ignore the information registered by one detector, and they point out that these operators yield different answers for $\phi_2-\phi_1$. But Eqs. (2) represent a different measurement scheme, based on selective use of the data, and must be expected to yield different answers for a quantum field.

Barnett and Pegg [1] have focused on the special case in which one photon enters at input port 1 and the vacuum enters at input port 2, which we have treated in some detail [4] in terms of Eqs. (1). Because only one detector can register a count in this case, while the other three register zero, it becomes particularly important not to ignore any one detector. Because these four possible outcomes of one measurement correspond to four different values of the phase difference $\phi_2-\phi_1$, the resulting probability density $p(\phi_2-\phi_1)$ has the form of four equally spaced $\delta$ functions [4], whereas a continuous distribution over the range 0 to $2\pi$ is expected. However, there is nothing to prevent us from repeating the measurements many times with a phase shifter $\Delta\phi$ inserted at one input port, and with $\Delta\phi$ ranging over 0 to $2\pi$ in small steps. The values of $\phi_2-\phi_1$ can then be extracted from the measured values of $\phi_2-\phi_1-\Delta\phi$, and the resulting ensemble of $\phi_2-\phi_1$ for Fock states will be found to be distributed uniformly over the interval 0 to $2\pi$, as one would expect for a Fock state.

Barnett and Pegg state that "$\cdots$ if the experiment of Noh, Fougères, and Mandel had a consistent interpretation as a quantum phase-difference measurement, then this should apply to any choice of input states." Our answer is that it does, when the operators in Eqs. (1) to which we are led are used in conjunction with the phase shifter $\Delta\phi$. Indeed, the procedure of repeating all measurements with different phase shifts $\Delta\phi$ inserted at one interferometer input port could with advantage be adopted as a general feature of our measurement scheme 2, as we have recently demonstrated [5].

[1] S. M. Barnett and D. T. Pegg, preceding Comment, Phys. Rev. A **47**, 4537 (1993).
[2] J. W. Noh, A. Fougères, and L. Mandel, Phys. Rev. A **45**, 424 (1992).
[3] J. W. Noh, A. Fougères, and L. Mandel, Phys. Rev. Lett. **67**, 1426 (1991).
[4] J. W. Noh, A. Fougères, and L. Mandel, Phys. Rev. A **46**, 2840 (1992).
[5] J. W. Noh, A. Fougères, and L. Mandel, Phys. Scr. (to be published).

 