Reply to "Comment on 'Operational approach to the phase of a quantum field'"

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In response to Hradil's Comment [preceding Comment, Phys. Rev. A **47**, 4532 (1993)], we point out that his prescription for putting both the cosine and sine equal to zero when the data yield $n_3=n_4$ and $n_5=n_6$ can lead to meaningless results. Moreover, Hradil's Eq. (5) for obtaining D^2 from the measured variance leads to disagreement with experiment. Finally, it is possible to obtain a continuous spectrum of phases with our technique.

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In his Comment Hradil [1] points out that for noncommuting cosine and sine operators $\hat{C}(\phi),\hat{S}(\phi)$ the so-called "dispersion" D^2 given by $D^2\equiv 1-|\langle e^{i\phi}\rangle|^2$ is a more appropriate measure of the uncertainty of the phase ϕ than the variance $\langle(\Delta\hat{C})^2\rangle+\langle(\Delta\hat{S})^2\rangle=(\langle\hat{C}^2\rangle+\langle\hat{S}^2\rangle)-\langle\hat{C}\rangle^2-\langle\hat{S}\rangle^2$. As $\langle\hat{C}_M^2\rangle+\langle\hat{S}_M^2\rangle=1$ for our measured scheme-2 phase operators, there is no difference between the two cases [2–4]. We agree that for non-commuting $\hat{C},\hat{S}$ operators, such as those of Susskind and Glogower [5], D^2 is more appropriate, although the use of the term "dispersion" for D^2 may add to the confusion, because dispersion is widely used as a synonym for variance.

The reason why the curve for the dispersion of the Susskind-Glogower [5] and the Pegg-Barnett [6] phase operators lies to the right of the curve for our operators in Fig. 5 of Ref. [2], whereas it lies to the left in Fig. 1 of the Comment [1], is connected with our rescaling of the incident light intensity. In our experiment each input is split by a 50%-50% beam splitter before the two inputs come together and mix. Our theory for measurement scheme-2 automatically allows for this splitting, whereas the Susskind-Glogower and Pegg-Barnett theories do not. We therefore rescaled their light intensities by a factor 2 in order to allow a meaningful comparison with our experimental data to be made.

Because we define the measured cosine and sine operators in terms of the photon counts $\hat{n}_3,\hat{n}_4,\hat{n}_5,\hat{n}_6$ registered by four different detectors [2–4],

$$\hat{C}_M=(\hat{n}_4-\hat{n}_3)/[(\hat{n}_4-\hat{n}_3)^2+(\hat{n}_6-\hat{n}_5)^2]^{1/2}\,,$$
$$\hat{S}_M=(\hat{n}_6-\hat{n}_5)/[(\hat{n}_4-\hat{n}_3)^2+(\hat{n}_6-\hat{n}_5)^2]^{1/2}\,, \tag{1}$$

we do not use data for which $n_4-n_3=0=n_6-n_5$, when the equations yield 0/0. We have pointed out that similar difficulties already exist in the domain of classical optics [3]. Such data play a negligible role (fewer than 1%) in the results shown in Fig. 5 of Ref. [2] or in Fig. 1 of the Comment [1], because the average photon number $\langle n_2\rangle$ was large (50). Moreover, they have nothing explicitly to do with the vacuum state. It is true that for very low counts the renormalization introduces some distortion both in the classical and quantum domains and we introduced the so-called "inferred" moments of the sine and cosine [2–4] partly for the purpose of correcting for the distortion. However, Hradil's prescription of arbitrarily putting $C=0=S$ when the right-hand sides of Eq. (1) are undefined makes little sense to us. There is no phase angle whose cosine and sine are both zero, and making this substitution certainly breaks the connection between what is measured and what is usually understood by phase.

In any case the use of Hradil's Eq. (5) to extract the dispersion D^2 from our formula for $V^2=\langle(\Delta\hat{C}_M)^2\rangle+\langle(\Delta\hat{S}_M)^2\rangle$ does not bring either the predictions of our theory or the experimental results any closer to the predictions of Pegg and Barnett [6] or Susskind and Glogower [6]. This is apparent from Fig. 1, where we have plotted D^2 as a function of $\langle n_1\rangle=|v|^2$ when the input is the coherent state $|v\rangle_1|v\rangle_2$.

Finally, we wish to respond to the last and "more fundamental" objection in the Comment by Hradil [1], concerning the continuous spectrum of the phase operator or

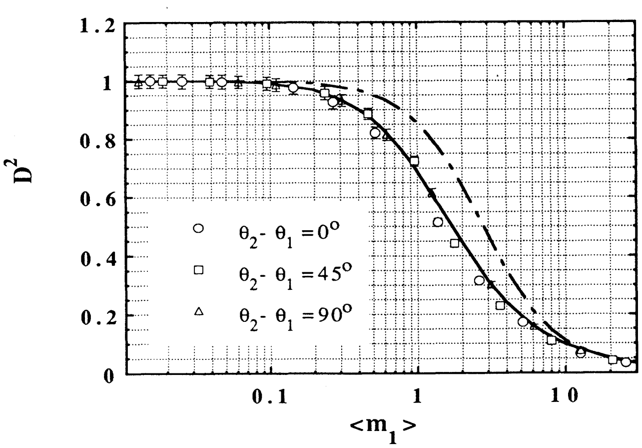

FIG. 1. Dispersion D^2 given by Hradil's relation $\langle(\Delta\hat{C}_M)^2\rangle+\langle(\Delta\hat{S}_M)^2\rangle=1-(1-D^2)/(1-P_0)^2$ as a function of mean photon counts $\langle\hat{n}_1\rangle=\langle\hat{n}_2\rangle$. The experimental points are taken from Ref. [2]. The full curve is based on our theory for $\langle(\Delta\hat{C}_M)^2\rangle+\langle(\Delta\hat{S}_M)^2\rangle$. The dashed curve corresponds to the theory of Pegg and Barnett [6] or Susskind and Glogower [5] with $D^2=1-|\langle(\exp i\phi_1)(\exp i\phi_2)^\dagger\rangle|^2$.

of our $\hat{C}_M(\phi)$ and $\hat{S}_M(\phi)$. It is true that in any measurement made with photon counters only a discrete set of outcomes is possible, corresponding to a discrete set of values of $\hat{C}_M, \hat{S}_M$ or of the phase. We have discussed the special case of an input state $|1\rangle_1|0\rangle_2$ in some detail in Ref. [4]. But it is not difficult to adapt the measurement procedure so as to yield an almost continuous spectrum of phase angles.

For this purpose we only need to place a variable phase shifter $\Delta\theta$, whose values can range between 0 and 2π, in front of the input port 1. Then the measurements yield values of $\hat{C}_M(\phi_2-\phi_1+\Delta\theta)$ and $\hat{S}_M(\phi_2-\phi_1+\Delta\theta)$, from which $\phi_2-\phi_1$ can be extracted. By repeating the measurements for many different $\Delta\theta$ in the range 0 to 2π, we can obtain values $\phi_2-\phi_1$ that cover almost the continuous range. It is only a matter of time and patience how small the adjustable increments of $\Delta\theta$ are made. The objection raised by Hradil is therefore readily met.

In conclusion, we emphasize again that we do not claim that our measured $\hat{C}_M, \hat{S}_M$ operators are the "correct" ones whereas other ones are "wrong." We claim only that $\hat{C}_M, \hat{S}_M$ come close to what is typically measured in the classical domain, where the concept of phase originates.

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