Guide to fabricating bistable-soliton-supporting media

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A theoretical guide to fabricating materials which could support bistable bright optical solitons in the sense of Kaplan's definition is presented. The approach is to build up the requisite nonlinearity by combining different media which are known to display saturable Kerr behavior. Conditions on the Kerr coefficients and saturation intensities for bistability to occur are derived.

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The fabrication of materials which could support bistable bright optical solitons in the sense of Kaplan's [1, 2] definition (i.e., solitons characterized by the same energy but distinctly different profiles) would allow the development of an alternative approach to all-optical switching using solitons [3], optical logic operations and optical computing. For such one-dimensional bistable solitons to be possible, Enns, Rangnekar, and Kaplan [2] have shown that media are required which are Kerr-like at low intensity ($\mathcal{I}$), have a sufficiently steep jump (behaving like $\mathcal{I}^n$ with $n \geq 3$) at intermediate $\mathcal{I}$, and either saturate or become Kerr-like again at larger $\mathcal{I}$. To date, the only medium in which bistable optical soliton behavior has been reported [4] is pure gaseous SF_6; however, the nature of the nonlinearity was not completely determined. Multiphoton effects were ruled out (an $\mathcal{I}^n$ contribution would result from the onset of an n-photon process) and multilevel resonant transitions were hypothesized to account for the observed saturation of the refractive index.

As we shall report in this paper, instead of engaging in a somewhat difficult search for single media with the requisite "exotic" refractive index behavior, a more viable experimental approach (which could lead to a practical fabrication technology) is to build up the desired nonlinearity by combining different media displaying saturable Kerr behavior. A saturable Kerr refractive index is one of the form $n = n_0 + n_2\mathcal{I}/(1+\mathcal{I}/\mathcal{I}_{\rm sat})$, where $\mathcal{I} \equiv |\phi|^2$ with ϕ the (complex) electric field, $\mathcal{I}_{\rm sat}$ is proportional to the saturation intensity, and the nonlinear coefficient n_2 can be positive or negative depending on the medium and the light frequency. Many gaseous (e.g., SF_6), liquid, and solid (e.g., organics and semiconductor-doped glasses [5–9]) media are known to have an intensity-dependent refractive index of this form.

As an illustrative example of the procedure, we shall build up a nonlinearity which is Kerr-like at small $\mathcal{I}$, has an $\mathcal{I}^3$ jump at larger $\mathcal{I}$, and saturates for sufficiently large $\mathcal{I}$. It is also easy to build up a refractive index where Kerr-like behavior is regained at large $\mathcal{I}$. The procedure can readily be extended to create media which can support bistable light bullets [10, 11], i.e., three-dimensional spheroidal optical bistable solitons. To attain an $\mathcal{I}^m$ jump, with m a positive integer, requires combining m saturable Kerr media. From the fabrication viewpoint this suggests that m should have the minimum value for bistability to occur. Thus, taking $m = 3$ (i.e., mixing three saturable gases or liquids, or adding three different semiconductor dopands to a glass), we consider the resultant refractive index $n = n_0 + n_2F(\mathcal{I})$ with

$$F(\mathcal{I}) = \frac{\mathcal{I}}{1+\mathcal{I}/\mathcal{I}_{\rm sat}^{(A)}} + \frac{(1-\epsilon)\mathcal{I}}{1+\mathcal{I}/\mathcal{I}_{\rm sat}^{(B)}} - 2\frac{(1-\epsilon)\mathcal{I}}{1+\mathcal{I}/\mathcal{I}_{\rm sat}^{(C)}} \quad (1)$$

and $\epsilon < 1$. The parameter ϵ controls the relative size of the Kerr coefficients, viz. $n_2^{(A)} = n_2$, $n_2^{(B)} = n_2(1-\epsilon)$, and $n_2^{(C)} = -2n_2(1-\epsilon)$. Relating the Kerr coefficients in this way anticipates one of the guiding rules on media selection that shall be derived. Selecting specific media that will satisfy these guidelines is beyond the scope of this paper and may not be a trivial task. Expanding Eq. (1) for small $\mathcal{I}$, we have

$$F(\mathcal{I}) = \epsilon\mathcal{I} - \left[\frac{1}{\mathcal{I}_{\rm sat}^{(A)}} + \frac{(1-\epsilon)}{\mathcal{I}_{\rm sat}^{(B)}} - 2\frac{(1-\epsilon)}{\mathcal{I}_{\rm sat}^{(C)}}\right]\mathcal{I}^2 + \left[\frac{1}{{\mathcal{I}_{\rm sat}^{(A)}}^2} + \frac{(1-\epsilon)}{{\mathcal{I}_{\rm sat}^{(B)}}^2} - 2\frac{(1-\epsilon)}{{\mathcal{I}_{\rm sat}^{(C)}}^2}\right]\mathcal{I}^3 + \cdots . \quad (2)$$

For sufficiently small $\mathcal{I}$, $F = \epsilon\mathcal{I}$, i.e., is Kerr-like with ϵ setting the magnitude of the slope. To guarantee $\mathcal{I}^3$ behavior at larger $\mathcal{I}$, we set the coefficient of the $\mathcal{I}^2$ term equal to zero, yielding the relation $1/\mathcal{I}_{\rm sat}^{(C)} = \frac{1}{2}[1/(1-\epsilon)\mathcal{I}_{\rm sat}^{(A)} + 1/\mathcal{I}_{\rm sat}^{(B)}]$ between the saturation intensities. For a positive $\mathcal{I}^3$ jump, we further require for given $\mathcal{I}_{\rm sat}^{(A)}$, $\mathcal{I}_{\rm sat}^{(B)}$ that $\epsilon < \epsilon_0$, ϵ_0 being a cutoff value above which the jump is negative. From (2) we find that $\epsilon_0 = (r-1)^2 + r - r\sqrt{(r-1)^2+1}$ with $r = \mathcal{I}_{\rm sat}^{(B)}/\mathcal{I}_{\rm sat}^{(A)}$. For large $\mathcal{I}$, $F(\mathcal{I})$ saturates to the value $F_{\rm sat} = \mathcal{I}_{\rm sat}^{(A)} + (1-\epsilon)\mathcal{I}_{\rm sat}^{(B)} - 2(1-\epsilon)\mathcal{I}_{\rm sat}^{(C)}$, which is positive for $\epsilon < \epsilon_0$. Provided these two conditions on $\mathcal{I}_{\rm sat}^{(C)}$ and ϵ are met, we have constructed an intensity-dependent refractive index of the desired form. This is still not sufficient, however, to guarantee a bistable energy curve with two stable solution branches in the same energy range. Further restrictions must be imposed on the range of the

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parameters $\mathcal{I}_{\rm sat}^{(A)}$ and $\mathcal{I}_{\rm sat}^{(B)}$ (which then determines the required $\mathcal{I}_{\rm sat}^{(C)}$) and on the size of ϵ. To see what these conditions are we must obtain the energy curve relevant to the nonlinearity given by Eq. (1). Consider the generalized nonlinear Schrödinger equation (GNLSE), viz.

$$2ik\left(\phi_{z_1}+\frac{\phi_{t_1}}{v_g}\right)+k|D|\phi_{t_1t_1}+2\frac{k^2n_2}{n_0}F(|\phi|^2)\phi=0, \tag{3}$$

which is derived from Maxwell's wave equation by making the slowly varying envelope approximation in the direction (spatial coordinate z_1) of propagation. Here k is the wave number, v_g the group velocity, t_1 the time, D the group velocity dispersion, and the subscripts denote differentiation. For "bright" solitons we have $n_2 > 0$ and have assumed anomalous dispersion, i.e., $D < 0$. Substituting Eq. (1) into (3) and introducing the dimensionless quantities $z \equiv \tau^{-2}|D|f_{\rm sat}\, z_1$, $t \equiv \tau^{-1}\sqrt{f_{\rm sat}}\,(t_1 - z_1/v_g)$, $E \equiv \tau\sqrt{kn_2/(|D|n_0)}\phi$, $(a,b,c) \equiv |D|n_0/\tau^2kn_2(\mathcal{I}_{\rm sat}^{(A)},\mathcal{I}_{\rm sat}^{(B)},\mathcal{I}_{\rm sat}^{(C)})$, and $f_{\rm sat} \equiv a^{-1}+(1-\epsilon)b^{-1}-4(1-\epsilon)^2[a+(1-\epsilon)b]^{-1}$, with τ a temporal scale factor which sets the pulse duration, Eq. (3) becomes

$$iE_z+\tfrac{1}{2}E_{tt}+f(|E|^2)E=0 \tag{4}$$

with

$$f(I\equiv|E|^2)=\frac{I}{f_{\rm sat}}\left[\frac{1}{1+aI}+\frac{(1-\epsilon)}{1+bI}-\frac{2(1-\epsilon)}{1+cI}\right], \tag{5}$$

where $c=\frac{1}{2}[a/(1-\epsilon)+b]$. We now seek bright solitary wave solutions to Eqs. (4) and (5) by assuming a solution of the form $E(z,t)=U(t)\exp(i\beta z)$ with U a real function satisfying the asymptotic boundary conditions U_t, $U_{tt}\to 0$ as $|t|\to\infty$ and β the real positive propagation constant. Substituting the assumed form of E into Eq. (4) yields

$$U_{tt}+2U[f(U^2)-\beta]=0. \tag{6}$$

For specified ϵ, a, and b, we numerically integrate Eq. (6) to find $U(t)$ as a function of β. With the $U(t)$ profile known, we then numerically calculate the energy $P=\int_{-\infty}^{\infty}|E|^2dt=\int_{-\infty}^{\infty}U^2dt$. Bistability occurs [1, 2] when there are two or more values of β corresponding to a given value of P. Different β values generate profiles of different heights and widths. For certain ranges of ϵ, a, and b, the $P(\beta)$ curve will be N shaped with the positive-slope legs of the N corresponding to "robust" solitary-wave solutions (i.e., solitons) and the intermediate negative-slope region to absolutely unstable solitary waves. The N-shaped $P(\beta)$ curve follows from the readily derived rule [2, 10, 3] that for $f=I^n$ in d dimensions $dP/d\beta$ is positive for $n<2/d$, zero for $n=2/d$, and negative for $n>2/d$. Here d=1. The stability of the solitary waves was examined in Ref. [2].

The values of ϵ, a, b, and c will depend on which liquid or gaseous media are combined or which semi-conductor dopands are added to a given glass. As a concrete example, for a typical semiconductor doped glass [5], $n_0\simeq 1.5$, $|D|\simeq 10^{-26}-10^{-25}{\rm s}^2/{\rm m}$, $n_2\mathcal{I}_{\rm sat}\equiv\Delta n_{\rm sat}\simeq 10^{-6}-10^{-4}$, and $\lambda\simeq 1-10\ \mu$m for anomalous dispersion. Thus, $a\simeq(1.6\times10^{-5}–0.16)/\tau^2$ with τ given in ps. For picosecond duration pulses, we set $\tau = 1$ ps so that $a\simeq 1.6\times10^{-5}–0.16$. Note that a scales as $1/\tau^2$ so larger a values are possible by decreasing τ. However, for $\tau \ll 1$ ps, higher-order contributions to the GNLSE cannot be neglected [12].

The value of ϵ must be below some small critical value $\epsilon_{\rm cr}<\epsilon_0$ to ensure that the I^3 contribution becomes *sufficiently large compared to the Kerr term* before saturation sets in. With these guidelines in mind, let us consider the representative values (i.e., within the above-determined range) $a = 0.01$, $b = 0.05$, and further take $\epsilon = 0.01$. The reason for the small value of ϵ will become apparent

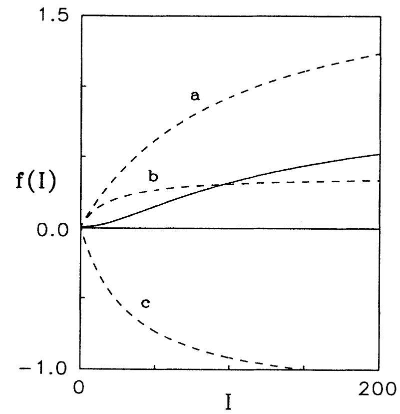

FIG. 1. Solid curve: $f(I)$ given by Eq. (5) for $\epsilon = 0.01$, $a = 0.01$, $b = 0.05$. Dashed curves: individual saturable Kerr contributions corresponding to $a = 0.01$, $b = 0.05$, and $c=\frac{1}{2}[a/(1-\epsilon)+b]\simeq 0.03$, respectively.

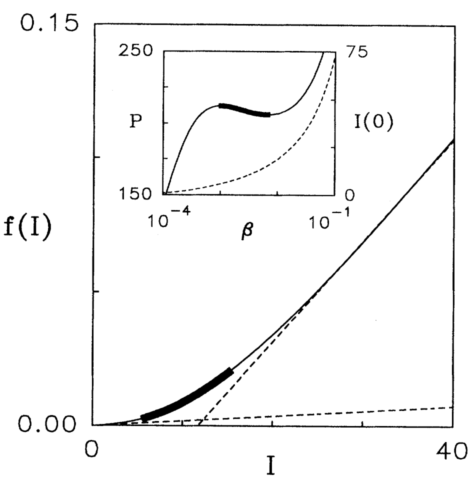

FIG. 2. Solid curve: $f(I)$. Lower dashed line: leading Kerr term, $f=\epsilon I/f_{\rm sat}$. Upper dashed line: linear asymptote to $f(I)$ curve in the range I=25–40. Thickened curve: region of $f(I)$ which corresponds to negative slope (unstable) region of energy curve in inset. Inset: solid curve: $P(\beta)$. Dashed curve: soliton intensity amplitude $I(0)$ vs propagation constant β.

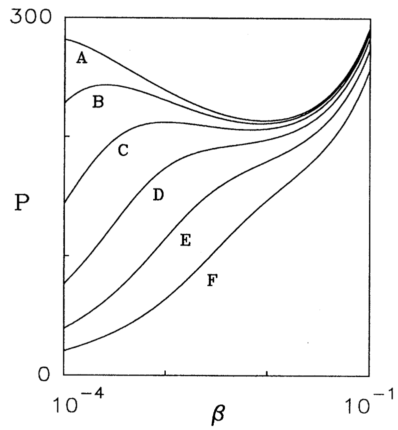

FIG. 3. Energy $P(\beta)$ for $a = 0.01$, $b = 0.05$, and (a) $\epsilon = 0.0025$, (b) $\epsilon = 0.005$, (c) $\epsilon = 0.01$, (d) $\epsilon = 0.02$, (e) $\epsilon = 0.04$, (f) $\epsilon = 0.08$.

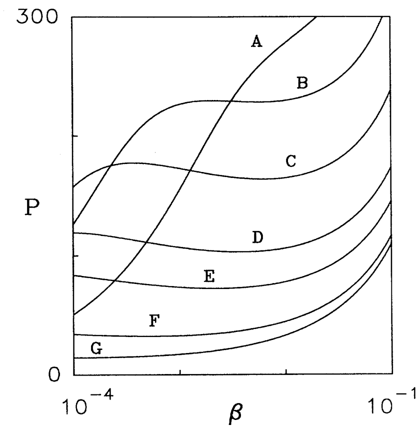

FIG. 4. Energy $P(\beta)$ for $\epsilon = 0.01$, $a = 0.01$, and (a) $b = 0.02$, (b) $b = 0.04$, (c) $b = 0.08$, (d) $b = 0.2$, (e) $b = 0.4$, (f) $b = 2$, (g) $b = 10$.

shortly. In this case, $c \simeq 0.03$ and $\epsilon_0 = 0.58$. The corresponding $f(I)$ is shown in Fig. 1 by the solid curve. Also shown are the three saturable Kerr contributions (dashed curves) which sum to give $f(I)$. For $n_0 = 1.5$, $\lambda = 1\ \mu$m, $|D| \simeq 10^{-25}$ s^2/m, $\tau = 1$ ps, and $|n_2| \simeq 10^{-16}$ m^2/V^2 (doped glass)-10^{-22} m^2/V^2 (silica glass), the intensity $I = 10$ corresponds to $\phi \simeq 4 \times 10^4 - 4 \times 10^7$ V/m, a readily attainable range of electric-field strengths. In terms of I, one need not be anywhere near saturation for bistability to occur. This is illustrated in Fig. 2 where we have focused on the behavior of $f(I)$ below $I = 40$. For $I \lesssim 4$, $f(I)$ given by the solid curve in the main figure is Kerr-like being tangent to the straight (lower dashed) line $f = \epsilon I/f_{\rm sat}$. Recalling the rule mentioned earlier, the associated energy curve should have positive slope in this region. As I increases, $f(I)$ deviates from linear behavior as the cubic contribution dominates. For the range $4 \lesssim I \lesssim 20$, the energy curve should have negative slope corresponding to unstable solitary waves. For $20 \lesssim I \lesssim 40$, $f(I)$ is approximately linear again (as seen by the upper dashed tangent line) and P should have positive slope. At still higher I, saturation begins to set in (Fig. 1) and $dP/d\beta$ will remain positive. These qualitative predictions are borne out as seen in the inset where the numerically obtained N-shaped energy curve (solid curve) is plotted as well as the maximum intensity $I(t = 0)$ (dashed curve) of the solitary wave. The unstable intermediate region, indicated by the thickened line in the main figure and inset, spans the range $I \simeq 5$–16. For fixed a and b, the importance of keeping ϵ below a critical value $\epsilon_{\rm cr}$ can be seen in Fig. 3. With a and b having the same values as in Figs. 1 and 2, ϵ has been varied from 0.0025 to 0.08. We see that $\epsilon_{\rm cr} \simeq 0.01$–0.02, which is well below ϵ_0. For $\epsilon > \epsilon_{\rm cr}$, the intermediate negative-slope region vanishes. One no longer has bistability since only one positive-slope branch is present. The leading small intensity Kerr contributions of the three saturable species must nearly cancel (but stay positive) for bistability to occur. In other words, the sum of the Kerr coefficients $(n_2^{(A)} + n_2^{(B)} + n_2^{(C)})/n_2 = \epsilon < \epsilon_{\rm cr} \ll 1$. Since $\epsilon_{\rm cr} \ll 1$, one has $c = \frac{1}{2}[a/(1-\epsilon) + b] \simeq \frac{1}{2}(a+b)$ so that the reciprocal of the saturation intensity of the medium with the negative Kerr coefficient must be approximately the average of the reciprocal saturation intensities of the two media with positive Kerr coefficients.

Taking $\epsilon = 0.01$ and $a = 0.01$, we have varied b in Fig. 4. Increasing b corresponds to lowering $\mathcal{I}_{\rm sat}^{(B)}$ and $\mathcal{I}_{\rm sat}^{(C)}$ (see Fig. 1). For sufficiently small b (e.g., $b = 0.02$ in Fig. 4), $f(I)$ rises too gently and bistability is lost. In other words, bistability is not possible if the saturation intensities are too high. On the other hand, if the saturation intensities are too low (e.g., for $b = 10$ in Fig. 4), $f(I)$ rises rapidly due to the early onset of saturation and the cubic contribution is wiped out, as is bistability. The permissible range of b (similar remarks applying to a) for bistability to occur spans two orders of magnitude, a desirable feature from the fabrication viewpoint.

In conclusion, we trust that the guidelines developed in this paper will prove beneficial to experimentalists attempting to fabricate materials which can support bistable solitons. As pointed out in the introduction and the cited references, this could lead to some important device applications.

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