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Nonlocal theory of accelerated observers

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A nonlocal theory of accelerated observers is developed on the basis of the hypothesis that an electromagnetic wave can never stand completely still with respect to an observer. In the eikonal approximation, the nonlocal theory reduces to the standard extension of Lorentz invariance to accelerated observers. The validity of the nonlocal theory would exclude the possibility of existence of any basic scalar field in nature. The observational consequences of this theory are briefly discussed.

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The standard relativistic theory of gravitation [1] is based on the notion that an accelerated observer is instantaneously equivalent to a hypothetical momentarily comoving inertial observer. This hypothesis of locality [2] is clearly satisfied for phenomena involving classical particles and rays of radiation. It has been suggested that a theory of pointlike coincidences encounters difficulties when extended to the quantum domain [3,4]. However, even the measurement of classical wave characteristics cannot be considered pointlike except in the eikonal limit. For instance, the determination of the frequency of an electromagnetic wave involves the measurement of the field over at least several periods. It appears, therefore, that the standard theory of accelerated observers may not be completely adequate to deal with classical wave phenomena. In this paper, I sketch a nonlocal theory which reduces to the standard theory in the eikonal limit.

Consider a classical measuring device $\mathcal{D}$; the quantum theory of measurement has been founded on the assumption that such a device is always inertial, i.e., that it is at rest in an inertial frame in Minkowski spacetime. Imagine now that the device $\mathcal{D}$ is accelerated; this is a realistic situation since all actual observers are accelerated. A theory of accelerated observers is thus necessary in order to interpret the physical measurements recorded by a measuring device. To establish contact with the basic laws of physics, the measurements of an accelerated observer may be expressed in terms of the measurements of a class of hypothetical inertial observers each momentarily comoving with the accelerated observer. This important assumption appears to be very reasonable in view of the fact that in most practical circumstances the deviations from the basic laws of physics due to inertial effects are small. Furthermore, the required relationship may be assumed to be *linear* in order to preserve the superposition principle for accelerated observers. This is essential since all available experimental data—which are necessarily obtained by accelerated observers—are consistent with the superposition principle.

If the interaction between the device $\mathcal{D}$ and the elementary phenomena under consideration is instantaneous as in the classical mechanics of point particles, the influence of inertial forces affecting $\mathcal{D}$ can be neglected at the instant when the measurement takes place and hence $\mathcal{D}$ is instantaneously equivalent to the hypothetical comoving inertial device. This is the basis for the description of inertial effects in Newtonian mechanics; it also forms the foundation for the standard extension of Lorentz invariance to accelerated systems and gravitational fields [1]. On the other hand, the influence of inertial effects may not be negligible for the measurement of classical wave phenomena which require an extended period of time; in this case the measurements of the accelerated observer may thus be related to the measurements of a *class* of comoving inertial observers. The instantaneous equivalence of an accelerated observer with a hypothetical momentarily comoving inertial observer is thus valid for a theory of *coincidences,* i.e., the pointwise interactions of classical particles and electromagnetic rays which correspond to waves of zero wavelength; beyond such idealizations, the presumed equivalence amounts to a *hypothesis of locality* in the standard theory. The theoretical significance and possible limitations of this hypothesis have been dealt with at length in a number of publications [4]. It suffices to state here that deviations from the hypothesis of locality are in general expected when the intrinsic reduced wavelength ($\lambda\!\!\!^{-}$) of the phenomenon under observation is no longer small compared to the acceleration length ($\mathcal{L}$) of the observer. Here $\mathcal{L}$ is a characteristic scale such that $\mathcal{L}=c^2/g$ for linear motion of constant acceleration g, while $\mathcal{L}=c/\Omega$ for rotation of constant frequency Ω. The notion of acceleration length is more precisely defined (and developed) in Ref. [2].

Let us illustrate these ideas by means of an example. Consider an electromagnetic radiation field specified by the components of the Faraday tensor $F_{\mu\nu}(x)$, which incorporates the electric and magnetic fields as measured by static inertial observers in Minkowski spacetime. The accelerated observer is characterized by its worldline $x^{\mu}(\tau)$, where τ is the proper time along the path. The observer refers its measurements to an orthonormal tetrad frame $\lambda^{\mu}_{(\alpha)}$ along its path such that the four-velocity $\lambda^{\mu}_{(0)}=dx^{\mu}/d\tau$ is the time axis and $\lambda^{\mu}_{(i)}$, $i=1,2,3$, are the spatial axes of the observer's local reference frame. At each instant τ, let the electromagnetic field measured by the accelerated observer be given by $\mathcal{F}_{\alpha\beta}(\tau)$. The standard extension of the theory of relativity (Lorentz invariance) to observers in accelerated motion and gravitational fields is based on the hypothesis of locality, namely, the assumption that the accelerated observer is at each instant locally equivalent to a comoving inertial observer which momentarily has the same tetrad as the accelerated observer. Thus according to the standard theory $\mathcal{F}_{\alpha\beta}(\tau)$ should be equal to $F'_{\alpha\beta}(\tau)$,

$$F'_{\alpha\beta}(\tau)=F_{\mu\nu}\lambda^{\mu}_{(\alpha)}\lambda^{\nu}_{(\beta)} , \quad (1)$$

which is the field measured by the instantaneously comoving inertial observer. The frequency content of the radiation field as measured by the accelerated observer can be determined from the Fourier analysis of the time series $\mathcal{F}_{\alpha\beta}(\tau)$; this result should reduce to the standard Doppler formula in the eikonal limit ($\lambda/\mathcal{L}\to 0$). It is interesting to consider the implications of Eq. (1) for the case of a plane monochromatic radiation of frequency ω incident on an observer rotating about the axis of propagation of the wave with frequency Ω. The frequency of the wave as measured by the rotating observer is $\omega'=\gamma(\omega\mp\Omega)$, where the upper (lower) sign indicates radiation of positive (negative) helicity. If $\lambda\Omega/c\to 0$, $\omega'=\gamma\omega$ and the transverse Doppler formula is recovered. On the other hand, for positive helicity radiation of $\omega=\Omega$ the wave stands completely still with respect to the observer; however, in this case $\lambda\Omega/c=1$ and there is no basic reason why the hypothesis of locality—i.e., $\mathcal{F}_{\alpha\beta}=F'_{\alpha\beta}$—should hold. Moreover, this surprising prediction of the standard theory has no observational basis at present; in fact, $\lambda/\mathcal{L}$ is so small in practice that any wave effect beyond the eikonal limit has been below the level of sensitivity of all experiments performed thus far [5]. It is interesting to consider a natural generalization of the standard theory that would go beyond the hypothesis of locality in accordance with the notion of complementarity.

Consider an observer moving with constant velocity in Minkowski spacetime. In the absence of acceleration, the electromagnetic radiation field measured by the observer is given by Eq. (1). Now imagine that the acceleration is turned on at a certain instant and the observer is forced to move on an accelerated path; after a while, the acceleration is turned off and subsequently the observer moves with constant velocity again. What is the most general relationship between $\mathcal{F}_{\alpha\beta}$ and $F_{\mu\nu}$ that would be consistent with all available data? The hypothesis of locality—which is essentially the principle of minimal coupling—asserts that $\mathcal{F}_{\alpha\beta}=F'_{\alpha\beta}$; however, this simple relation could be generalized in a number of ways. Such a generalization must be consistent with linearity as well as the fundamental principle of *causality*. These considerations lead to the ansatz

$$\mathcal{F}_{\alpha\beta}(\tau)=F'_{\alpha\beta}(\tau)+\int_{\tau_0}^{\tau}K_{\alpha\beta}{}^{\gamma\delta}(\tau,\tau')F'_{\gamma\delta}(\tau')d\tau' , \quad (2)$$

where τ_0 is the instant at which accelerated motion begins and the kernel $K_{\alpha\beta\gamma\delta}$ is antisymmetric in its first and second pairs of indices and is expected to depend on the acceleration of the observer. This ansatz is consistent with Lorentz invariance and the superposition principle. Furthermore, it follows from the theory of integral equations originally developed by Volterra [6] that in the space of continuous functions the relationship between $\mathcal{F}_{\alpha\beta}$ and $F_{\mu\nu}$ is unique [7]. If the kernel vanishes in the absence of any acceleration, then Eq. (2) would imply the standard result of the Lorentz-invariant theory while in the JWKB approximation the nonlocal part of Eq. (2) would be expected to be proportional to $\lambda/\mathcal{L}$ so that in the eikonal limit ($\lambda/\mathcal{L}\to 0$) the hypothesis of locality would be recovered in the case of geometric optics. Thus the relationship between $\mathcal{F}_{\alpha\beta}$ and $F'_{\alpha\beta}$ given by Eq. (2) would be a natural *nonlocal* generalization of the standard theory [8].

It now remains to determine the kernel and discover its precise relationship with the acceleration of the observer. A consistent theory can be developed on the basis of the fundamental hypothesis that *no observer can stay at rest with a wave.* This assumption is a natural generalization of an important consequence of Lorentz invariance, namely, that an electromagnetic wave cannot lose its temporal variation with respect to any inertial observer; only when the $\omega=0$ limit is actually considered would the "wave" appear to stand still with respect to *all* inertial observers. This is an elementary consequence of the relativistic Doppler effect. In this way, the notion of a photon acquires observer-independent status for inertial observers. This work aims to extend these ideas to all observers [9]. If the accelerated observer measures a constant electromagnetic field, then the electromagnetic field according to inertial observers must be constant [10]. That is, a constant $F_{\mu\nu}$ should result in a constant $\mathcal{F}_{\alpha\beta}$ regardless of the motion of the observer. The implementation of this requirement essentially specifies the kernel once the further assumption is introduced that the kernel is of the *Faltung* type, i.e., it depends on τ and τ' only through $\tau-\tau'$. To see this in detail, it is advantageous to represent the electromagnetic field as a six-vector and the kernel as a 6×6 matrix. That is, let $F_{\mu\nu}\to F_A$, where $(F_A)=(-\mathbf{E},\mathbf{B})$ and the index A ranges over the set (01, 02, 03, 23, 31, 12). The metric of Minkowski spacetime is assumed to have a signature of $+2$. In this notation Eq. (1) can be written as $F'=\Lambda F$, where $\Lambda(\tau)$ is a 6×6 matrix that can be expressed at each instant τ in terms of an instantaneous Lorentz transformation connecting the global frame of static observers in Minkowski spacetime to the local frame of the accelerated observer. Thus Eq. (2) can be written in the form

$$\mathcal{F}=\Lambda F+\int_{\tau_0}^{\tau}K(\tau-\tau')\Lambda(\tau')F(\tau')d\tau' ; \quad (3)$$

moreover, for the sake of simplicity we set $\tau_0=0$ in the following. Let F be a constant field, then the requirement that $\mathcal{F}$ be a constant as well implies that $\mathcal{F}=\mathcal{F}(0)=\Lambda_0 F$. Hence the kernel K must satisfy the equation

$$\Lambda(\tau)+\int_{0}^{\tau}K(\tau-\tau')\Lambda(\tau')d\tau'=\Lambda_0 . \quad (4)$$

It follows from the theory of Volterra integral equations that a *resolvent kernel R exists* such that

$$\Lambda_0+\int_{0}^{\tau}R(\tau-\tau')\Lambda_0 d\tau'=\Lambda(\tau) . \quad (5)$$

The resolvent kernel can thus be obtained by differentiating Eq. (5) with respect to τ, i.e.,

$$R(\tau)=\frac{d\Lambda(\tau)}{d\tau}\Lambda_0^{-1}\ . \tag{6}$$

This is a basic result of the theory presented here: If the observer is inertial, $\Lambda(\tau)$ is a constant and hence the resolvent kernel as well as the kernel vanishes and the standard result of the Lorentz-invariant theory is recovered. On the other hand, the resolvent kernel is directly proportional to the acceleration of the observer and thus the determination of the kernel can proceed via the standard methods of the theory of Volterra equations. In particular, it is possible to use Laplace transformations since Eq. (3) is of convolution type. If $\bar{K}(s)$ is the Laplace transform of $K(t)$, then Eqs. (4) and (5) imply that $(I+\bar{K})\bar{\Lambda}=\bar{\Lambda}_0$, and $(I+\bar{R})\bar{\Lambda}_0=\bar{\Lambda}$, where I is the unit (6×6) matrix. It follows that $(I+\bar{K})(I+\bar{R})=I$. Once $\bar{K}(s)$ is determined, $K(t)$ can be computed via the inverse Laplace transform. These results are in accordance with the intuitive considerations that motivated this analysis; moreover, once the acceleration of the observer is turned off, the nonlocal part of Eq. (3) merely adds to the result of the standard theory a *constant* part as the residue of past acceleration of the observer. This constant term only affects the boundary conditions since the addition of a constant field to a solution of the electromagnetic field equations in inertial spacetime simply produces a new solution. In any measuring device the influence of past accelerations would be canceled when the device is reset; hence any conflict that might arise with the interpretation of data as a consequence of past movements of the device is avoided.

It is interesting to consider anew the problem of appearance of a circularly polarized electromagnetic wave with respect to a uniformly rotating observer in the (x^1,x^2) plane. The observer rotates about the x^3 axis on a circle of radius r with frequency Ω and uniform speed $c\beta=r\Omega$. With respect to the Cartesian coordinates of static inertial observers, the natural coordinate axes of the rotating observer are

$$\lambda^{\mu}_{(0)}=\gamma(1,-\beta\sin\phi,\beta\cos\phi,0)\ , \tag{7}$$

$$\lambda^{\mu}_{(1)}=(0,\cos\phi,\sin\phi,0)\ , \tag{8}$$

$$\lambda^{\mu}_{(2)}=\gamma(\beta,-\sin\phi,\cos\phi,0)\ , \tag{9}$$

$$\lambda^{\mu}_{(3)}=(0,0,0,1)\ , \tag{10}$$

where $\phi=\Omega t=\gamma\Omega\tau$. It can be shown that the kernel can be written as

$$K=\gamma^2\Omega\begin{bmatrix} I_3 & -\beta I_1 \\ \beta I_1 & I_3 \end{bmatrix}\ , \tag{11}$$

where I_i is proportional to the operator for infinitesimal rotation about the x^i axis, $(I_i)_{jk}=-\epsilon_{ijk}$. Imagine a plane wave of frequency ω propagating along the x^3 axis; the electric and magnetic fields can be obtained, respectively, from the real parts of $\mathbf{E}=\mathcal{A}\mathbf{e}_{\pm}\exp(-i\omega t+i\omega x^3/c)$ and $\mathbf{B}=\mathcal{A}\mathbf{b}_{\pm}\exp(-i\omega t+i\omega x^3/c)$, where $\mathcal{A}$ is a complex amplitude,

$$\mathbf{e}_{\pm}=2^{-1/2}(1,\pm i,0)\ ,\quad \mathbf{b}_{\pm}=\mp i\mathbf{e}_{\pm}\ , \tag{12}$$

and the upper (lower) sign refers to right (left) circularly polarized radiation. The hypothesis of locality implies that $\mathbf{E}'=\mathcal{A}\mathbf{e}'_{\pm}\exp(-i\omega'\tau)$ and $\mathbf{B}'=\mathcal{A}\mathbf{b}'_{\pm}\exp(-i\omega'\tau)$, where $\omega'=\gamma(\omega\mp\Omega)$ and

$$\mathbf{e}'_{\pm}=2^{-1/2}(\gamma,\pm i,\pm i\beta\gamma)\ ,\quad \mathbf{b}'_{\pm}=\mp i\mathbf{e}'_{\pm}\ , \tag{13}$$

with respect to the coordinate axes of the rotating observer. Let the experiment in the rotating frame begin at $t=\tau=0$. It follows from the nonlocal theory that the field as measured by the rotating observer is given by the real part of

$$\mathcal{F}=\frac{\gamma\omega}{\omega'}\left[1\mp\frac{\Omega}{\omega}e^{i\omega'\tau}\right]F'\ , \tag{14}$$

so that the measured frequency of the wave is again ω'; however, when $\omega=\Omega$ for a right circularly polarized wave the measured field is not constant but varies linearly with proper time as might be expected for resonance. Moreover, for $\omega'\neq 0$ the amplitude of the wave as measured by the rotating observer is different from the prediction of the standard theory by a factor of $\gamma\omega/\omega'=(1\mp\lambda\!\!\!^{-}\Omega/c)^{-1}$ due to nonlocal effects; the amplitude is larger (smaller) for positive (negative) helicity radiation. It would be interesting to search for such effects experimentally; the characteristic amplitude of the deviation from the standard theory is $\lambda\!\!\!^{-}\Omega/c\simeq 10^{-7}$ for radio waves of frequency 10 GHz incident upon an observer rotating at 10^3 rounds per second.

It is important to note that the nature of nonlocality proposed in this paper is fundamentally different from the nonlocality that appears in the phenomenological theories of media with memory of past motions and electromagnetic fields. This is clear, in particular, from the constancy of the kernel in the simple case considered above while memory is expected to fade in the case of nonlocal media.

Finally, it should be pointed out that the ideas developed here for the electromagnetic field are quite general and are therefore applicable to any basic wave function [11]. Thus Eq. (3) may be thought of as expressing the *generic* relationship between a field according to static observers in the inertial frame (F) and the field as measured by an accelerated observer ($\mathcal{F}$). If the spin of the field is nonzero, the transformation $F\rightarrow\mathcal{F}$ is nonlocal and the kernel is well defined; otherwise, the transformation is local. That is, $K=0$ for a scalar field since $\Lambda=1$ and it follows from Eq. (6) that $R=0$. For a scalar field, $\omega'=\gamma(\omega-m\Omega)$ where $m=0,\ \pm1,\pm2,\ldots$ is the component of the angular momentum parameter along the axis of rotation of the observer (cf. Ref. [5]); hence it is in principle possible to have $\omega'=0$ for a scalar field of frequency ω such that $\omega/\Omega=m$. This contradicts the basic hypothesis upon which the nonlocal theory has been constructed: A basic radiation field cannot lose its temporal variation with respect to any observer. The absence of a fundamental scalar field is thus an important consequence of the nonlocal theory presented here. This prediction appears to be consistent with the observational data available at present.

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[5] More generally, $\omega' = \gamma(\omega - M\Omega)$, where M is the wave's total angular momentum parameter along the axis of rotation of the observer. In the eikonal approximation this relation reduces to the Doppler effect together with the spin-rotation coupling. Thus $\omega' = 0$ can occur for any field whenever $\omega/\Omega = M$. In connection with the possibility of observing the spin-rotation coupling, see B. Mashhoon, Phys. Rev. Lett. **61**, 2639 (1988); **68**, 3812 (1992); Phys. Lett. A **139**, 103 (1989); F. W. Hehl and W. T. Ni, Phys. Rev. D **42**, 2045 (1990); J. C. Huang, Cambridge and University of Missouri-Columbia Report (unpublished); M. P. Silverman, Phys. Lett. A **152**, 133 (1991); Y. Q. Cai and G. Papini, Phys. Rev. Lett. **66**, 1259 (1991); **68**, 3811 (1992); J. Audretsch and C. Lämmerzahl, Appl. Phys. B **54**, 351 (1992); J. Audretsch, F. W. Hehl, and C. Lämmerzahl, in *Relativistic Gravity Research,* edited by J. Ehlers and G. Schäfer, Lecture Notes in Physics Vol. 410 (Springer, Berlin, 1992), p. 368; B. Mashhoon, in *Quantum Gravity and Beyond, Essays in Honor of Louis Witten on His Retirement,* edited by F. Mansouri and J. Scanio (World Scientific, Singapore, 1993).

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[7] This uniqueness property can be essentially extended to the space of square-integrable functions as well (cf. F. G. Tricomi, Ref. [6]).

[8] In the standard theory the field determination is based on the immateriality of acceleration at any given instant of time. This is reasonable to the extent that the influence of inertial effects can be neglected over the time scale necessary for a proper determination of the field. According to the ansatz (2), the deviation from the standard theory for a radiation field is in the form of a weighted "average" of the field consistent with the requirement of causality.

[9] The hypothesis that an electromagnetic wave can never completely stand still with respect to an observer may be extended to a principle of complementarity of absolute and relative motion. In this connection, see B. Mashhoon, F. W. Hehl, and D. S. Theiss, Gen. Rel. Grav. **16**, 711 (1984); B. Mashhoon, Phys. Lett. A **126**, 393 (1988); University of Missouri-Columbia Report, 1991 (unpublished); University of Missouri-Columbia Report, 1992 (unpublished).

[10] It is very important to note here that the constant field under discussion is the $\omega = 0$ limit of a radiation field. It is clear that for an electrostatic or a magnetostatic field Eq. (1) holds, i.e., $\mathcal{F}_{\alpha\beta} = F'_{\alpha\beta}$.

[11] A detailed treatment will be published elsewhere.