Violation of Bell's inequality by macroscopic states generated via parametric down-conversion

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We show that correlated photon-number states predicted to violate a Bell's inequality of the Clauser-Horne-Shimony-Holt type might be generated from parametric amplification. The test is potentially one of macroscopic quantum mechanics in that violations are predicted for situations where a large number of photons are detected in a single detector at one time. The traditional amplifier and detectors allow violation only for regimes where the probability of generation of a macroscopic state is very small. We suggest that this probability may be dramatically improved with the use of squeezing and photon-number-state-preparation techniques.

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I. INTRODUCTION

The correctness of quantum mechanics has been verified in numerous experimental situations. A particularly strong test of quantum mechanics was suggested by Bell [1] who showed that the predictions of all classical theories, based as they are on the "common-sense" assumptions of local realism, contradict those of quantum mechanics. Experimental tests that have been performed support quantum mechanics [2]. However, such tests have been so far restricted to microscopic systems in the sense that a measurement is made on one particle at a time. There has been a developing interest in tests of quantum mechanics for macroscopic systems. These include proposals by Leggett and co-workers [3] to test whether one can generate a superposition of two macroscopically distinct states. There have been recent suggestions to prepare similar states in optical systems [4]. So far, however, there has been no experimental confirmation.

Our interest here is in tests of quantum mechanics against local realistic theories in macroscopic or mesoscopic systems where there is a significant number of particles incident on each measurement apparatus. In these tests, quantum mechanics predicts a violation of a Bell's inequality which is derivable from the assumptions of local realism. Such a test for an optical system was originally proposed by Drummond [5] who considered states generated by cooperative atomic fluorescence. Mermin [6] first presented, we believe, a related Bell's test for higher-spin states and, more recently, Braunstein and Caves [7] have proposed new tests. However, the obvious experimental situations corresponding directly to these proposals are currently difficult to realize. Oliver and Stroud [8] suggested using correlated Rydberg atoms, which has the advantage of excellent detection efficiencies, but the atomic state is not so readily produced. Here we suggest producing macroscopic quantum states which violate a Bell's inequality in optical parametric down-conversion or similar four-wave-mixing processes, thus obtaining a test of quantum mechanics against classical theories for situations of large particle numbers. We point out that the multiparticle tests described in this paper are different from those proposed recently by Greenberger *et al.* [9], and Mermin [9], and Yurke and Stoler [9] in that we have large particle numbers at each analyzer or measurement apparatus. Thus impinging on each analyzer is a wave packet with N quanta. In the Greenberger-Horne-Zeilinger proposals, the particles are each spatially separated so that there is only one particle incident on each analyzer, although we have recently considered [9] how one might achieve the Greenberger-Horne-Zeilinger phenomenon with more than one particle per analyzer.

Correlated signal-idler photon pairs have been generated via parametric down-conversion [10] and shown to give violation of Bell's inequalities in the experiments of Ou and Mandel [11], Shih and Alley [12], and Rarity and Tapster [13]. Here we show that correlated multiphoton and even macroscopic states which violate a Bell's inequality are also generated in parametric down-conversion and that experimental arrangements similar to the above can be used in principle to contradict the classical predictions. Our analysis here relates primarily to the scheme used by Ou and Mandel [11]. We show that with the traditional arrangements the probability of generation and detection of these quantum states decreases with increasing number of particles detected, making true macroscopic experiments difficult. The achievement, however, of a violation of a Bell's inequality for situations of N quanta per wave packet where $N=2$ may well be possible with current techniques and would represent a test of quantum mechanics in a new regime. We discuss how the use of quantum-noise-reduction techniques such as squeezed light and photon-number-state preparation may improve the feasibility of generating nonlocal quantum states with still higher particle numbers.

II. VIOLATIONS OF BELL'S INEQUALITIES USING CORRELATED PHOTON-NUMBER STATES

We being by examining the properties of a correlated photon-number state

$$|\varphi\rangle = |n\rangle|n\rangle \ . \tag{2.1}$$

We will show that this state exhibits distinctive quantum features even for large n by way of a violation of Bell's inequality, and then discuss how a similar state may be generated using parametric down-conversion. The orthogonal modes are referred to as signal and idler modes with boson operators a_1 and a_2 respectively. These fields have a differing frequency or polarization or k direction so that they may be spatially separated. The parametric interaction has been used to generate correlated photon-number states which correspond to (2.1) with $n=1$ and to demonstrate violations of Bell's inequalities in this microscopic regime [11–17]. A schematic interpretation of the scheme used by Ou and Mandel is depicted in Fig. 1. The final detected modes are c_+, c_-, d_+, d_-, where the polarizers and/or beam splitters produce the following transformations of the signal and idler output modes:

$$\begin{aligned} c_+ &= a_+\cos\theta + a_-\sin\theta, \quad d_+ = b_+\cos\phi + b_-\sin\phi \ , \\ c_- &= -a_+\sin\theta + a_-\cos\theta, \quad d_- = -b_+\sin\phi + b_-\cos\phi \ , \\ a_+ &= (a_1 + ic_1)/\sqrt{2}, \quad a_- = (a_2 + ic_2)/\sqrt{2} \ , \\ b_+ &= (ia_1 + c_1)/\sqrt{2}, \quad b_- = (ia_2 + c_2)/\sqrt{2} \ . \end{aligned} \tag{2.2}$$

Here the c_1 and c_2 are modes for the input vacuum states at the 50-50 beam splitters. These transformations used in conjunction with the parametric amplifier were predicted originally by Reid and Walls [14] to give violations of a Bell's inequality. In the apparatus of Ou and Mandel, the nonoverlapping modes a_1 and a_2 are incident at different input ports of the same beam splitter, and the emerging $a_\pm$ and $b_\pm$ are transformed using two spatially separated polarizers.

The original experiments measure the joint photon-count probability for detecting a photon at the spatially separated locations A and B corresponding to c_+ and d_+, respectively. Here we follow the idea introduced by Drummond [5] and consider the joint probability $P_N(\theta,\phi)$ of detecting N photons at A and N photons at B. The probability may be written in terms of the correlation functions by way of the photon-count formula

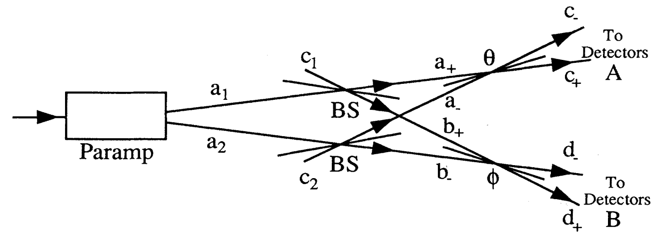

FIG. 1. Schematic diagram of the transformations involved in the experimental arrangement to test Bell's inequalities (BS denotes beam splitter).

developed by Mandel [18] and Kelley and Kleiner [19]. We examine here the simplest and experimentally relevant situation where the detection time T and the detection efficiency are sufficiently small that only the lowest-order terms in the formulas contribute. The probability then becomes

$$P_N(\theta,\phi) = \eta\langle :c_+^{\dagger N} c_+^N d_+^{\dagger N} d_+^N: \rangle \ , \tag{2.3}$$

where η is an efficiency factor assumed small and the $::$ denote normal ordering. We also define the joint probability of detecting N photons at A and N photons at B if the polarizer preceding B is removed. Quantum mechanics predicts

$$P_N(\theta,-) = \eta\langle :c_+^{\dagger N} c_+^N (d_+^\dagger d_+ + d_-^\dagger d_-)^N: \rangle \ . \tag{2.4}$$

Similarly, if the polarizer preceding A is removed, we have

$$P_N(-,\phi) = \eta\langle :(c_+^\dagger c_+ + c_-^\dagger c_-)^N d_+^{\dagger N} d_+^N: \rangle \ . \tag{2.5}$$

The joint probabilities can be measured experimentally and results compared to the predictions of local realistic theories. The classical assumptions of local realism along the lines first discussed by Bell lead to the following Clauser-Horne-Shimony-Holt (CHSH) [1] inequalities:

$$B_N = \frac{P_N(\theta,\phi) - P_N(\theta,\phi') + P_N(\theta',\phi) + P_N(\theta',\phi')}{P_N(\theta',-) + P_N(-,\phi)} \leq 1 \ . \tag{2.6}$$

All classical theories are constrained by this inequality. We investigate in this paper the quantum-mechanical predictions for B_N, beginning with the idealized correlated photon-number state (2.1).

The moments are readily calculated using the transformations (2.2):

$$\begin{aligned} P_N(\theta,\phi) \sim \langle c_+^{\dagger N} d_+^{\dagger N} c_+^N d_+^N \rangle = \tfrac{1}{4}^N \sum_{r=0}^{N} \sum_{r'=0}^{N} \sum_{r''=0}^{N} \sum_{r'''=0}^{N} \Bigg\{ & \begin{bmatrix} N \\ r \end{bmatrix} \begin{bmatrix} N \\ r' \end{bmatrix} \begin{bmatrix} N \\ r'' \end{bmatrix} \begin{bmatrix} N \\ r''' \end{bmatrix} (\cos\theta)^{2N-r-r''} (\sin\theta)^{r+r''} \\ & \times (\sin\phi)^{r'+r'''} (\cos\phi)^{2N-r'-r'''} \langle a_2^{\dagger 2N-r-r'} a_1^{\dagger r+r'} a_2^{2N-r''-r'''} a_1^{r''+r'''} \rangle \Bigg\} \ , \end{aligned} \tag{2.7a}$$

$$P_N(\theta,-)\sim\langle :c_+^{\dagger N}c_+^N(d_+^\dagger d_+ + d_-^\dagger d_-)^N:\rangle = \tfrac{1}{4}^N \sum_{r=0}^{N}\sum_{r'=0}^{N}\sum_{r''=0}^{N}\begin{bmatrix}N\\r\end{bmatrix}\begin{bmatrix}N\\r'\end{bmatrix}\begin{bmatrix}N\\r''\end{bmatrix}(\cos\theta)^{2N-r-r''}(\sin\theta)^{r+r''}$$
$$\times\langle a_2^{\dagger N-r+r'}a_1^{\dagger N+r-r'}a_2^{N-r''+r'}a_1^{N+r''-r'}\rangle\ , \tag{2.7b}$$

$$P_N(-,\phi)=P_N(\phi,-)\ .$$

The correlated photon-number state (2.1) gives for the correlated moments in the original modes

$$\langle a_1^{\dagger i}a_1^{j}a_2^{\dagger m}a_2^{k}\rangle=\begin{cases}\dfrac{n!}{(n-i)!}\,\dfrac{n!}{(n-k)!}, & \text{if } i=j,\ m=k,\ i\le n, \text{ and } k\le n\\ 0, & \text{otherwise}\ .\end{cases} \tag{2.8}$$

This puts a constraint on r''' ($r'''=r+r'-r''$) in (2.7a) and r'' ($r''=r$) in (2.7b).

Results for B_N for optimal choices of angles θ, ϕ, θ', and ϕ' are plotted in Fig. 2 for various n and N (see Table I). A contradiction with the classical result is possible even for large particle numbers, provided the number of particles N detected at A and B is equal to n, the number of particles incident initially on each beam splitter. As N reduces below n, the violation of the classical inequality is lost. Although violation with $N=n$ is possible with a particular choice for angle for large n, the range of angles allowing a violation reduces as n increases (Fig. 2). A similar effect was pointed out by Mermin [6] in his study of higher spin states violating Bell's inequality.

These predictions are very similar to those of Drummond [5] who considered a correlated four-mode state

$$|\varphi_D\rangle=\frac{1}{n!\sqrt{n+1}}(a_+^\dagger b_+^\dagger + a_-^\dagger b_-^\dagger)^n|0\rangle|0\rangle|0\rangle|0\rangle\ . \tag{2.9}$$

Here one calculates joint probabilities for detecting N photons associated with mode c_+ at A and N photons associated with mode d_+ at B, where

$$c_+=a_+\cos\theta+a_-\sin\theta,\quad d_+=b_+\cos\phi+b_-\sin\phi\ ,$$
$$c_-=-a_+\sin\theta+a_-\cos\theta,\quad d_-=-b_+\sin\phi+b_-\cos\phi\ . \tag{2.10}$$

The + and − modes typically designate orthogonal po-

TABLE I. The optimal angles $\theta,\theta',\phi,\phi'$ needed to obtain the maximum violation of the Bell inequality.

		Nth-order inequality			
$n=N$	θ	θ'	ϕ	ϕ'	B_N (max)
1	0.393	1.178	0.785	0.000	2.414
2	0.365	3.050	−1.662	1.955	2.617
3	0.263	−0.062	−1.615	−1.260	2.717
4	0.235	−0.033	−1.604	1.800	2.780
5	0.194	−0.023	−1.548	−1.375	2.820
6	0.170	−0.017	−1.590	−1.400	2.846
7	0.140	−0.014	−1.580	−1.420	2.863
8	0.140	−0.009	−1.580	−1.450	2.880
9	0.140	−0.007	−1.580	−1.480	2.890
10	0.130	−0.005	−1.580	−1.482	2.898
30	0.017	−0.002	−1.571	−1.487	2.917
100	0.007	0.000	−1.571	−1.550	2.922

		$(N-1)$th-order inequality			
$n=N-1$	θ	θ'	ϕ	ϕ'	B_{N-1} (max)
2	0.13	2.39	2.05	2.53	1.745
3	−0.04	0.69	1.15	0.74	1.681
4	−0.56	−0.84	−0.83	−0.53	1.694
5	−0.60	−0.83	−0.82	−0.57	1.717
6	−0.65	−0.84	−0.80	−0.56	1.722
7	−0.92	−0.74	−0.75	−0.97	1.731
8	−0.91	−0.74	−0.76	−0.96	1.739
9	−0.92	−0.75	−0.76	−0.92	1.745
10	−0.93	−0.77	−0.75	−0.90	1.749
30	0.71	0.80	0.81	0.72	1.781
100	0.82	0.78	0.78	0.82	1.793

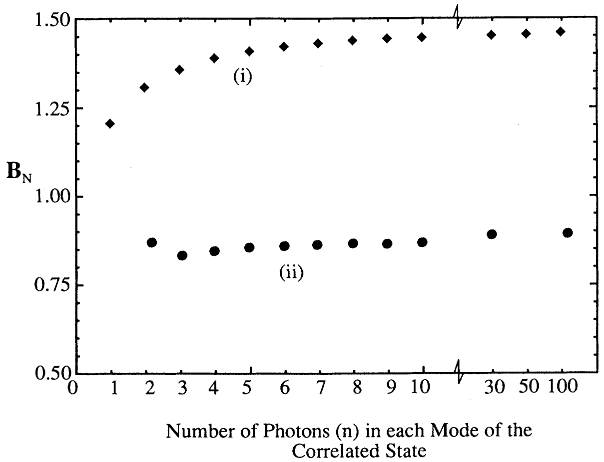

FIG. 2. Plot of the maximum value of B_N vs n, the number of photons in the correlated state $|n\rangle_1|n\rangle_2$ for (i) $N=n$, (ii) $N=n-1$. A violation of Bell's inequality is obtained where $B_N>1$.

larizations although they may also refer to different directions. The best violation of the Bell's inequality is possible if one detects all n photons at A and all n photons at B. For N sufficiently small compared to n, the violation is lost. We present the results of Drummond in Fig. 3.

For the system we discuss here as depicted in Fig. 1, one may use the unitary transformations equivalent to the beam splitter and polarizer transformations to write down the correlated state actually detected at A and B, with the signal and idler input state being (2.1). For $n=1$, this state is a linear combination of the states

$$|1\rangle_{d_-}|0\rangle_{c_-}|1\rangle_{d_+}|0\rangle_{c_+},\quad |0\rangle_{d_-}|1\rangle_{c_-}|0\rangle_{d_+}|1\rangle_{c_+}\ ,$$
$$|2\rangle_{d_-}|0\rangle_{c_-}|0\rangle_{d_+}|0\rangle_{c_+},\quad |0\rangle_{d_-}|0\rangle_{c_-}|0\rangle_{d_+}|2\rangle_{c_+}\ ,$$
$$|0\rangle_{d_-}|0\rangle_{c_-}|2\rangle_{d_+}|0\rangle_{c_+},\quad |0\rangle_{d_-}|2\rangle_{c_-}|0\rangle_{d_+}|0\rangle_{c_+}\ ,$$
$$|1\rangle_{d_-}|0\rangle_{c_-}|0\rangle_{d_+}|1\rangle_{c_+},\quad |0\rangle_{d_-}|1\rangle_{c_-}|1\rangle_{d_+}|0\rangle_{c_+}\ ,$$
$$|0\rangle_{d_-}|0\rangle c_-|1\rangle_{d_+}|1\rangle_{c_+},\quad |1\rangle_{d_-}|1\rangle_{c_-}|0\rangle_{d_+}|0\rangle_{c_+}\ ,$$

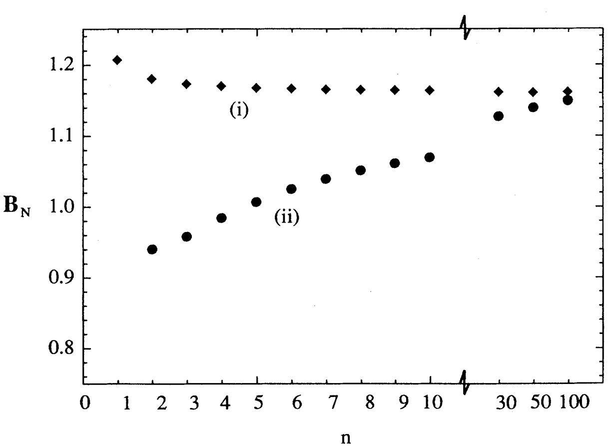

FIG. 3. Plot of the maximum value of B_N for the four-mode state $(a_+^\dagger b_+^\dagger + a_-^\dagger b_-^\dagger)^n|0\rangle|0\rangle|0\rangle$: (i) $N=n$, (ii) $N=n-1$. A violation of Bell's inequality occurs for $B_N>1$.

where the appropriate coefficients are dependent on θ and ϕ. The moments required for B_N are readily calculated for the $n=1$ case directly from the transformations (2.2). For example,

$$\langle :c_+^\dagger c_+ d_+^\dagger d_+ :\rangle = \tfrac{1}{4}\{\langle a_1^\dagger a_1 a_2^\dagger a_2\rangle \sin^2(\theta+\phi) + \langle a_1^{\dagger 2} a_1^2\rangle \sin^2\theta \sin^2\phi + \langle a_2^{\dagger 2} a_2^2\rangle \cos^2\theta\cos^2\phi\}\ , \tag{2.11}$$

$$\langle :c_+^\dagger c_+ (d_+^\dagger d_+ + d_-^\dagger d_-):\rangle = \tfrac{1}{4}\{\langle a_1^\dagger a_1 a_2^\dagger a_2\rangle + \langle a_1^{\dagger 2} a_1^2\rangle \sin^2\theta + \langle a_2^{\dagger 2} a_2^2\rangle \cos^2\theta\}\ . \tag{2.12}$$

Here we have assumed that $\langle a_1^\dagger a_1 a_2^\dagger a_2\rangle$ and $\langle a_1^{\dagger 2} a_1^2\rangle$ are the only nonzero moments of the original beams. For the $n=1$ case here, where $\langle a_i^{\dagger 2} a_i^2\rangle = 0$, we get for our final relative probabilities results, such as

$$P(\theta,\phi)/P(\theta,-) = \sin^2(\theta+\phi)\ . \tag{2.13}$$

This solution gives a violation of the classical inequality (2.6), for example, for $\theta=\pi/8$, $\theta'=3\pi/8$, $\phi=\pi/4$, and $\phi'=0$. The angular dependence appearing is a result of nonzero interference terms of the type $\langle a_+^\dagger b_-^\dagger a_- b_+\rangle$. This term is nonzero, because the state after the beam splitters may be written, according to quantum mechanics, as the product of two superposition states:

$$\tfrac{1}{2}(|0\rangle_{a_+}|1\rangle_{b_+} + |1\rangle_{a_+}|0\rangle_{b_+})(|0\rangle_{b_-}|1\rangle_{a_-} + |1\rangle_{b_-}|0\rangle_{a_-})\ . \tag{2.14}$$

The interpretation of each superposition state is along the lines of Dirac's interpretation of the two-slit experiment—that each photon goes through both slits (in this case; through both arms) of the beam splitter. The resulting interference here is reflected in the violation of the Bell inequality (2.6), which is predicted from classical theories where the photon must go through one slit or the other. One could demonstrate the interference by introducing a phase shift along the arm a_+, as indicated in Fig. 1. Such a shift for fixed choices of θ and ϕ can destroy the Bell-inequality violations.

The state generated for $n=1$ from the input state (2.1) followed by the transformations (2.2) is in fact a correlated superposition state similar to the correlated two-photon polarization states used in experiments of Aspect *et al.* [2] [and equivalent to (2.9) with $n=1$],

$$(a_+^\dagger b_+^\dagger + a_-^\dagger b_-^\dagger)|0\rangle|0\rangle|0\rangle|0\rangle\ . \tag{2.15}$$

If we choose $\theta=\phi=0$, then the probability of joint detection of a photon in d_- and c_+ is proportional to the joint correlation function $\langle a_1^\dagger a_1 a_2^\dagger a_2\rangle$ of the original beams, while the probability for joint detection of photons in d_- and c_- is proportional to the second-order correlation function $\langle a_1^{\dagger 2} a_1^2\rangle$ of the single beam. For the photon pair $|1\rangle|1\rangle$ state, and similar correlated states where

$$\langle a_1^\dagger a_1 a_2^\dagger a_2\rangle \gg \langle a_1^{\dagger 2} a_1^2\rangle\ ,$$

we have therefore that the detection of a photon in d_- is much more strongly correlated with the detection of a photon in c_+ than in c_-. Careful examination of our quantum state tells us that the conditional probability of detecting a photon at c_+ given that one is detected at d_- has only a maximum value of 0.5. This is because the paired photons may go through the second arm of the beam splitter to produce a photon at d_+. The implication of this is that the stronger inequality [1] derived originally by Bell cannot automatically be violated from an experiment of this type (such as the Ou and Mandel [11] experiment), even with perfect detection efficiencies. However, the modified CHSH inequalities which we use in (2.6) involve measurement of a relative probability. The joint probabilities $P(\theta,\phi)$ are normalized with respect to the joint probabilities of the type $P(\theta,-)$ which involve detection of two photons but with one polarizer removed (as compared to normalization with respect to the true marginal probability where only one photon is detected). This relative probability relates to the probability of detecting a photon at c_+, given that a photon is detected at d_- and that a photon will be detected at either c_+ or c_-. The measured correlation is then equivalent to the correlation of the state (2.15) of Aspect *et al.*, and (2.9) with $n=1$:

$$|0\rangle_{a_+}|1\rangle_{b_+}|1\rangle_{a_-}|0\rangle_{b_-}+|1\rangle_{a_+}|0\rangle_{b_+}|0\rangle_{a_-}|1\rangle_{b_-} . \tag{2.16}$$

This explains why the violation of the Bell-CHSH inequality is identical for $n=1$ for these cases (compare Figs. 2 and 3).

The solutions for $n=1$ suggest to us why the violations of classical inequalities diminish or vanish for $N=1$ if we use an input field $|n\rangle|n\rangle$ where $n>1$. The self-correlation $\langle a_1^{\dagger 2}a_1^2\rangle$ increases relative to the cross correlation $\langle a_1^\dagger a_1 a_2^\dagger a_2\rangle$, and there is a significant joint probability for detecting photons at c_+ and d_+ as well as at c_+ and d_-.

For larger initial photon numbers n, the quantum state after the first set of beam splitters is a superposition of a "macroscopic" number of states. Some of these states are macroscopically distinct but most are only microscopically distinct. We may write this state as

$$a_1^{\dagger n}a_2^{\dagger n}|0\rangle=\frac{1}{2^n}(a_+^\dagger-ib_+^\dagger)^n(a_-^\dagger-ib_-^\dagger)^n|0\rangle , \tag{2.17}$$

where $|0\rangle$ denotes the product of the vacuum state for all relevant modes. The number of photons detected at each output can range from 0 through to $2n$ in steps of one photon. The optimal situation ($N=n$) examines the subset of results where n photons are detected at c_+ and c_- combined, and n photons are detected at d_+ and d_- combined. The terms in the state (2.17) contributing to this measured subensemble are

$$\sum_{x=0}^{n}\begin{bmatrix} n \\ x\end{bmatrix}^2 a_+^{\dagger x}a_-^{\dagger n-x}b_+^{\dagger n-x}b_-^{\dagger x}|0\rangle . \tag{2.18}$$

We note now that this state for $n>1$ is similar yet different to the state (2.9) considered by Drummond [5]. Here again a strong correlation exists between the generation of n photons at a_+ (or a_-) and the generation of n photons at b_- (or b_+). The state (2.9) considered by Drummond can be expressed as

$$\sum_{x=0}^{n}\begin{bmatrix} n \\ x\end{bmatrix} a_+^{\dagger x}a_-^{\dagger n-x}b_+^{\dagger x}b_-^{\dagger n-x}|0\rangle , \tag{2.19}$$

and we see that the same terms contribute (at least for $N=n$) upon exchanging b_- and b_+, although with different weightings. The cross terms such as $a_+^\dagger a_-^\dagger b_+^\dagger b_-^\dagger$ are more significant compared to terms such as $a_+^{\dagger 2}b_-^{\dagger 2}$ for the situation (2.18) we discuss primarily in this paper. This explains the quantitative differences for $n>1$ between the results presented in Figs. 2 and 3. In the case (2.9) of Drummond, the probabilities (2.3)–(2.5) depend only on the difference $\varphi=\theta-\phi$ so that the violation may be parametrized with respect to a single angle φ. At large n, the violation (always obtainable) is obtained at small values of φ. The optimal angular choice for the state (2.18) generated via the apparatus of Fig. 1 is tabulated in Table I.

To conclude this section, our calculations show that the interference produced from the quantum superposition states (2.18) is sufficient to contradict all classical interpretations of this experiment even at large n where large numbers of photons are detected. This macroscopic set of superposition states may be generated with the correlated number state $|n\rangle|n\rangle$ incident as the modes a_1 and a_2 on the apparatus depicted in Fig. 1.

III. GENERATION OF THE MACROSCOPIC QUANTUM STATES VIA PARAMETRIC DOWN-CONVERSION

The correlated number state (2.1) itself is an idealized state and we must consider how it might be prepared in an experimental situation. We begin here by considering parametric down-conversion. The simplest model interaction Hamiltonian for this process is written

$$H_I=-\hbar\chi(a_3^\dagger a_1 a_2+a_3 a_1^\dagger a_2^\dagger) . \tag{3.1}$$

Here χ is a coupling coefficient proportional to the $\chi^{(2)}$ nonlinear susceptibility of the medium, and the a_i are boson operators for modes at frequencies ω_i, respectively, where $\omega_3=\omega_1+\omega_2$ for frequency matching. For down-conversion, mode a_3 is the pump and a_1,a_2 the signal and idler fields, respectively. This Hamiltonian applies most rigorously if the interaction occurs inside an optical cavity resonant at all three frequencies. It is an approximate description only of traveling-wave amplification [20].

The traditional parametric down-conversion employs a laser field for the pump. This is analogous to a classical pump of amplitude ε which is of sufficiently large intensity that it may be assumed to be undepleting. The prototype Hamiltonian is

$$H_I=-\hbar\chi\varepsilon(a_2 a_1+a_2^\dagger a_1^\dagger) . \tag{3.2}$$

This simplistic Hamiltonian has been used extensively to model the parametric process. Here the signal and idler are initially in vacuum states. The equations of motion derivable from (3.2) are readily solved to give

$$a_2 = a_2(0)\cosh r + i a_1^\dagger(0)\sinh r ,$$
$$a_1 = a_1(0)\cosh r + i a_2^\dagger(0)\sinh r , \qquad (3.3)$$

where $r=\chi\varepsilon t$. We find it useful to solve for the evolution of the state itself in the interaction picture. The solution is

$$\exp\{i\chi\varepsilon t(a_1a_2 + a_1^\dagger a_2^\dagger)\}|0\rangle_1|0\rangle_2 = \sum_{n=0}^{\infty} c_n|n\rangle|n\rangle , \qquad (3.4)$$

where

$$c_n = (-i\tanh r)^n/\cosh r .$$

We see that the state produced from the classical parametric amplifier is a linear combination of the infinite correlated number states. Our solutions are a simple model for the output signal and idler state after an interaction time t with the parametric medium. The solutions (3.3) and (3.4) both enable ready calculation of the correlation functions of the output field. The joint probabilities $P_N(\theta,\phi)$, $P_N(\theta,-)$ are given by (2.7), but here we need to calculate the $\langle a_1^{\dagger i} a_1^j a_2^{\dagger m} a_2^k\rangle$ for the parametric amplifier,

$$\langle a_1^{\dagger i} a_1^j a_2^{\dagger m} a_2^k\rangle = \begin{cases} \displaystyle\sum_{n=\max(j,k)}^{\infty} c_{n-k+m}^* c_n \frac{n!(n-k+m)!}{(n-k)!(n-j)!} \\ 0 \ \text{ if } m+j \neq i+k . \end{cases} \qquad (3.5)$$

We note here that phase-sensitive moments can be nonzero, e.g., $\langle a_1 a_2\rangle$. However, close examination of our measured correlations (2.7) reveals that requirement of $m-k=i-j$ in fact implies that we need consider only $m=k$ and $i=j$. Our apparatus is sensitive only to the intensity correlations of the type $\langle a_1^{\dagger i} a_1^i a_2^{\dagger j} a_2^j\rangle$ of the original state. [This allows the simplification of $r'''=r+r'-r''$ in (2.7a) and $r''=r$ in (2.7b).]

Before discussion of the results for the CHSH inequality (2.6), a point is made about the calculation in relation to the detector efficiencies. With an ideal perfectly efficient photon detector, one can in principle detect all of the photons emerging from the apparatus and hence infer the total number $2n$ signal and idler photons at the inputs. One could then restrict attention to the subensemble relevant to a particular value of n, and the prediction of the violation of the CHSH inequality (2.6) would be as presented in Fig. 2. The realistic situation, however, is of poor photon detection efficiencies and the experiment cannot infer the incident signal-idler photon number n from the total number of photons N detected. All of the input states $|n\rangle|n\rangle$ with $n>N$ will contribute. The probability of detection of N photons is given by the expression (2.3) which we calculate.

The various B_N and also the mean output photon number $\langle a^\dagger a\rangle$ are plotted in Fig. 4(a) for various N. Violations of the Bell's inequality are possible at low r corresponding to low mean photon number. The first-order inequality $N=1$ corresponds to the calculations of Tan and Walls [21] and Reid and Walls [14] and the violation has been achieved experimentally in the Ou and Mandel experiment [11]. We see here that violations are also possible for larger N. Thus we have a test of quantum mechanics against the predictions of classical physics for a situation where more than one particle is detected at a time. Unfortunately, the violation is only possible for very low average photon numbers $\langle a^\dagger a\rangle$. Thus the prob-

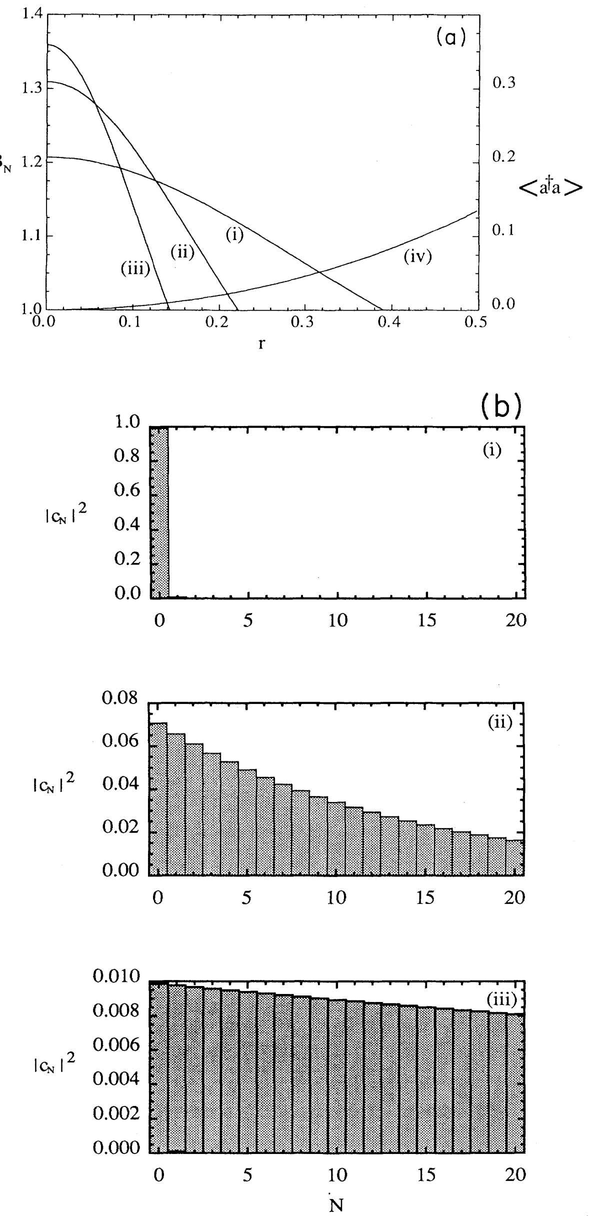

FIG. 4. The traditional amplifier with a classical pump: (a) B_N vs r for (i) $N=1$, (ii) $N=2$, (iii) $N=3$. Curve (iv) is the mean photon number $\langle a^\dagger a\rangle$. A violation of Bell's inequality is obtained for $B_N>1$. (b) Plot of $P=|c_N|^2$, the probability that the signal-idler outputs have exactly N photons, for various r: (i) $r=0.1$, (ii) $r=2$, (iii) $r=3$.

ability of actually detecting the N photons where $N > 1$ becomes small, and the experiment becomes more difficult. The maximum value of r allowing a violation for $N = 2$ corresponds to an experimental situation where the probability of detecting two photons simultaneously is 0.05 times the probability of one-photon detection. This is potentially compounded by the problem of poor detection efficiencies, which may further reduce the actual probability of detecting N, where $N > 1$, photons.

The expansion (3.4), as sketched in Fig. 4(b), gives some insight as to why violations of higher-order inequalities are lost. If we examine the infinite expansion for the parametric amplifier with the classical (coherent) pump at low pump intensities, we see that the first two terms dominate, giving the Bell-inequality violation at $N = 1$. Increasing ε brings into importance the higher n terms, yet the higher-order inequalities are not violated at higher ε because of the significant contribution, for fixed N, of the higher-order states $|n\rangle|n\rangle$ where $n > N$. We recall from Sec. II that violation of Bell's inequalities was lost for the $|n\rangle|n\rangle$ state if the number N of photons detected was smaller than n. For small ε, where violations of higher-order ($N > 1$) inequalities are possible, the reduction in magnitude of each successive c_n is significant.

Thus we have established that the use of the traditional amplifier directly may allow a feasible test of the higher multiparticle inequalities only for smaller N values. The above interpretations, however, suggest how one might modify the experiment to allow a better probability of generation of the state $|n\rangle|n\rangle$ where $n > 1$. First, as mentioned above, the use of ideal photon counters (along with double-channel polarizers or beam splitters) to count the total number of photons emerging at the outputs would enable determination of the signal-idler photon input number n. One could increase the probability of obtaining a larger number n of signal-idler photons by increasing χ, select a fixed $N = n$, and restrict attention to the reduced ensemble where n photons are detected at each output A and B. With the traditional amplifier, the fluctuations in the photon number of the output signal and idler fields are super-Poissonian. Thus one has inefficient preparation of the particular $N = n$ correlated number state. This efficiency may be improved by employing a squeezed intensity pump for the down-conversion process. A cavity configuration may be more favorable to increase the effective value of χ. Indeed, employment of a squeezed pump for nondegenerate parametric oscillation has been predicted recently [22] to generate high-intensity signal and idler beams which show reduced intensity fluctuations [23,24] in each beam in addition to an intensity correlation between the beams. It may also be possible to generate the $|n\rangle|n\rangle$ state using two down-conversions [9]. Preparation of a photon-number state has been achieved by Hong and Mandel [25] using the down-conversion process. The detection of n photons in an idler field k_1 prepares the corresponding signal field k_1' in a state of photon number n, since the photon numbers of the signal and idler are correlated. Using a second down-conversion, with signal and idler fields k_2' and k_2, and waiting until detection of n photons in the idler k_2, one may prepare an $|n\rangle|n\rangle$ state in both signal fields. Our calculation here, however, assumes simultaneous incidence of the two n-quanta wave packets on the apparatus, so care must be taken to ensure this. While one can use recently developed quantum-noise-reduction and photon-number-state-preparation techniques to improve the feasibility of generating the macroscopic quantum state, the current inefficiencies involved in photon counting will be a limiting factor. For $N = 2$, for example, one must wait until the joint detection of the two photons at each location A and B is achieved. In practice, one may be required to detect the two photons using a beam splitter and two single-photon detectors. This reduces the detection efficiency. For larger photon number n, one can measure the intensity at each output more efficiently. The predicted sensitivity (Figs. 2 and 3) of the results, however, to the detection of precisely n photons (with $n - 1$, the violation is diminished) indicates that determination of the intensity is required to the accuracy of a few photons in order to obtain a violation of this particular Bell's inequality.

The effect of dissipation as the fields propagate through the medium has been ignored in the calculations so far, and will clearly diminish the value of the probability of detecting n photons. We will discuss these effects in Sec. IV with the view that the parametric down-conversion experiment could be performed for smaller n values.

So far in this section we have discussed a two-mode output parametric amplifier. It is possible to consider interactions describable by the following Hamiltonian:

$$H = -\hbar\{a_3^\dagger(a_1a_2 + b_1b_2) + a_3(a_1^\dagger a_2^\dagger + b_1^\dagger b_2^\dagger)\} \tag{3.6}$$

as a means of generating the four-mode states (2.9) considered originally by Drummond [5]. Violation of Bell's inequalities using entangled states generated from such a four-mode Hamiltonian has been studied previously by Reid and Walls [14] and Horne Shimony, and Zeilinger [15]. Here a_3 is the pump mode. Calculations with a classical pump give similar results to the two-mode situation shown in Fig. 4. The two-mode case has been presented in this paper because it relates directly to a traditional amplifier down-converting into a particular frequency and k-vector signal-idler combination. The multimode signal-idler output interaction of the type (3.6) however, is a realistic description of the parametric process where there is a range of frequencies and k vectors available. It may be preferable to use a cavity configuration to enhance the down-conversion into the chosen set of modes. The four-mode interaction (3.6) is thus achievable; has the advantage of being able to test, at least in principle, the original stronger form of Bell's inequality directly; and appears less sensitive to detection loss since Fig. 3 shows that an $n - 1$ violation is possible for $n = 60$.

IV. THE EFFECT OF LOSS

The states and Hamiltonians discussed in Sec. III do not account for losses which for the most part are unavoidable in real physical systems. Loss is well known to rapidly destroy macroscopic quantum coherence [26] and its a major reason macroscopic superposition states or "Schrödinger-cat" states are difficult to prepare experimentally. Thus we must examine how loss may diminish or destroy the contradiction with the macroscopic classical inequalities.

In fact, this question is already important to Bell-inequality tests at the microscopic level, because the very poor photodetection efficiencies imply a very large effective loss of photons and a real reduction in correlation [1]. The implication of this is that Bell's original inequality has never been violated in any real system. This problem has been bypassed by the introduction of the modified inequalities of Clauser and Horne [1]. Although these inequalities are weaker in their test of quantum mechanics in that they employ auxiliary assumptions, they have been violated in experimental situations [2]. These weaker inequalities replace the true marginal probabilities (such as the probability of detecting N photons at A) with joint probabilities of the type $P_N(\theta,-)$ (the probability of detecting N photons at A and N photons at B with the polarizer at B removed). This results in a renormalization which enables violation of the classical inequalities to be possible in spite of the very poor photodetection efficiency. In fact, these comments are applicable to many other forms of loss. The use of the modified inequalities means that one can still expect violations even with significant loss. The poor photodetection efficiency is a significant form of loss in the experimental arrangements we consider here. The difficulty with the tests at large N is therefore not so much obtaining a violation, which one can do with the Clauser-Horne-Shimony-Holt (2.6) inequalities, but overcoming the fact that the probability of actually detecting the N photons, where N is macroscopic, becomes negligible. One needs sufficient data in order to calculate the appropriate averages.

We wish to illustrate these points with respect to our parametric systems. The coupling of our ideal modes to other modes, which is the mechanism of loss, brings about an uncorrelated loss of individual signal and idler photons, and hence the correlation which is implicit in solutions of the type (3.4) is reduced. We will now have included in our expansions asymmetrical terms such as $|n\rangle|n-1\rangle$. This means that if N photons are detected at A, one can no longer conclude that N photons will be detected at B. The implication is a reduction in the violation of Bell's original strong inequality. However, the modified Clauser-Horne inequalities may still be violated under certain circumstances because it is the relative size of the joint probability $\langle :c_+^{\dagger N}c_+^N d_+^{\dagger N}d_+^N:\rangle$ compared to

$$\langle :c_+^{\dagger N}c_+^N(d_+^\dagger d_+ + d_-^\dagger d_-)^N:\rangle$$

which is important.

The two-mode example (depicted by Fig. 1) is interesting in that it is reliant on the form of the CHSH inequalities to get a contradiction with quantum mechanics at all, i.e., one cannot obtain a violation of Bell's original inequalities with this arrangement. Put simply, the reason is that a photon detected at A does not imply a photon will be at B even with 100% detection efficiencies.

We present calculations from a model which incorporates the effect of loss as the fields propagate through the medium. Consider the traditional parametric amplifier with a classical pump. We rewrite the Hamiltonian as follows:

$$\begin{aligned} H &= H_I + \sum_{i=1}^{2} a_i\Gamma_i^\dagger + a_i^\dagger\Gamma_i \,, \\ H_I &= i\hbar E(a_2^\dagger a_1^\dagger - a_2 a_1) \,. \end{aligned} \tag{4.1}$$

Here we have chosen for convenience the pump phase so that E is real and we note that $|E|=|\chi\varepsilon|$. The Γ_i symbolize reservoir modes into which the modes a_2, a_1 may lose energy. The correlation functions are probably most readily calculated by solving operator equations of motion with noise included. However, because we later find it useful to actually calculate the probability for detecting n photons, we choose to use c-number techniques where one can obtain the solution to the density operator ρ itself. The master equation in the interaction picture in the Markovian approximation is

$$\begin{aligned} \frac{\partial\rho}{\partial t} &= \frac{1}{i\hbar}[H_I,\rho] + \frac{\gamma}{2}(2a_1\rho a_1^\dagger - a_1^\dagger a_1\rho - \rho a_1^\dagger a_1) \\ &\quad + \frac{\gamma}{2}(2a_2\rho_2^\dagger - a_2^\dagger a_2\rho - \rho a_2^\dagger a_2) \,, \end{aligned} \tag{4.2}$$

where γ is the damping constant, assumed equal for both modes. We choose to expand ρ in terms of a positive P representation [27],

$$\rho = \int P(\alpha_i,\alpha_i^\dagger,t)\frac{|\alpha_1,\alpha_2\rangle\langle\alpha_1^{\dagger *},\alpha_2^{\dagger *}|}{\langle\alpha_1^{\dagger *},\alpha_2^{\dagger *}|\alpha_1,\alpha_2\rangle} d^2\alpha_1 d^2\alpha_2 d^2\alpha_1^\dagger \alpha_2^\dagger \,. \tag{4.3}$$

Here $\alpha_i,\alpha_i^\dagger$ are independent complex c numbers corresponding to operators $a_i,a_i^\dagger$, respectively. The integrations are defined over the entire complex plane of each $\alpha_i,\alpha_i^\dagger$. The equation of motion for P can be derived readily from the master equation and the time-dependent solution evaluated (see the Appendix) for the vacuum initial condition for the signal and idler fields. It is seen that α_i and $\alpha_i^\dagger$ are constrained so that $\alpha_2=\alpha_1^*$ and $\alpha_2^\dagger=\alpha_1^{\dagger *}$ and the integrals are thus readily evaluated. The probability of detecting n photons in a_1 and m photons in a_2 is given by

$$\begin{aligned} P_{nm} &= {}_2\langle m|_1\langle n|\rho|n\rangle_1|m\rangle_2 \\ &= \int\int P(\alpha_1,\alpha_1^\dagger)\frac{\langle n|\alpha_1\rangle\langle\alpha_1^{\dagger *}|n\rangle\langle m|\alpha_2\rangle\langle\alpha_2^{\dagger *}|m\rangle}{|\langle\alpha_1^{\dagger *}|\alpha_1\rangle|^2} \\ &\quad \times d^2\alpha_1 d^2\alpha_1^\dagger \,, \end{aligned} \tag{4.4}$$

where the integrations are over the complex plane of α_1 and $\alpha_1^\dagger$, these being independent complex variables, and $P(\alpha_1,\alpha_1^\dagger)$ is given in the Appendix. The moments needed

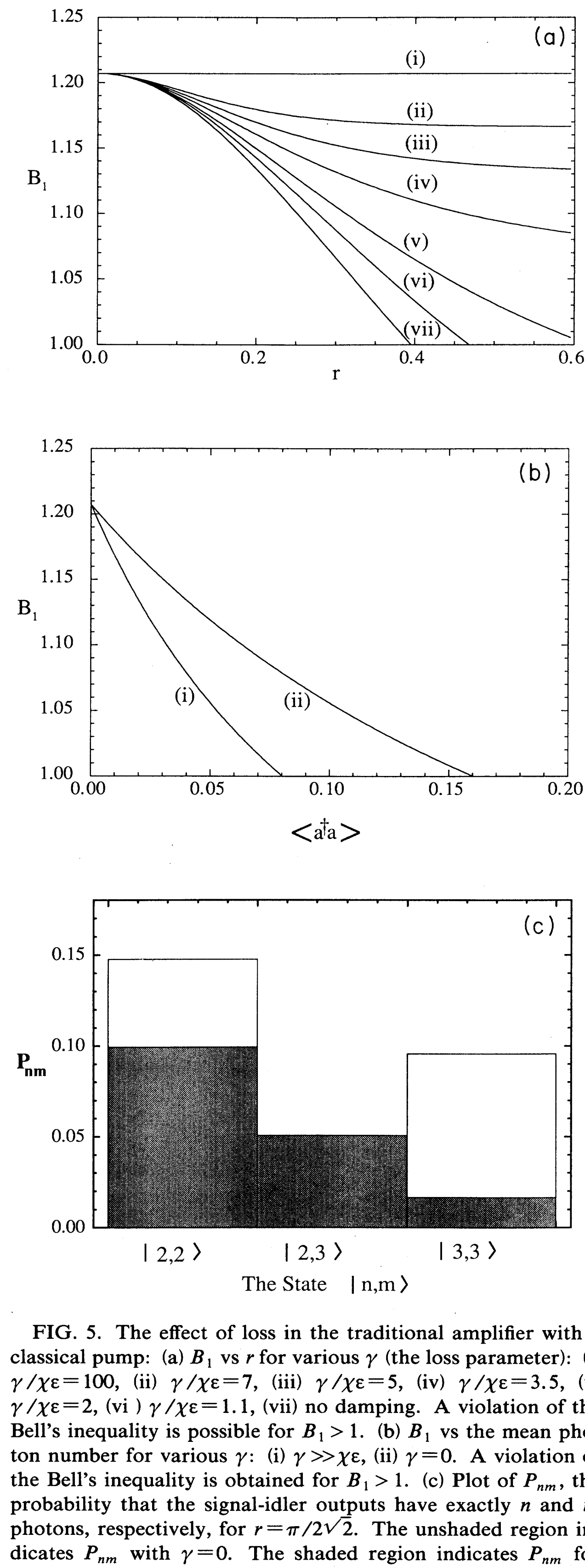

FIG. 5. The effect of loss in the traditional amplifier with a classical pump: (a) B_1 vs r for various γ (the loss parameter): (i) $\gamma/\chi\varepsilon=100$, (ii) $\gamma/\chi\varepsilon=7$, (iii) $\gamma/\chi\varepsilon=5$, (iv) $\gamma/\chi\varepsilon=3.5$, (v) $\gamma/\chi\varepsilon=2$, (vi) $\gamma/\chi\varepsilon=1.1$, (vii) no damping. A violation of the Bell's inequality is possible for $B_1>1$. (b) B_1 vs the mean photon number for various γ: (i) $\gamma \gg \chi\varepsilon$, (ii) $\gamma=0$. A violation of the Bell's inequality is obtained for $B_1>1$. (c) Plot of P_{nm}, the probability that the signal-idler outputs have exactly n and m photons, respectively, for $r=\pi/2\sqrt{2}$. The unshaded region indicates P_{nm} with $\gamma=0$. The shaded region indicates P_{nm} for $\gamma/\chi\varepsilon=1$.

to evaluate B_N and $\langle a^\dagger a\rangle$ may be calculated either by taking the appropriate averages over the Gaussian solution for $P(\alpha_1,\alpha_1^\dagger)$ or by evaluating averages from the solutions of the linear stochastic equations given by the (A1) in the Appendix, or by solving operator equations of motion derived directly from the Hamiltonian (4.1).

The values of B_N and the mean photon number $\langle a^\dagger a\rangle$ are presented in Fig. 5 for varying degrees of loss. As we expected, violations of the higher-order CHSH inequalities are still possible. Not surprisingly, the contradiction with the classical prediction, however, is lost at lower mean photon numbers. To obtain violations of the Nth-order inequality, we need to choose r so that the probability of the higher photon number states $|N+1\rangle|N+1\rangle,\ldots$, etc. is small compared to the probability P_{NN} of $|N\rangle|N\rangle$. The states with photon number lower than N do not contribute to the statistics since they give zero probability of detecting N photons. With loss present, the probabilities will change. Figure 5(a) illustrates the change in relative probabilities with the loss parameter γ. The higher-order probabilities are more sensitive to loss and for fixed r the violation is improved. The absolute probability of obtaining the higher-order state $|N\rangle|N\rangle$, however, has diminished and the mean photon number decreased [Figs. 5(b) and 5(c)].

V. CONCLUSION

We have shown how multiparticle macroscopic states predicted to violate a Bell's inequality may be generated from a correlated photon-number state $|n\rangle|n\rangle$. The inequality is tested by the measurement of joint probabilities where N photons are detected at one space-time point. The violation is evident for $N=n$ photons but reduces dramatically for $N<n$.

The correlated state $|n\rangle|n\rangle$ may be generated via parametric down-conversion. The violation of the Nth-order Bell's inequalities is predicted for the output of the amplifier operating at small gain. However, the sensitivity of the violation to the value of N means that at larger gains the output of the parametric amplifier gives no violation. This is because the probability of the amplifier generating $|n+i\rangle|n+i\rangle$ (where $i>0$), rather than $|n\rangle|n\rangle$, is significant. Hence, violations are predicted only for regimes where the probability of actually generating the $|n\rangle|n\rangle$ state where n is large is very small.

We point out that the use of intensity-squeezed pumps or photon-number-state-preparation techniques may improve the situation by allowing violations of Bell's inequalities were there is a significant probability of generation of the macroscopic state. Loss will effect a reduction in this probability.

APPENDIX

A Fokker-Planck equation for the positive P function may be derived from the master equation (4.2) assuming that certain boundary terms vanish. The Fokker-Planck equation is equivalent to the linear stochastic equations

$$\begin{aligned}\dot{\alpha}_1 &= -\gamma\alpha_1 + E\alpha_2^\dagger + \Gamma_{\alpha_1}(t) ,\\ \dot{\alpha}_1^\dagger &= -\gamma\alpha_1^\dagger + E\alpha_2 + \Gamma_{\alpha_1^\dagger}(t) ,\\ \dot{\alpha}_2 &= -\gamma\alpha_2 + E\alpha_1^\dagger + \Gamma_{\alpha_2}(t) ,\\ \dot{\alpha}_2^\dagger &= -\gamma\alpha_2^\dagger + E\alpha_1 + \Gamma_{\alpha_2^\dagger}(t) ,\end{aligned} \tag{A1}$$

where nonzero noise correlations are

$$\langle \Gamma_{\alpha_1}(t)\Gamma_{\alpha_2}(t')\rangle = \langle \Gamma_{\alpha_1^\dagger}(t)\Gamma_{\alpha_2^\dagger}(t')\rangle = E\delta(t-t') .$$

Careful examination [28] reveals to us that with the vacuum initial condition $\alpha_1=\alpha_1^\dagger=\alpha_2=\alpha_2^\dagger=0$, the variables are constrained to satisfy $\alpha_2=\alpha_1^*$ and $\alpha_2^\dagger=\alpha_1^{\dagger *}$. A reduction in the number of dimensions for the positive P representation was first pointed out by Wolinsky and Carmichael [29]. The Fokker-Planck equation has the following time-dependent solution for the positive P function:

$$P(\alpha_1,\alpha_1^\dagger,t) = N\exp\{2\bar{a}_-(\alpha_1^\dagger\alpha_1+\alpha_1^{\dagger *}\alpha_1^*) - 2\bar{a}_+(|\alpha_1^\dagger|^2+|\alpha_1|^2)\} , \tag{A2}$$

where $\bar{a}_\pm = a_\pm / E(a_+^2 - a_-^2)$,

$$a_\pm = \frac{(1-e^{-2\lambda_2 t})}{2\lambda_2} \pm \frac{(1-e^{-2\lambda_1 t})}{2\lambda_1} ,$$

and $\lambda_{\substack{1\\2}} = \gamma \pm |E|$. Here N is a normalization constant, given by

$$N^{-1} = \int \exp[2\bar{a}_-(\alpha_1^\dagger\alpha_1+\alpha_1^{\dagger *}\alpha_1^*) - 2\bar{a}_+(|\alpha_1^\dagger|^2+|\alpha_1|^2)]d^2\alpha_1 d^2\alpha_1^\dagger . \tag{A3}$$

The integration is over the entire complex plane of the independent variables α_1 and $\alpha_1^\dagger$. We see from the solution (A2) that the boundary terms do indeed vanish and the integrals (A3) are readily calculated to yield

$$N = \frac{\pi^2}{4}E^2(a_+^2 - a_-^2) . \tag{A4}$$

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