Accelerated clock principles in special relativity

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(Received 27 July 1992)

The accelerated clock principle in special relativity and possible alternatives are investigated. Alternative clock rates are proposed and their implications on the proper time of an observer with constant proper acceleration considered. The effect such alternatives have on the synchronization of spatially separated accelerated clocks is discussed and a review of experimental work is used to establish an upper bound on the magnitude of acceleration-dependent contributions to the rate.

PACS number(s): 03.30.+p

I. INTRODUCTION

The modern definition of special relativity (SR) is that of a theory of flat space-time [1]. As such it should provide a complete description of both inertial and accelerated motions through space-time devoid of gravitational fields. Such a description must include methods of transforming from one frame of reference to another. The Lorentz transformation (LT) provides such a relation between all inertial frames, and its derivation follows from the relativity principle and the postulate that the speed of propagation of information or energy in all inertial frames has a finite upper bound equal to the speed of light c [2]. To obtain a general transformation between accelerated and inertial observers it is necessary to make an additional postulate which relates an accelerated frame to the inertial frame with which it is instantaneously comoving. Whether or not this postulate is regarded as in some approaches as general relativistic [3] or, as in this work, special relativistic, its logical independence demands a separate experimental appraisal.

The standard approach regards the motion of an accelerating observer to be equivalent to an infinite sequence of instantaneously comoving inertial observers. This is formulated in the accelerated-clock principle (ACP), which states that the rate of an accelerated clock is identical to that of an instantaneously comoving inertial clock [4]. On the view imposed above, this is a further postulate required to complete the spectral theory of relativity. From this assertion it follows that the rate depends only on the instantaneous velocity and not on the acceleration *per se.* Since the LT relates each local inertial frame to an arbitrary inertial observer, the general transformation is obtained by integrating over all such local frames. In particular, the temporal transformation is given by the integrated proper time of the accelerated observer,

$$\tau_a = \int dt \, \left(1 - \dot{x}^2/c^2\right)^{1/2}. \tag{1}$$

Although most authors follow this standard approach and use the integrated proper time (and so implicitly assume the ACP) alternatives have been proposed. Other theories have been offered by Romain [5], Khan [6,7], and Kowalski [8]. In these it is suggested that the acceleration does in fact contribute to the rate of an accelerated clock. Mashhoon [9] refers to this presumed equivalence between the rates of the accelerated and instantaneously comoving inertial clocks as the "hypothesis of locality" and questions the validity of its application to physical phenomena which are not pointlike but instead have intrinsic time (T) and length (L) scales. He states that "the local immateriality of acceleration means, in terms of realistic measurements, that the influence of inertial effects can be neglected over the length and time scales characteristic of elementary local observers." He then further suggests that if the acceleration scales of the observer are of similar order to the intrinsic scale of the phenomena under observation, we may find locality violated. If this alternative point of view is taken, then the integrated proper time does not suffice to describe the temporal relation between accelerated and inertial frames and a new acceleration-dependent expression is necessary. Experimental tests of SR involving acceleration will provide an upper bound on the magnitude of any deviation from the ACP.

We denote a small increment in the time measured by an accelerated clock to be $d\tau_a$ and that of the instantaneously comoving inertial observer to be $d\tau_i$. The ACP asserts that these are equal. In this paper we investigate test theories in which this rate ratio $d\tau_a/d\tau_i$ is not unity. Suppose that any acceleration-dependent modification to the ACP can be expressed as a power series in the proper acceleration a. The rate ratio would then be

$$\frac{d\tau_a}{d\tau_i} = 1 - \lambda \sum_{n=1}^{\infty} g_n a^n, \tag{2}$$

where the coefficients g_n define the theory and have dimensions $[g_n] = L^{-n}T^{2n}$.

Initially we consider a modification containing only first-order acceleration terms. The first model studied (alternative 1) is an adaptation of a theory proposed by Khan [7] in which $g_1 = x/2c^2$, where x denotes the position of the accelerated observer in some preferred inertial frame relative to some preferred point. This choice of g_1 is by no means unique and dimensional analysis suggests that $g_1 = GM/c^4$ and $g_1 = (\hbar G/c^7)^{1/2}$ are also possible candidates but would not yield preferred frame theories. The former does require the stipulation of an arbitrary mass and so does not preclude an acceleration term dependent on particle mass; the latter contains only universal constants and so would introduce an acceleration term common to all observers. These possibilities will be discussed in Sec. II C.

The second modification considered (alternative 2) is again an adaptation of Khan's work but also serves to correct what we believe to be an inconsistency in Khan's analysis. This modification is then able to satisfy Khan's "principle of reciprocity" and with suitable choice of parameter λ reproduce the ACP result and a host of intermediate models. The "principle of reciprocity" summarizes a philosophy held by Khan that requires that two observers in relative motion make identical assessments of one another's motion through the use of their own measuring devices (rods and clocks.) The significance of this proposal becomes apparent when uniformly accelerated motion is analyzed in the context of the reciprocity principle. On doing this Khan deduced that there would be no "twin paradox," a conclusion that is at variance with the Hafele and Keating [10, 11] experiment in which caesium beam clocks were flown around the world and then compared with control clocks at the U.S. Naval Observatory. Khan derived a rate ratio of the form

$$\frac{d\tau_a}{d\tau_c} = 1 - \tfrac{1}{2}\lambda \boldsymbol{a} \cdot \boldsymbol{x}, \tag{3}$$

where x is the distance from a preferred position in a preferred frame, and in Khan's case $\lambda = 1$. We find that this rate ratio for uniformly accelerating observers did not in fact integrate to give the coordinate transformation necessary to satisfy the reciprocity hypothesis. The general expression for the rate ratio which would satisfy the reciprocity hypothesis is more complex than that suggested by Khan and may be expressed as a function of time in the case of uniformly accelerated motion:

$$\frac{d\tau_a}{d\tau_i} = 1 + 2\lambda \left(\frac{(\theta - ad)[(1+\theta^2)^{1/2} - \theta] - 1}{[\theta - (1+\theta^2)^{1/2} - 1]^2} \right), \tag{4}$$

where $\theta = at + b$. For a clock which is instantaneously at rest in the preferred frame (which occurs for this example at $t = -b/a$) and with $\lambda = 1$ this rate ratio reduces to Eq. (3). Both these proposals have inherent problems associated with the possibility that the rate may become negative. This predicts some bizarre results which place a limit on λ from physical grounds alone and reflects limitations in Khan's theory. These effects could be overcome by including higher-order terms or otherwise restricting the rate to be positive.

The third alternative is due to Kowalski [8] and stems from an important consequence of SR associated with the concept that two clocks synchronized in one frame will not be synchronized in a frame moving relative to the first. In Sec. III we review the prediction of SR that clocks which are slowly separated then undergo a period of acceleration before being slowly reunited will not be synchronized in their new rest frame [12]. We then apply the two alternative clock hypotheses to this problem and demonstrate that the degree of asynchronization will depend upon the theory and choice of parameter λ. Kowalski [8] hypothesizes that the clocks are synchronized in the new frame, and in so doing effectively demands an appropriately sized value of λ in his theory.

In Sec. IV we review experimental tests of SR that are applicable to the ACP and compare with the predictions of the modified theories to obtain upper bounds for the relevant parameter models.

II. UNIFORMLY ACCELERATED MOTION

We shall now use the results of the LT (using natural units) to discuss accelerated motions in flat space-time. An accelerated observer A' will at any instant be at rest in some inertial frame $S'(x', t')$, the instantaneously comoving rest frame. Any length and time measurements made with the coordinates of S' are related by the LT to our arbitrarily chosen inertial frame S. Let the *proper time* denoted by τ_i retain its standard meaning and refer to the time recorded by the clocks in the instantaneously comoving inertial rest frames. Following the nomenclature of Romain [5] the time registered by the accelerating clock will be called the *natural time* (τ_a). The ratio of the differentials of the natural time and the proper time $(d\tau_a/d\tau_i)$ will be referred to as the rate ratio.

A. Constant proper acceleration equations

We wish to establish the equation of motion of an observer A' moving with constant proper acceleration, that is, constant acceleration a with respect to the instantaneously comoving inertial frames. Following Desloge [12],

$$x = \frac{1}{a}[1 + (at+b)^2]^{1/2} + d, \tag{5}$$

where b and d are constants of integration related to the velocity and position of A' in S, respectively.

B. Natural time assuming the ACP

Under the ACP for each instantaneous rest frame of A' the natural time and proper time are equivalent, that is, $d\tau_a = d\tau_i$. From Eq. (5),

$$\dot{x} = \frac{at+b}{[1+(at+b)^2]^{1/2}}. \tag{6}$$

From the time dilation formula for differentials,

$$d\tau_i = (1 - \dot{x}^2)^{1/2} dt. \tag{7}$$

Integrating (7),

$$\tau_i = \frac{1}{a}[\sinh^{-1}(at+b) - \sinh^{-1} b]. \tag{8}$$

Equation (8) gives the relationship which the accelerated observer A' would find between the reading τ_a on its clock and the reading t on the clock of the observer in frame S with which it is at that moment coincident.

C. Natural time assuming alternative 1 to the ACP

As a first alternative theory, consider the proposal of Khan [6] motivated by the "principle of reciprocity." This principle requires that if A and A' are two observers in relative motion they will make identical assessments of each other's motion using their own respective clocks and rods. In contrast with the ACP the rate now has the proper acceleration and displacement as variables:

$$\frac{d\tau_a}{d\tau_i} = 1 - \tfrac{1}{2}\lambda a x. \tag{9}$$

We include the parameter λ to provide a family of theories which will include both the ACP (for $\lambda = 0$) and Khan (for $\lambda = 1$) models. The rate given in Eq. (9) clearly involves for the definition of x a *preferred frame* S and a *preferred point* in S. This notion of a preferred point has some important and unusual consequences.

An immediate consequence of the above rate is that the clocks of two spatially separated observers which initiate their acceleration simultaneously in S will be observed to be running at different rates by an observer in S at later times. This implies that the natural times of the two clocks as determined by S at any instant t will be different. In Sec. III it will be shown that this may be used to predict the degree to which two spatially separated, identically accelerating clocks will move out of synchronization when viewed from the original frame S. In contrast the ACP predicts that spatially separated identically accelerating clocks will remain synchronized in their initial rest frame.

Let us consider the behavior of the rate for a constant proper acceleration a. If x varies according to the equation for hyperbolic motion, the rate will increase (or decrease depending on the signs of λ and a) as x increases. If $\lambda a x > 2$, then the rate will be negative. This indicates that the instantaneously comoving inertial observer would measure the accelerated observers clock to be running backwards. This unusual prediction raises the question as to whether Eq. (9) violates causality. First we establish the natural time equation corresponding to this rate.

To calculate the natural time, first substitute Eq. (5) into Eq. (9) to obtain the rate as a function of t:

$$\frac{d\tau_a}{d\tau_i} = 1 - \frac{\lambda}{2}\{[1 + (at + b)^2]^{1/2} + ad\}. \tag{10}$$

The instantaneously comoving observer at time t has the value of $d\tau_i/dt$ given by Eq. (7). Hence,

$$\frac{d\tau_a}{dt} = \frac{1 - (\lambda/2)\{[1 + (at + b)^2]^{1/2} + ad\}}{[1 + (at + b)^2]^{1/2}}. \tag{11}$$

Integrating Eq. (11) with the boundary condition $\tau_i = 0$ at $t = 0$ we obtain the natural time of A':

$$\tau_a = \frac{1}{a}\left(1 - \frac{\lambda a d}{2}\right)[\sinh^{-1}(at + b) - \sinh^{-1} b] - \frac{\lambda t}{2}. \tag{12}$$

With the possibility of acausal effects it is not surprising that from Eq. (12) we find that τ_a is multivalued for $\lambda \neq 0$ and that for $\lambda > 0$, τ_a is a decreasing function in the limit as $t \to \pm\infty$.

The twin problem has been analyzed from the viewpoint of the accelerated observer by many authors [3, 12–15] through the construction of an accelerated reference frame by methods which invariably incorporate the ACP. Consider a version of the twin "paradox" in which two identical observers, one inertial and one accelerating, are initially spatially coincident. They move to some maximum separation and then return so that they are again spatially coincident. Uniformly accelerated motion satisfies this form of the twin problem as the world line is a hyperbola, and so an accelerated observer may coincide twice with an inertial observer. If the times registered between meetings are t and τ_a for the inertial and accelerated observers, respectively, then from Eq. (8) we see that on the ACP the noninertial observer will record the shorter time. This is a general feature of discussions of the twin paradox even when the motion is largely inertial [16, 17].

Next consider the possibilities offered by applying the twin-paradox scenario to the natural time equation resulting from our modified rate. We assume without loss of generality that at $t = 0$, $x = L$. This implies that $d = L - (1 + b^2)^{1/2}/a$. Substituting d into Eq. (5) and solving for t when $x = L$ yields $t = -2b/a$ as the other time at which A' passes $x = L$. The natural time of A' at $t = -2b/a$ is then by Eq. (12),

$$\tau_a = \frac{-2}{a}\left(1 - \frac{\lambda}{2}[aL - (1 + b^2)^{1/2}]\right)\sinh^{-1} b + \frac{\lambda b}{a}. \tag{13}$$

Next we establish when this natural time is zero. This occurs trivially if $b = 0$, so we seek solutions for $b \neq 0$. Suppose the initial velocity is restricted to be much less than the speed of light: $\dot{x} \ll 1$. From Eq. (6) with $t = 0$ it is apparent that $b = \pm\dot{x}/(1 - \dot{x}^2)^{1/2} = \pm\dot{x} + O(\dot{x}^3)$, so that $|b| \ll 1$. Hence Eq. (13) reduces to $\tau_a \approx b\,\{-2/a[1 + (\lambda/2)(1 + b^2/2 + \cdots) - \lambda L a]\} + \lambda b/a$ implying $\lambda \approx 2/aL$ when $\tau_a = 0$.

Note that on comparison with Eq. (9) this is the same value for λ which marks the transition at which the rate becomes negative. This result provides an estimate of the range of accelerations and distances which may be used for a given λ, and also demonstrates that there is a limit at which the theory becomes acausal. If we reintroduce the speed of light c we see that $\lambda \approx 2c^2/aL$ is the maximum value λ can have before the natural time for a return trip becomes negative. The above thought experiment illustrates that beyond this limit there exists the possibility of traveling away and subsequently returning younger than on setting out. This result is sufficiently bizarre to discredit a theory where the rate depends linearly on the acceleration and separation. One method of overcoming this is to postulate a modification, which makes the rate non-negative, such as

$$\frac{d\tau_a}{d\tau_i} = \left(1 - \frac{\lambda a x}{4}\right)^2 . \tag{14}$$

For sufficiently small λ this reduces to the original linear theory which we will continue to use in subsequent sections.

We next briefly discuss implications of the other position-independent rate ratios mentioned in the Introduction. The two possible candidates $g_1 = GM/c^4$ and $g_1 = (\hbar G/c^7)^{1/2}$ may be substituted into the rate-ratio equation to first order in the acceleration which is

$$\frac{d\tau_a}{d\tau_i} = 1 - \lambda g_1 a. \tag{15}$$

Both these proposals suggest that the rate ratio is dependent upon the sign and magnitude of the acceleration. For large enough positive accelerations the above rate ratio may be negative and so leads to some bizarre effects. This occurs when $a > 10^{-52}/\lambda \ \ \mathrm{ms}^{-2}$ (approximately.) Unlike the Khan-type theories there is no cumulative position dependence on these effects so the latter would not become apparent just by accelerating for a sufficiently long time.

Putting these two proposals through the same machinery as was used in the above case yields

$$\tau_a = \frac{1}{a}(1 - \lambda g_1 a)[\sinh^{-1}(at + b) - \sinh^{-1} b], \tag{16}$$

where the significant change occurs in the first set of parentheses. Physically this implies that an arbitrary inertial observer would deduce the accelerated clock to be running faster, slower, or indeed not running at all depending upon the sign and magnitude of a and the magnitude of λ. For an accessible example consider the same twin arrangement as given previously.

Suppose we have two observers, one at rest at the origin of an inertial frame S and the other accelerating with proper acceleration a. Assume that at $t = 0, x = 0$ the observers are at the same position. Their next coincidence is at $t = -2b/a, x = 0$; all coordinates are in S. At $t = -2b/a$ the natural time registered by the accelerated clock is

$$\tau_a = -\frac{2}{a}(1 - \lambda g_1 a) \sinh^{-1} b, \tag{17}$$

while at $t = 0, \tau_a = 0$. Thus we are able to compare the time interval logged up on the accelerated observer's and inertial observer's respective clocks between meetings for various values of λ and a. The inertial clock always registers $t_2 - t_1 = 2b/a$ irrespective of the theory. On the other hand the time registered by the accelerated clock (as judged by the inertial observer) is

$$\tau_2 - \tau_1 = \frac{2}{a}(1 - \lambda g_1 a) \sinh^{-1} b. \tag{18}$$

This provides for a number of scenarios.

- $\lambda g_1 a = 1$: The acelerated observer measures no change in time between the two meetings. The inertial observer concludes that time has stopped for the accelerated observer.
- $\lambda g_1 a > 1$: The accelerated observer measures a time loss between meetings. The inertial observer concludes that time is running backward for the accelerated observer.
- $\lambda g_1 a < 1$: Here a range of possibilities exists depending upon the magnitude of a. It includes the standard ACP result for $\lambda = 0$ and also contains a theory in which the accelerated clock is deduced to have measured the longer time interval.

This type of theory does not provide for Kowalski's variety of synchronization demand as discussed in Sec. III. This is due to the exclusion of any position-dependent terms, without which spatially separated, identically accelerated clocks age at the same rate.

D. Natural time assuming alternative 2 to the ACP

The procedure by which Khan obtains the transformation Eq. (21) is outlined below and circumvents the integration over inertial frames required by the ACP. Having obtained the transformation we derive the rate ratio $d\tau_a/d\tau_i$ implied by the model, then generalize to a family of alternatives to the ACP which may then be compared with experimental tests of SR.

Suppose that A' moves in hyperbolic motion in frame S and that A is an observer at rest in S at $x = 0$ (see Fig. 1). Let P_1 be the event when A sends a light signal to A' at time t_1 and P' the event A' receives the signal at time t. A' immediately returns the signal to A. Let P_2 be the event when A receives the reflected signal at time t_2. All coordinates refer to the frame S. Observer A deduces that event P' occurs at a time $t = \frac{1}{2}(t_1 + t_2)$

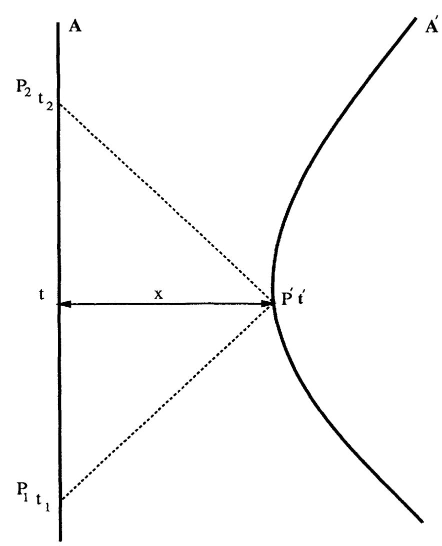

FIG. 1. World lines for discussing the coordinate transformation derived by Khan between the accelerated observer and an inertial frame which is constructed from light signals and time measurements carried out by one of the observers in accordance with the reciprocity hypothesis.

and at a distance $x = \frac{1}{2}(t_2 - t_1)$. Suppose t' is the time recorded on the clock carried by A' at the instant when the light signal originating from P arrives; we wish to obtain a transformation,

$$\phi : A \longrightarrow A', \tag{19}$$

such that $t' = \phi(t_1)$. The reciprocity hypothesis would then require that $t_2 = \phi(t')$ so that $t_2 = \phi(\phi(t_1))$. Khan establishes the transformation by finding an iterate of order $\frac{1}{2}$ for the function $t_2(t_1)$, the results of which are

$$\phi(t_1) = \frac{pt_1 + q}{t_1 + r}, \tag{20}$$

where p, q, and r are constants expressed in terms of a, b, and d as $p = \pm 1/a + d - b/a$, $q = d^2 - 1 + b^2/a^2$, $r = \pm 1/a + d + b/a$.

Next we express ϕ in terms of the original constants by substituting for p, q, and r and choose the lower value of p and r so that ϕ remains finite at $t = -b/a$:

$$\phi(x,t) = \frac{(-a + ad - ab)(t - x) + a^2d^2 - 1 - b^2}{a[a(t - x) - 1 + ad + b]}. \tag{21}$$

This result gives an explicit transformation between the coordinate time of S and the natural time of observer A' in terms of the initial conditions.

We differentiate $\phi(x,t)$ with respect to t and use the LT to find the rate of A' with respect to its instantaneously comoving inertial frame:

$$\frac{d\phi}{d\tau_i} = \frac{(pr - q)(1 - \dot{x})}{(t - x + r)^2(1 - \dot{x}^2)^{1/2}}, \tag{22}$$

and evaluate this rate when A' is stationary in the preferred frame S. This requires $\dot{x} = 0$ which in turn implies that $t = -b/a$ by Eq. (6) and hence [from Eq. (5)] we arrive at the expression derived by Khan [his (3.15)]:

$$\frac{d\phi}{d\tau_i} = \tfrac{1}{2}(1 - ad) = 1 - \tfrac{1}{2}ax. \tag{23}$$

Now introduce a dimensionless parameter λ into the rate given by Eq. (22). This enables us to obtain both the ACP [Eq. (8)] (for $\lambda = 0$) and the Khan [Eq. (27)] transformation (for $\lambda = 1$) between the coordinate time of S and the natural time of A'. Rearranging Eq. (22) and introducing λ we have

$$\frac{d\tau_a}{d\tau_i} = 1 + \lambda\left(\frac{(pr - q)(1 - \dot{x})}{(t - x + r)^2(1 - \dot{x}^2)^{1/2}} - 1\right). \tag{24}$$

To solve Eq. (24) for τ_a first substitute for x so that the expression is in terms of t only, then substitute in the expressions for p, q, and r. To simplify the algebra we make the substitution $\theta = at + b$ and with some calculation obtain

$$\frac{d\tau_a}{d\tau_i} = 1 + 2\lambda\left(\frac{(\theta - ad)[(1 + \theta^2)^{1/2} - \theta] - 1}{[\theta - (1 + \theta^2)^{1/2} - 1]^2}\right). \tag{25}$$

Using the methods outlined above and the LT we rearrange this into a first-order differential equation in τ_a and t. The resulting equation is separable. Specifying the same boundary conditions as before and with further simplification we obtain

$$\begin{aligned}\tau_a = {} & (1/a)(1 - \lambda)(\sinh^{-1}\theta - \sinh^{-1} b) \\ & + (\lambda/a\theta)\{(1 - ad)[(1 + \theta^2)^{1/2} - 1] - b\theta\}.\end{aligned} \tag{26}$$

To obtain ϕ, the transformation which denies the asymmetric aging of twins, we set $\lambda = 1$ in Eq. (26),

$$\phi = \frac{(1 - ad)[(1 + \theta^2)^{1/2} - 1] - b\theta}{a\theta}. \tag{27}$$

Although it is not immediately obvious, Eq. (21) with Eqs. (5) and (27) are equivalent.

The rate ratio given by Eq. (25) is consistent with the transformation derived by Khan whereas that given by Eq. (23) does not integrate to give the function ϕ. We believe that this point may have been overlooked by Khan and feel that the above result provides a more complete description of the relation between the accelerated observer and inertial observer within the framework of the reciprocity hypothesis. The problems associated with negative rates are still apparent in Eq. (25) which again indicates that it is necessary to include higher-order terms to ensure a physically sensible solution in the limit of large accelerations and separations.

III. CLOCK SYNCHRONIZATION

Suppose we have two identical clocks A and B which are spatially coincident, synchronized, and at rest in an inertial frame $S(x,t)$ at some past time $-T$. Clock B is then slowly transported so that at $t = t_0$ the clocks are separated by a distance L. At t_0 both clocks initiate identical constant proper acceleration which ceases at coordinate time $t = t_1$. Clock A maintains a constant velocity with respect to S while B is slowly transported back to A so that at $t = t_1 + T$ the clocks are again spatially coincident and stationary but now in the rest frame S' of A. We compare the natural times of the two clocks before and after their separation, acceleration, and rejoining for each of the models discussed in Sec. II. Figure 2 displays the world lines in frame S of these events; a detailed calculation using the ACP model is given in the Appendix.

At event 6 the natural times of A and B, respectively, are

$$\begin{aligned}\tau_{+A} &= (1/a)\sinh^{-1}(at_1) + T/\gamma_V, \\ \tau_{+B} &= \tau_{+A} + \gamma_V VL + O(1/T).\end{aligned} \tag{28}$$

Comparing A and B subsequent to their being reunited we find that $\tau_{+A} - \tau_{+B} = -\gamma_V VL + O(1/T)$, so that for the ACP model at least, they are no longer synchronized. The terms $O(1/T)$ are negligible provided slow transport is over a sufficiently long time.

We next consider how alternative models will affect the synchronization. The first alternative leads, through essentially similar calculations to that used in the ACP case, to natural times at event 6 which are

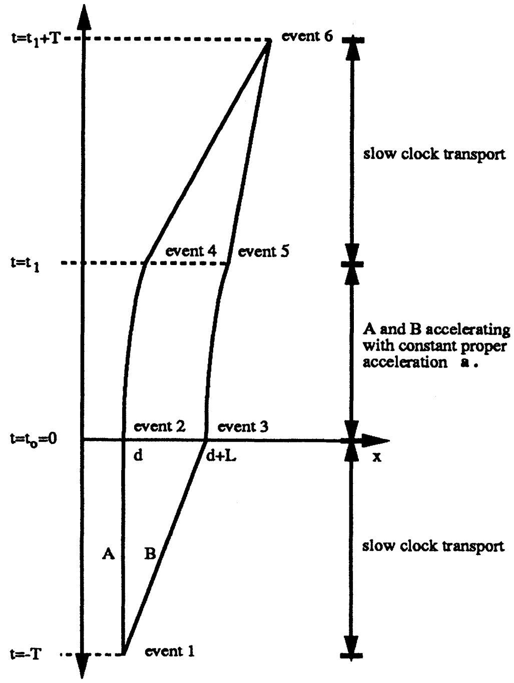

FIG. 2. World lines in frame S of two clocks A and B as they are separated by slow transport, identically accelerated, and then reunited.

$$\tau_{+A} = \frac{1}{a}\left(1 - \frac{\lambda}{2}a(d - 1/a)\right)\sinh^{-1}(at_1) - \frac{\lambda t_1}{2} + \frac{T}{\gamma_V}, \tag{29}$$

$$\tau_{+B} = \tau_{+A} + \frac{\lambda L}{2}\sinh^{-1}(at_1) + \gamma_V VL.$$

The difference between the natural times of A and B after they have been reunited now depends upon how long they were accelerated for, and is

$$\tau_{+A} - \tau_{+B} = (\lambda L/2)\sinh^{-1}(at_1) - \gamma_V VL. \tag{30}$$

The parameter λ determines to what extent the clocks are asynchronized in S'. In fact it is possible to contrive the parameter so that clocks will be synchronized in the new rest frame. This idea is supported by Sachs [18] and a similar proposition has been made by Kowalski [8] in his hypothesis of "phase invariance." This hypothesis effectively denies the existence of a Boughn-type [19] twin "paradox" just as Khan denies the usual twin paradox in which one twin is not accelerated. In each case these authors will require Eq. (9) to hold at first order and so require a preferred frame theory.

If clocks initially synchronized in S are to be synchronized in the new rest frame S' after a period of uniform acceleration, then Eq. (30) must be equal to zero. Since $\gamma_V V = at_1$,

$$\lambda = \frac{2at_1}{\sinh^{-1}(at_1)}. \tag{31}$$

Hence the appropriate λ value for the Kowalski-type synchronization is strictly a function of the time t_1 for which the clocks are accelerated although reducing to $\lambda = 2$ as $t_1 \to 0$. Also worthy of note is the fact that λ is independent of L so that the one choice of parameter is sufficient to synchronize all clocks in S accelerated identically to a new frame. For $at_1 \ll 1$ λ is approximately 2. Thus Kowalski's demand for the preservation of synchronization may be satisfied in such cases with the appropriate choice of parameter in a preferred point theory.

Applying the above argument to the second alternative theory we find that again the natural times of accelerated clocks have terms dependent on the initial position. The λ value necessary to ensure synchronization in the new frame is obtained as before except that we use Eq. (25) for the natural time. Hence,

$$\tau_{+A} - \tau_{+B} = \frac{\lambda L[(1 + (at_1)^2]^{1/2} - 1}{at_1} - \gamma_V VL, \tag{32}$$

and so,

$$\lambda = \frac{(at_1)^2}{[1 + (at_1)^2]^{1/2} - 1}. \tag{33}$$

For $at_1 \ll 1$ we find that λ is approximately 2. We therefore propose a simplified rate-ratio hypothesis of the form of Eq. (9) as an approximate model of the ACP ($\lambda = 0$), Khan's ($\lambda = 1$), and Kowalski's ($\lambda \approx 2$) test theory to be tested against experiment.

IV. EXPERIMENTAL TESTS OF THE ACP

There has been a number of experimental tests of SR incorporating the Mössbauer effect in a rotating system, notably those of Hay *et al.* [20] and Turner and Hill [21]. We wish to establish whether or not these experiments constitute a test of the theory under consideration and so bound the value of λ.

The apparatus essentially consists of a ^{57}Fe source on the axis of a rotating disk and an iron absorber on the perimeter. A detector is situated beyond the perimeter but is not attached to the disk and remains at rest in the laboratory frame. The absorption is monitored as a function of the angular velocity of the disk. The absorber and source are maintained at a constant temperature to eliminate thermal effects (see Pound and Rebka [22]). We regard the disk to be rotating in an inertial frame with the proviso that Earth's rotation be included for a complete solution. If the angular velocity of the disk is $\dot{\theta}$ and the radial vector to a point on the perimeter is $\boldsymbol{r}$, then the acceleration at that point is $\boldsymbol{a} = -\boldsymbol{r}\dot{\theta}^2$.

To apply the rate given by Eq. (9) to this motion we must establish the preferred point and the distance from this point to the accelerated observer (see Fig. 3). Suppose that the center of the rotating frame is located at $(x = h, y = k)$ in the preferred frame S and let $\boldsymbol{R}$ denote a vector from the preferred point chosen as the origin of S to a point on the rim of the rotating disk. Substi-

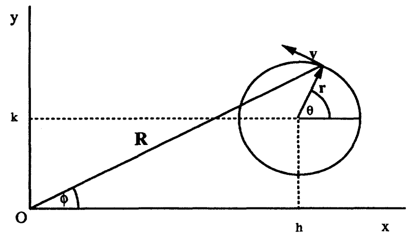

FIG. 3. The rotor experiment diagrammatically, a source is placed at $x = h, y = k$ and an absorber rotates with a uniform angular velocity at a radial distance r. The modified rate proposed is dependent on the distance to the preferred point (selected as the origin) and the acceleration of the observer. The distance d depends sinusoidally on θ.

tuting the appropriate parameters into the modified rate equation gives

$$\begin{aligned}\frac{d\tau_a}{d\tau_i} &= 1 - \frac{\lambda\, \boldsymbol{a}\cdot\boldsymbol{R}}{2} \\ &= \tfrac{1}{2}1 + \lambda\dot{\theta}^2 rR\cos(\theta-\phi),\end{aligned}$$

where

$$R = \sqrt{(h + r\cos\theta)^2 + (k + r\sin\theta)^2}, \qquad \tan\phi = \left(\frac{k + r\sin\theta}{h + r\cos\theta}\right). \tag{34}$$

We consider the two limiting cases.

1. As $\boldsymbol{R}\to \boldsymbol{0}$ the modified rate reduces to $d\tau_a/d\tau_i = 1 + \lambda r^2\dot{\theta}^2/2$. Thus we would expect to observe a count rate which was different from that predicted by the ACP by a factor proportional to the square of the tangential velocity.

2. If $\boldsymbol{R}\gg \boldsymbol{r}$ the modified rate is $d\tau_a/d\tau_i \approx 1 + (\lambda\dot{\theta}^2 r/2)\sqrt{h^2+k^2}\cos(\theta-\phi)$ where the phase factor ϕ is nearly constant. The count rate would now be dependent on the angular position of the detector.

Turner and Hill [21] established that any angular dependence of the rate ratio is less than 6% of the mean rate ratio which implies that

$$\left|\frac{\lambda\dot{\theta}^2 rR/c^2}{1-(\lambda\dot{\theta}^2 rR/2c^2)}\right| < 0.06. \tag{35}$$

In the analysis of this experiment Hay uses the standard relativistic Doppler equations for a source with uniform velocity u and radial component u_r,

$$D = \frac{1+u_r}{(1-u^2)^{1/2}} = 1 + u_r + \tfrac{1}{2}u^2 + O(u^3). \tag{36}$$

There has been some criticism of the application of this result to accelerated observers and some discussion as to the effect on redshift calculations [23, 24]. The contribution to D from modifications to the ACP will be in addition to any terms obtained from a more detailed derivation of the Doppler equation and we retain this simple model as a first approximation. Modifying the method of Rindler [4] used to derive Eq. (36) we get $D_{\rm alt} = d\tau_i/d\tau_a D$. Since the correlation between theory and experiment is of the order of 2% we have

$$\left|\frac{D - D_{\rm alt}}{D}\right| < 0.02. \tag{37}$$

In order to establish an upper bound for λ we need an estimate of R. Suppose we take the preferred frame to be the frame in which the background microwave radiation is isotropic and the preferred point the position of Earth at A.D. 1. Then take the relative velocity of Earth to the microwave background [29] as 600 km/s, and the distance from our present position to the preferred point as $\sim 10^{16}$ m. Given that $r \sim 0.13$ m and $\dot{\theta} \sim 1000\pi$ rad/s we have $\lambda < 10^{-7}$.

Pound and Rebka [22] using the 14.1-keV nuclear γ-ray transition of ^{57}Fe observed that a difference in temperature ΔT between the source and absorber produced a shift $\Delta\nu$ in the absorption line. The standard explanation is that the frequency shift is a result of second-order Doppler shift caused by the random thermal vibrations of the nucleus in the crystal lattice. Sherwin [25] interprets this as confirmation of the ACP. However Mashhoon [26] points out that since the lifetime of the excited state is very long compared to the period of the motion, effects which are linear in acceleration and velocity cancel. Consequently any acceleration-dependent contributions to the rate would also be of second order. This experiment does not differentiate clearly between the ACP and the first alternative theory which has linear dependence. As suggested in Sec. II C a preferred frame rate must contain higher-order terms to avoid nonphysical solutions and so we expect the appearance of second-order effects. The maximum shift $\Delta\nu/\nu = v^2/c^2$ implied by the standard theory agrees with experiment (to within about 10%). This implies that any specific acceleration-dependent effects will be at least one order of magnitude less.

Bömmel [27] conducted an experiment in which a spatially separated source and absorber were accelerated along the line joining them by oscillating piezoelectric transducers. The source and absorber were given identical accelerations so that in the laboratory frame their separation was constant. Using oscillations with frequencies between 1 and 5 MHz the change in the counting rate $\Delta n/n$ was found to be linearly dependent on the separation. These results were interpreted as a redshift and compared with the prediction $\Delta\nu/\nu = aL/c^2$.

More recently Baryshevsky [28] explained the effect as a property of the ultrasonic excitation of Mössbauer sources and absorbers. An analysis of Bömmel's experiment reveals that first-order Doppler effects will be observed. Assume the source and absorber are undergoing simple harmonic motion with frequency ω and are in phase. Let the position of the source and absorber be $x_s = A\sin(\omega t)$ and $x_a = x_s + L$, respectively. Let us also assume that the separation L is much greater than the amplitude A so that the propagation time of a γ ray between absorber and source is $\approx L/c$. If the source emits a γ ray at time t, then the velocity of the source with respect to the laboratory frame is $v_s = A\omega\cos(\omega t)$ and the velocity of the absorber upon reception of the γ ray is consequently $v_a = A\omega\cos(\omega t + \omega L/c)$. Hence

$$v_s - v_a \approx \frac{A\omega^2 L}{c}\sin\left(\omega t + \frac{\omega L}{2c}\right), \tag{38}$$

which is not in general zero. Thus there are time-dependent first-order Doppler effects and since the emission time is random this will broaden the linewidth rather than give a definite shift in the absorption line. This effect will obscure any relativistic contributions which we might hope to observe. This leaves some doubt as to the extent which the above experiment constitutes a test of the ACP. Unfortunately it does not appear that similar experiments have been carried out with collinear motion as they would provide the most suitable framework in which to test the rate under consideration.

We consider the Hafele-Keating experiment [10, 11]. The objective of this experiment was to provide a macroscopic test of the kinematic time-dilation effect of SR. Four caesium-beam atomic clocks were transported in commercial aircraft twice around the world, once in each direction. The calculation of the expected time again is fairly straightforward and includes both kinematic and gravitational terms. The agreement between the observed time gain and that predicted is of the order of 10%. Let us consider the repercussions of an alternative model of the type parametrized by Eq. (9) for the Hafele-Keating observation.

The most stringent tests of a preferred frame theory and of Eq. (3) are those in which the absolute distance x to the preferred point appears. This in turn demands comparison of clocks with relative acceleration or velocity. The Hafele-Keating experiment data, as registered, did not do this, and only a less stringent test is possible. Caesium-beam clocks are known to undergo short fluctuations in rate due to shot noise and also spontaneous quasipermanent changes in rate over a longer period (typically two to three days.) These changes are normally independent for a collection of clocks. Hafele and Keating used a method of correlated rate change during the trip. Here the rates of each of the four clocks are compared and consequently any change in the rate of an individual clock may be ascertained. They did not compare the rate of the accelerated clocks with a ground-based clock at the U.S. Naval Observatory during the flight time. Hafele and Keating say explicitly that the clock rates aboard the aircraft were within three standard deviations (15%) of the correlated mean. This places an upper bound on any acceleration-dependent and position-dependent rate ratio term. If R_1 and R_2 are the rate ratios of two clocks separated in the aircraft, $\bar{R}$ the mean rate ratio, and ΔR the deviation of the rate ratio from the mean, then

$$\left|\frac{\Delta R}{\bar{R}}\right| < 0.15. \tag{39}$$

However, everyday experience is enough to constrain λ far more stringently. Take a preferred frame theory of the form of Eq. (3) with either Khan's or Kowalski's value of λ (1 and 2, respectively) and again take the preferred frame to be the frame in which the background microwave radiation is isotropic and the preferred point the position of earth at A.D. 1 (now $\sim 10^{16}$ m distant). Then the rate ratio deviates from unity in the alternative theory by $\sim \lambda a_\parallel/9$ ($a_\parallel$ = component $\boldsymbol{a} \parallel \boldsymbol{x}$, 9 arises from c^2 in the denominator) and the cumulative effect of a linear acceleration to speed $\boldsymbol{v}$ is a synchrony change in the clock of order

$$\Delta T = \frac{\lambda v_\parallel}{9}\ \text{s}. \tag{40}$$

Hence an airplane during takeoff ($\Delta v \approx 250$ m/s) would experience in such a theory a change in absolute synchronization of its clock of order 28λ s. Since in practice a change in synchronization is not detected to within fractions of a second, λ must be very much less than 1 in such a theory. On the Khan and Kowalski values of λ all passengers would note dramatic changes in their watches during takeoff for such a value of x.

V. CONCLUSION

Motivated by the work of Khan [6, 7] and Kowalski [8] we investigated alternatives to the ACP of SR which included acceleration and position-dependent terms. It was shown that these proposals are preferred frame, preferred position theories. Further we introduced a dimensionless parameter λ into various rate-ratio equations, the variation of which provided a family of theories including both the alternatives and the ACP.

Through considering uniformly accelerated motion it was found that there exist some problems associated with these alternative proposals. In particular, if the rate ratio becomes negative we find acausal results in "twin paradox" type thought experiments.

For the problem of spatially separated, identically accelerated clocks we find that the amount by which the clocks are out of synchronization on completion of acceleration depends upon the model selected. It is shown that a Kowalski-type theory, whereby clocks initially synchronized will remain so after acceleration, can be obtained by a suitable choice of λ (which with suitable conditions is approximately 2).

Finally a review of experimental work may be used to place an upper bound on λ. As the alternate theories are preferred frame, preferred point theories it is necessary to choose some inertial reference frame. If this is taken to be the frame in which the microwave background radiation is isotropic, both the rotating Mössbauer and everyday experience lead us to conclude that $\lambda \ll 1$. This would then rule out both the Khan ($\lambda = 1$) and Kowalski ($\lambda = 2$) alternatives on phenomenological grounds.

ACKNOWLEDGMENT

We acknowledge gratefully copies of correspondence with Professor Frank Kowalski regarding his phase invariance hypothesis and for several references.

APPENDIX: CLOCK SYNCHRONIZATION

We calculate the natural times of two clocks A and B after they have been slowly separated, identically accelerated, and then slowly reunited using the ACP prescription. The coordinates are of the initial rest frame S and all velocities are relative to this. Terms $O(1/T)$ are considered negligible.

Event 1: At $t=-T$ the clocks A and B are at rest and coincident at $x=d$. Separation of the clocks by slow transport begins and the natural times of A and B are $\tau_{-A}=\tau_{-B}=-T$.

Event 2: At $t=t_0=0$, A begins constant proper acceleration a from rest in S. A has position $x_{0A}=d$ and registers a natural time $\tau_{0A}=0$.

Event 3: At $t=t_0=0$, B begins constant proper acceleration a from rest in S having been slow transported to position $x_{0B}=d+L$. B registers a natural time $\tau_{0B}=0+O(1/T)$.

Event 4: At $t=t_1$, A ceases acceleration and continues with constant velocity $V=at_1/[1+(at_1)^2]^{1/2}$. A has position $x_{1A}=(1/a)[1+(at_1)^2]^{1/2}+d-1/a$ and registers a natural time dependent on the model selected; for the ACP this is $\tau_{1A}=(1/a)\sinh^{-1}(at_1)$.

Event 5: At $t=t_1$, B ceases acceleration and begins slow transport (relative to frame S') back to A. B has position $x_{1B}=x_{1A}+L$ and again the natural time depends upon the model; for the ACP $\tau_{1B}=(1/a)\sinh^{-1}(at_1)+O(1/T)$. In order that A and B are spatially coincident at $t=t_1+T$ the velocity of B relative to S is $V_B=V-L/T$.

Event 6: At $t=t_1+T$, A and B are spatially coincident and at rest in S'. If $\gamma_V=1/(1-v^2)^{1/2}$ then the natural time registered by A for the ACP model is $\tau_{+A}=(1/a)\sinh^{-1}(at_1)+T/\gamma_V$. Since $1/\gamma_{V_B}=1/[1-(V-L/T)^2]^{1/2}$ we expand this as a series in $1/T$. We then find the natural time of B for the ACP model is $\tau_{+B}=(1/a)\sinh^{-1}(at_1)+T/\gamma_V+\gamma_V VL+O(1/T)$.

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