COMMENTS

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Comment on "Gravity as a zero-point-fluctuation force"

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A paper by H. Puthoff [Phys. Rev. A **39**, 2333 (1989)], which claims to derive Newtonian gravity from stochastic electrodynamics, contains a serious computational error. When the calculation is corrected, the resulting force is shown to be nongravitational and negligible.

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In Ref. [1], Puthoff claims to derive Newtonian gravity within the framework of stochastic electrodynamics (SED). That paper is an attempt to implement Sakharov's suggestion [2] that gravity is a residual effect of zero-point fluctuations of other fields. In SED, zero-point fluctuations of the electromagnetic field are assumed to produce a (physical) stochastic electromagnetic background with a prescribed spectrum, and Puthoff argues that massive particles distort this background in a way that gives rise to a residual inverse square force.

The purpose of this Comment is to point out a computional error that invalidates this result. When correctly calculated, Puthoff's potential corresponds to a force that falls off as the inverse *fourth* power of distance, with a magnitude that is completely negligible at measurable distances.

Puthoff derives a van der Waals–type two-particle interaction potential of the form

$$U=\frac{\gamma}{2}\frac{1-\cos(2\mathcal{R})}{\mathcal{R}^3}\ , \tag{1}$$

where γ is a constant and $\mathcal{R}=\omega_c R/c$ (R is the distance between the two particles and ω_c is a cutoff frequency on the order of the inverse Planck time). Intuitively, one would expect this potential to lead to a force that falls off as R^{-4}—for large R, the cosine term in the numerator varies much faster than the denominator, and should average to zero. In Ref. [1], it is claimed that a suitable averaging procedure results in an R^{-2} force. This conclusion, however, is based on an incorrect approximation that keeps some terms and neglects others of the same order.

In Eqs. (B5)–(B7) of Ref. [1], Puthoff evaluates the integral

$$I=\int_{\pi n}^{\pi(n+1)}\frac{1-\cos(2\mathcal{R})}{\mathcal{R}^4}d\mathcal{R}\ , \tag{2}$$

and obtains

$$I\sim\frac{2\pi}{3(\pi n)^2} \tag{3}$$

for large n. This is clearly incorrect: the integrand in (2) is non-negative and is bounded above by $2/\mathcal{R}^4$, so

$$0\le I\le -\frac{2}{3}\frac{1}{\mathcal{R}^3}\Big|_{\pi n}^{\pi(n+1)}\sim\frac{2\pi}{(\pi n)^4}\ . \tag{4}$$

This bound will be violated by (3) for $\mathcal{R}\sim 1$, i.e., as soon as the distance between the two particles is larger than a number on the order of the Planck length.

The error in Ref. [1] comes in evaluating the series

$$\left[2\mathcal{R}-\frac{(2\mathcal{R})^3}{3\times 3!}+\frac{(2\mathcal{R})^5}{5\times 5!}-\cdots\right]_{\pi n}^{\pi(n+1)}=\mathrm{Si}(2\mathcal{R})\Big|_{\pi n}^{\pi(n+1)}\ , \tag{5}$$

where $\mathrm{Si}(2\mathcal{R})$ is the sine integral

$$\mathrm{Si}(z)=\int_0^z\frac{\sin t}{t}dt\ . \tag{6}$$

This series appears as part of Eq. (B5) of Ref. [1], and by using the approximation $(n+1)^p\approx n^p+pn^{p-1}$ Puthoff effectively replaces the difference $\mathrm{Si}(2\pi(n+1))-\mathrm{Si}(2\pi n)$ by the first term in its Taylor expansion. But such an approximation is only valid up to terms of order n^{-2}, which are of the same order as other terms kept in Eq. (B5).

The expression (5) can be evaluated correctly for large n by using the asymptotic expansion [3]

$$\mathrm{Si}(2\pi n)\sim\frac{\pi}{2}-\frac{1}{2\pi n}\left[1-\frac{2!}{(2\pi n)^2}+\frac{4!}{(2\pi n)^4}-\cdots\right]\ . \tag{7}$$

By inserting this expression into Puthoff's Eq. (B5), one can easily show that the R^{-2} force found in [1] disappears, and that the average net force from the potential (1) is

$$F=-\left[\frac{\omega_c}{c}\right]\left[\frac{3\gamma}{2\mathcal{R}^4}+O(R^{-5})\right], \tag{8}$$

exactly as one would naively expect. The same result can be obtained, although with more effort, by keeping all orders in the binomial expansion of $(n+1)^p$ in Eq. (5).

The magnitude of this force can be estimated from Puthoff's (independently derived) value for the cutoff frequency ω_c. It is not hard to see that

$$F \sim F_{\text{grav}}\left[\frac{L_p}{R}\right]^2, \tag{9}$$

where $L_p \sim 10^{-33}$ cm is the Planck length. The residual SED force is thus completely negligible at measurable distances.

Puthoff has recently suggested that this situation might be salvaged by a rather *ad hoc* lowering of the cutoff frequency ω_c. For

$$\mathcal{R}=\frac{\omega_c R}{c} \ll 1 \tag{10}$$

it is indeed true that the potential U of Eq. (1) approximates a $1/R$ potential. But the required cutoff is difficult to justify. U is approximately Newtonian only at scales small compared to $\lambda_c = c/\omega_c$. To obtain Newtonian gravity in the Solar System, one would require a cutoff on the order of 10^{14} cm. (It should be emphasized that this is a *lower* limit on wave lengths contributing to Puthoff's proposed residual "gravitational" force.) For Newtonian gravity on the scale of galactic clusters, this lower limit would again have to be of the same order or larger, i.e., megaparsecs. Note that the potential U falls off faster than $1/R$ at distances greater than λ_c, and is therefore not relevant to proposals to modify Newtonian gravity to explain galactic rotation curves.

I know of no plausible mechanism that would provide a cutoff of a stochastic electromagnetic background at such a large scale. Moreover, such a cutoff would be inconsistent with Puthoff's model for mass. If gravity is to be interpreted as a residual effect of stochastic electromagnetic fields, one should expect only charged particles to be affected. To explain the mass of particles such as the neutron, Puthoff postulates a substructure of charged "partons." It is easy to see that these cannot simply be the ordinary constituent quarks—mass ratios come out wrong—so a further substructure is required. The basic calculations of Ref. [1] are based on the assumption that frequencies of the stochastic background are high enough that such partons interact as free particles.

But quarks and leptons are observed to be pointlike to scales of 10^{-17} cm [4], so only wavelengths smaller than this should contribute to Puthoff's gravitational interaction. More precisely, no substructure is seen at energies of 1–2 TeV, so if stochastic background radiation is to probe charged constituents of quarks and leptons, its energy must be at least that great. Fluctuations with wavelengths of Solar System size—or atomic size, for that matter—will see neutrons as neutral, and thus "massless," particles. While a more careful analysis might give rise to additional numerical factors, it is hard to see how to bridge the gap between 10^{-17}-cm subatomic scales and 10^{21}-cm astrophysical scales.

Puthoff's proposed method for mass generation presents another problem as well. It is true that charged constituents could lead to an interaction of neutral particles with stochastic electromagnetic background radiation (although not, I am arguing, an inverse square force at large distances). But Eötvös-type experiments have confirmed to a high degree of accuracy that electrostatic energy and nuclear binding energy also contribute to mass [5]. It is not at all easy to see how these observations could be explained in terms of electromagnetic interactions.

It should be stressed that these problems do not invalidate other efforts to generate gravity dynamically from vacuum fluctuations (see, for instance, [6] for a review of computations in ordinary quantum field theory). However, the attempt to derive gravity from stochastic electrodynamics appears to have failed.

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[5] See, for instance, C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973), Chap. 38.
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