Topological phase due to electric dipole moment and magnetic monopole interaction

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We show that there is a dual Aharonov-Casher topological effect [Phys. Rev. Lett. **53**, 319 (1984)] on a neutral particle with electric dipole moment interacting with a magnetic field produced by magnetic monopoles.

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In 1984 Aharonov and Casher (AC) [1] showed that there is a topological effect on a neutral particle with magnetic dipole moment when the particle is moving on a plane under the influence of an electric field produced by a uniformly charged infinitely long filament perpendicular to the plane. When the particle moves around the filament, the wave function will develop a phase equal to $\mu\lambda_e$. Here μ is the magnetic dipole moment of the particle and λ_e is the linear electric-charge density of the filament. In the configuration discussed here, the particle is classically force free. This effect is called the AC effect. Several groups have considered this effect [2, 3]. A more general derivation and the specific conditions for the AC effect are given in Ref. [2]. The AC effect has also been observed experimentally using neutrons [4].

In this Brief Report we remark that should magnetic monopoles exist, the dual of the AC effect would be possible. A neutral particle with an electric dipole moment d will experience a topological effect due to the magnetic field $\mathbf{B}$ produced by magnetic monopoles. The equation of motion for a spin-$\frac{1}{2}$ neutral particle with an electric dipole moment is

$$\left(\partial\!\!\!/ + m + i\frac{d}{2}\sigma_{\mu\nu}\gamma_5 F^{\mu\nu}\right)\psi = 0\ . \tag{1}$$

Assuming that there is only a static magnetic field $\mathbf{B}$, this becomes

$$(\partial\!\!\!/ + m + id\boldsymbol{\sigma}\cdot\mathbf{B}\gamma_5)\psi = 0\ , \tag{2}$$

which can be further rewritten as

$$(\partial\!\!\!/ + m - d\boldsymbol{\gamma}\cdot\mathbf{B}\gamma_4)\psi = 0\ . \tag{3}$$

This equation has the same form as the equation of motion for a neutral particle with a magnetic dipole moment μ moving in a static electric field $\mathbf{E}$. Indeed changing $-d$ to μ and $\mathbf{B}$ to $\mathbf{E}$ we obtain the equation of motion for the later. We immediately anticipate that Eq. (2) implies the existence of a topological phase for the particle wave function when appropriate conditions are satisfied. One of the conditions for topological AC phase in the wave function is that there are regions where $\nabla\cdot\mathbf{E}\neq 0$. In our case one would have to have regions where $\nabla\cdot\mathbf{B}$ is not zero in order to generate a topological phase. But in the Maxwellian electrodynamics the magnetic field $\mathbf{B}$ is always divergence-less, and one would not have any phase. However, if Dirac magnetic monopoles exist, one would have $\nabla\cdot\mathbf{B} = \rho_m$, where ρ_m is magnetic-monopole density. Although magnetic monopoles have not been observed it is interesting to speculate along these lines, and to determine the conditions for developing a topological phase in the wave function.

Our discussion is modeled on that for the AC effect in Ref. [2]. The basic idea is to find a solution of Eq. (1) which can be written as

$$\psi = \exp\left(-id\mathbf{a}\gamma_4\int^{\mathbf{r}}\boldsymbol{\Gamma}\cdot dl\right)\psi'\ , \tag{4}$$

where ψ' is a solution of free Dirac equation of motion. The matrix $\mathbf{a}$ and the field $\boldsymbol{\Gamma}$ are determined by substituting Eq. (1) in Eq. (4). One also must make sure that $\nabla\times\boldsymbol{\Gamma} = 0$ in the region of space where Eq. (4) holds. We find that a solution of the form (4) exists when

$$-d\boldsymbol{\gamma}\cdot\mathbf{B} = i\boldsymbol{\gamma}\cdot\boldsymbol{\Gamma}\mathbf{a};\quad \mathbf{a}\gamma_4\gamma_\mu = \gamma_\mu\mathbf{a}\gamma_4. \tag{5}$$

These equations have consistent solutions only in two or less spatial dimensions. The one-dimensional solution is of no interest to us, so we consider the motion of a particle in a plane. Let the plane be the x-y plane. We find $\mathbf{a} = \sigma_{12}$, $\boldsymbol{\Gamma} = (-B_y, B_x, 0)$ and on this plane $\partial_z B_z = 0$. In the $\nabla\cdot\mathbf{B} = 0$ region, we have

$$\psi = \exp\left(id\sigma_{12}\gamma_4\int^{\mathbf{r}}(\mathbf{B}\times\mathbf{z})\cdot dl\right)\psi'\ , \tag{6}$$

where the $\mathbf{z}$ is the unit vector along the z direction. In this region $\nabla\times\boldsymbol{\Gamma}$ does vanish.

If we let the particle move along a closed path in a plane in the $\nabla\cdot\mathbf{B} = 0$ region which encloses some region where $\nabla\cdot\mathbf{B}$ is not zero, the wave function of the particle will develop a phase

$$\phi = -d\lambda_m\ , \tag{7}$$

where λ_m is the linear magnetic-monopole density. The phase is with respect to ψ'. We have shown that there is an analogous AC effect as stated earlier.

There is a 1.5-standard-deviation discrepancy between the experimental value and the theoretical prediction for the AC effect. $\phi_{\rm expt}/\phi_{\rm theor} = 1.46\pm 0.35$ and $\phi_{\rm expt} = 2.19\pm 0.52$ mrad. While we would not wish to encourage

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the idea that this discrepancy is more than an indication that this difficult experiment should be repeated, we have checked on its implications for the present effect. Taking into account the limit on the electric dipole moment of the neutron ($d < 1.2 \times 10^{-25} e$ cm [5]), the linear magnetic-monopole density would have to be at least $10^{22}/e\,\mathrm{cm}$ to bring theory and experiment into agreement. Needless to say, such large magnetic-monopole densities are ruled out by other experiments [6].

In the literature sometimes the Aharonov-Bohm effect [7] is regarded the dual of the AC effect. We claim that this is not the correct relationship between these effects because the electric charge and magnetic dipole are not related by a dual rotation. In our opinion, none of the topological effects previously discussed in the literature [1, 4, 8] can legitimately be called the dual to the AC effect. The physical quantities in the effect we consider in this paper and the AC effect are related by a dual rotation. In this sense, the effect discussed in this paper is the only situation which should truly be called the dual to the AC effect.

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