Detection of a weak external signal via the switch-on-time statistics of a semiconductor laser

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We calculate the passage-time statistics of a gain-switched single-mode semiconductor laser with a weak, detuned injected signal. Results are given in terms of a relevant scaling parameter. We find that a weak injected signal can be detected by measurements of the mean passage time or its variance whenever its frequency is close to the one of the semiconductor lasers in the on state. In this case, the external signal is resonant with the laser field at the time when amplification becomes possible, and hence it triggers the switch-on more efficiently. The detection bandwidth is of the order of 50 GHz for the case considered.
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I. INTRODUCTION

Following the proposal by Vemuri and Roy [1] it was demonstrated [2] that the switch-on time of a Q-switched class-A laser can be largely reduced when a weak external signal is injected in the laser. This result implies that measurements of the switch-on time provide an interesting technique, with high-resolution bandwidth, for the detection of weak signals. The idea of the technique is closely related to other studies of light amplification aimed to get insight into the nature of fundamental laser noise [3]. More recently it has also been shown that by introducing a frequency shift to the light in each round trip in the cavity, the steady-state output of the laser can be controlled by the strength of a weak injected signal [4]. In this situation the weak signal is detected by a steady-state measurement, while in Refs. [1] and [2] the detection is based on measurements of the transient dynamics.

The transient dynamics considered in Refs. [1] and [2] was characterized by a passage-time (PT) calculation in [5]. The PT is defined as the time at which the laser output power reaches for the first time a reference value after the laser is switched on. It can be identified with the switch-on time. The statistics of the PT in an ensemble of different switch-on events were calculated in [5] by a method [6, 7] which does not rely on a Fokker-Planck equation. This method made it possible to obtain explicit analytical results for the PT statistics for any value of the frequency of the injected signal and to estimate the bandwidth resolution for the detection of the signal. It was also shown in [5] that the variance of the PT distribution was much more sensitive than the mean PT to the presence of the injected signal. An alternative to the PT method in [5] is the calculation of the mean output power of the laser as a function of time [8]. Information on a characteristic switch-on time is obtained when this relation is inverted to obtain time as a function of mean output power. However, this inversion cannot be carried out analytically when the injected signal is not on resonance. In addition, the calculation in Ref. [8] gives no information on the dispersion of values of switch-on times for different switch-on events. Another interesting theoretical study [9] is the analysis of the effect of the phase fluctuations of the injected signal on the switch-on time.

All the above-mentioned studies [1, 2, 5, 8, 9] of switch-on times in the presence of an injected signal consider a class-A laser, which is a laser that can be described by a single equation for the electric field after adiabatic elimination of the atomic variables. In this paper we consider this technique of detection of weak signals for lasers in which such adiabatic elimination is not possible. In this context we present here the calculation of PT statistics for the gain switching of a single-mode semiconductor laser with an injected signal. Semiconductor lasers are known to have a relatively large intrinsic noise level that may make doubtful its suitability for the detection of weak signals. However, it is worth considering this possibility due to their great practical advantages. A further motivation of this calculation is of general fundamental interest and it is the analysis of the role played by the linewidth enhancement factor α of semiconductor lasers [10]. The α factor introduces special features in the frequency dynamics which have not been explored in detail during the transient regime in the presence of an injected signal. We will analyze the combined effect of the α factor and frequency mismatch between the injected signal and oscillation frequency of the laser in the transient dynamics.

We show in this paper that for a single-mode semiconductor laser the mean PT, and to a much larger extent, its variance are considerably reduced by a weak injected signal. These results corroborate previous work for class-A lasers [5]. The main differences with the analysis of class-A lasers appear in the frequency dynamics through the scaling variable which gives the relevant combination of the different parameters in the model: the detection of the weak signal is possible within a bandwidth of several gigahertz centered around the steady-state frequency of the laser. We recall that the optimum detection for a class-A laser [5] does not occur for the case of frequency resonance between the external signal and the steady-state laser field of the solitary laser. In our case, the α dependence of the cw frequency offers a possibility for the determination of the value of α through a PT measurement. In our calculation we will follow the same general ideas as in [5] and [11]. In [11] the PT statistics were calculated in the absence of an external field. Therefore an equation for the laser intensity decoupled from the field phase could be used. In [5] a class-A laser in which the dynamics of the field is decoupled from the dynamics of atomic variables was considered. Here we study the joint effect of the external field and carrier number in the dynamics of the electric field, which implies some crucial modifications of the calculation in [11].

The paper is organized as follows. In Sec. II a general PT calculation based on the rate equations for a semiconductor laser is summarized. The implications of this calculation and a parametric analysis of results is given in Sec. III. Our calculation is compared with the results of numerical simulations. Finally, the Appendix contains some mathematical details of the theoretical analysis.

II. PASSAGE-TIME CALCULATION

Our calculation is based on the usual rate equations for a single-mode semiconductor laser with an extra term describing the influence of the external field. In the system of reference where the laser frequency is zero at transparency, these equations read [12]

$$\dot{E} = \frac{1}{2}\left[-\gamma + \frac{g(N-N_0)}{1+s\mid E\mid^2} + \frac{i\alpha g(N-N_0)}{1+s\mid E\mid^2}\right]E + k_e F_e e^{i\gamma_2 t/2} + (2\beta' N)^{1/2}\xi(t'), \quad (1)$$

$$\dot{N} = -\gamma_e N + C - \frac{g(N-N_0)}{1+s\mid E\mid^2}\mid E\mid^2, \quad (2)$$

where $E_e = F_e e^{i\gamma_2 t/2}$ is the complex external field (whose amplitude and frequency are F_e and $\gamma_2/2$, respectively) with coupling parameter k_e, g gives the gain rate per carrier, γ is the inverse cavity lifetime, γ_e is the carrier recombination rate, $C = \gamma_e \bar{N}_0$ is the injection current, β' is the spontaneous-emission rate per carrier, N_0 is the carrier number at transparency, s is the nonlinear gain saturation parameter, and α is the linewidth enhancement factor [10]. Spontaneous-emission noise is modeled by a complex Gaussian white noise $\xi(t')$ with zero mean and correlation

$$\langle \xi(t')\xi^*(t'')\rangle = 2\delta(t'-t''). \quad (3)$$

The calculation of passage times after the laser is gain switched involves only the first stages of evolution when the laser intensity is small. In addition, the deterministic drift in the carrier number N due to the injection current completely dominates the evolution of N [5,7,11] so that we neglect the saturation of the gain due to the intensity in (1) and the nonlinear coupling term in (2). In addition, we change our frame of reference so that the injected field is real by making

$$E = F e^{i\gamma_2 t/2}. \quad (4)$$

It is convenient to write the resulting equations in the new frame of reference and for dimensionless variables [11]

$$\dot{z} = \frac{\mu}{2\epsilon}\left[(n-1)(1+i\alpha) + i\frac{\bar{\alpha}}{\mu}\right]z + s_e + \sqrt{n\eta}\,\xi(t), \quad (5)$$

$$\dot{n} = \epsilon(\lambda + 1 - n), \quad (6)$$

where we have defined

$$n = \frac{N}{N_{\rm st}}, \quad z = \frac{F}{I_{\rm st}^{1/2}}, \quad t = (\gamma\gamma_e)^{1/2}t'. \quad (7)$$

The normalizing factors

$$I_{\rm st} = \mid F_{\rm st}\mid^2 = \frac{\gamma_e}{\gamma}\frac{g(\bar{N}_0 - N_0) - \gamma}{g}, \quad (8)$$

$$N_{\rm st} = N_{\rm th} = N_0 + \frac{\gamma}{g}, \quad (9)$$

are the steady-state values of the intensity and carrier number for an injection current $C = \gamma_e \bar{N}_0$ above its threshold value $C_{\rm th} = \gamma_e N_{\rm th}$ (when gain saturation is neglected). We have also introduced the dimensionless parameters

$$\epsilon = \left(\frac{\gamma_e}{\gamma}\right)^{1/2}, \quad f_e = \frac{F_e}{I_{\rm st}^{1/2}}, \quad n_0 = \frac{N_0}{N_{\rm st}}, \quad \bar{n}_0 = \frac{\bar{N}_0}{N_{\rm st}},$$
$$\bar{k_e} = \frac{k_e}{(\gamma\gamma_e)^{1/2}}, \quad s_e = \bar{k_e} f_e, \quad \lambda = \bar{n}_0 - 1, \quad \bar{\gamma_2} = \frac{\gamma_2}{\gamma}, \quad (10)$$
$$\eta = \frac{\beta'}{\epsilon^2\lambda(\gamma\gamma_e)^{1/2}}, \quad \mu = \frac{gN_{\rm st}}{\gamma}, \quad \bar{\alpha} = \alpha - \gamma_2/\gamma.$$

Finally, the spontaneous-emission noise $\xi(t)$ has the correlation given by (3).

Equations (5) and (6) are the starting point for our calculation of the PT statistics. It is worth remembering that, due to the approximations involved, these equations are valid only in the initial stages of evolution. In this time interval, the evolution of the carrier number is almost independent of that of the electric field because the laser intensity is very weak. The semiconductor laser is initially prepared in the off state associated with the bias injection current $C_b < C_{\rm th}$. At $t = 0$ the laser is gain switched by changing the injection current to a value $C > C_{\rm th}$ and at the time T the laser reaches for the first time a fixed reference value $|z|^2 = i_r = 0.1$. This time is a random magnitude which depends on the spontaneous-emission events triggering the switch-on. Our strategy

[5–7,11] to calculate the statistical properties of T is to determine $z(t)$ for a given sequence of spontaneous-emission events, that is, a realization of the process $\xi(t)$. Next we invert the relation obtaining $T(z)$, which gives T as a known function of a random variable.

Following [11], we consider that noticeable laser emission cannot occur until the carrier number has crossed its threshold value $n_{\rm th} = 1$, which happens at a time $\bar{t}$ given by

$$\bar{t} = \frac{1}{\epsilon} \ln \left(\frac{1 + \lambda - n(0)}{\lambda} \right) , \tag{11}$$

with $n(0)$ known as a function of the bias current C_b, $n(0) = C_b/(\gamma_e N_{\rm st})$. We therefore integrate Eqs. (5) and (6) with initial conditions $n(\bar{t}) = 1$, $z(\bar{t}) = z_0$. We find

$$n(t) = 1 + \lambda \left[1 - e^{-\epsilon(t - \bar{t})} \right] . \tag{12}$$

Equation (5) can be solved for times later than $\bar{t}$ as

$$z(t) = h(t) e^{A(t)}, \tag{13}$$

where

$$A(t) = \frac{\mu}{2\epsilon} \int_{\bar{t}}^{t} dt' \left\{ [n(t') - 1] (1 + i\alpha) + i \frac{\bar{\alpha}}{\mu} \right\} \tag{14}$$

and

$$h(t) = \int_{\bar{t}}^{t} \left[s_e + \sqrt{n(t')\eta}\, \xi(t') \right] e^{-A(t')}\, dt' + h_0 , \tag{15}$$

with $n(t)$ given by (12).

The function $h(t)$ contains the stochasticity of the process $z(t)$. It plays the role of a time-dependent random initial condition which is exponentially amplified in time. The initial value of the field h_0 is a random variable associated with the small fluctuations around zero field in the initial off-state of the laser. Unless C_b is very close to threshold the effect of h_0 in the statistical properties of the passage time is negligible [13], and in the following we set $h_0 = 0$. The random process $h(t)$ is Gaussian with the mean value and variance given by

$$\langle h(t) \rangle = s_e \int_{\bar{t}}^{t} e^{-A(t')}\, dt', \tag{16}$$

$$\Sigma^2(t) = \langle |\, h(t)\, |^2 \rangle - |\, \langle h(t) \rangle\, |^2 = 4\eta \int_{\bar{t}}^{t} n(t') e^{-2A_1(t')}\, dt', \tag{17}$$

where $A_1(t)$ is the real part of $A(t)$. Equations (16) and (17) are formally identical to the result obtained for a class-A laser [5], but in the present case the slow dynamics of the carrier number determines a completely different scenario. For class-A lasers, $A(t)$ is simply a linear function of time due to the fact that the population inversion reaches its asymptotic value almost immediately [5]. On the contrary, in class-B lasers n evolves much more slowly, the turn-on time occurring before n has reached its asymptotic value. The evolution of n yields a corresponding evolution of the index of refraction and the laser frequency (frequency chirping), hence we have a time-dependent detuning between the injected signal and the laser field. Therefore, the efficient detection of the external signal is due to a transient resonance between the external signal and the laser field, as we discuss later. Analytical expressions for $\langle h(t) \rangle$ and $\Sigma^2(t)$ can be found (see the Appendix), but it is worth noting that due to the external signal, the approximations made in a previous work for the calculation of the PT of a class-B laser in the absence of an injected signal [11] yield completely unphysical results. The reason is that in order to study the role of the detuning between the laser field and the external signal, the growth of the carrier number n cannot be approximated by a linear function, but instead the full exponential growth must be considered. From the Appendix we have that, for times of interest and typical semiconductor laser parameters, Eqs. (16) and (17) are approximately independent of t, yielding

$$\langle h(t) \rangle \approx \frac{s_e}{\epsilon} \sqrt{\frac{2\pi}{z_0}} e^{-i\theta} \left[1 + i \frac{\theta}{z_0} \right]^{z_0 + i\theta - 1/2} , \tag{18}$$

$$\Sigma^2(t) \approx 4\eta \left(\frac{\epsilon}{\mu} + \sqrt{\frac{2\pi}{\lambda\mu}} \right) , \tag{19}$$

where we have defined $z_0 = \mu\lambda(1 + i\alpha)/(2\epsilon^2)$ and $\theta = \bar{\alpha}/(2\epsilon^2)$. As a consequence, we can substitute $h(t)$ by h, a Gaussian variable whose mean value and variance are given by Eqs. (18) and (19), respectively.

The first passage time T is determined by the condition

$$i_r = |\, z(T)\, |^2 = |\, h\, |^2\, e^{2A_1(T)} , \tag{20}$$

which yields T as a function of a Gaussian random variable h,

$$T - \bar{t} = \sqrt{\frac{2}{\lambda\mu} \ln \frac{i_r}{|\, h\, |^2}}. \tag{21}$$

The statistical properties of T are most easily calculated through the generating function $W(\rho)$, which can be determined through the same approximations as in [5] and [11],

$$W(\rho) = \langle e^{-\rho(T - \bar{t})} \rangle \simeq e^{-\rho\left(\frac{2\tau}{\lambda\mu}\right)^{1/2}} e^{-\beta}\, \Gamma \left(\frac{\rho}{(2\tau\lambda\mu)^{1/2}} + 1 \right) \times M \left(\frac{\rho}{(2\tau\lambda\mu)^{1/2}} + 1, 1, \beta \right) , \tag{22}$$

where $M(a, b, z)$ is the confluent hypergeometric function [14,15]

$$\beta = \frac{|\, \langle h \rangle\, |^2}{\Sigma^2} \tag{23}$$

is the natural scaling parameter which appears in the calculation, and

$$\tau = \ln \frac{i_r}{\Sigma^2} \gg 1 . \tag{24}$$

III. RESULTS

The above results for the generating function are similar to those obtained for class-A lasers [5]. The mean and the variance of the PT can be obtained from (22) as in [5], and they are decreasing functions of the relevant scaling parameter β. It turns out that for typical semiconductor laser parameters (as those in the caption of Fig. 1), the mean PT is not very sensitive to β: less than 5% decrease in three decades of variation of β. However, the variance of the PT distribution shows a strong dependence on β, with a 20% reduction for $\beta = 1.9$. The interesting physical differences between class-A and class-B lasers appear in this context through the scaling parameter β. Namely, the dependence of $\langle h \rangle$ on the detuning of the external signal is very different for class-A and class-B lasers, reflecting the slow dynamics of the laser frequency associated with the evolution of n. For class-A lasers the dependence of β on the detuning is simply a Lorentzian with respect to the laser frequency in the "off" state [5]. Instead, for semiconductor lasers we have that

$$\beta = \beta_0 G(\bar{\alpha}), \tag{25}$$

where

$$\beta_0 = \frac{s_e^2}{\Sigma^2} \frac{4\pi}{\mu\lambda\sqrt{1+\alpha^2}}$$

is the value of β when $\bar{\alpha} = 0$ and $G(\bar{\alpha})$ is a function which carries the dependence on the frequency mismatch γ_2. From the Appendix we have that

$$G(\bar{\alpha}) = \left| \left[1 + i \frac{\bar{\alpha}}{\mu\lambda(1+i\alpha)} \right]^{[\mu\lambda(1+i\alpha)+i\bar{\alpha}-\epsilon^2]/2\epsilon^2} \right|^2 . \tag{26}$$

For usual laser parameters, $\bar{\alpha} \ll \mu\lambda$, so

$$G(\bar{\alpha}) \approx \exp\left[\frac{-\bar{\alpha}^2}{\epsilon^2 \mu\lambda(1+\alpha^2)} \right] . \tag{27}$$

Since the mean PT and its variance are monotonously decreasing functions of β [5], their minima as a function of γ_2, which give the optimum frequency mismatch for detection, occur for the values of γ_2 that maximize $G(\bar{\alpha})$. This maximum is for $\bar{\alpha} = 0$, which implies $\gamma_2 = \alpha\gamma$, i.e., the cw frequency of the laser. This result is different from that obtained for a detuned class-A laser, where the aforementioned minima occurred for an external signal resonant with the laser field in the "off" state, whose frequency is not the cw frequency of the laser [15]. For a semiconductor laser, the frequency evolves in time due to the finite relaxation time of the carrier number, but since amplification is not possible until $t = \bar{t}$, when $n = 1$ and the frequency of the laser (except for phase noise) is $2\pi f = \alpha\gamma/2$, we find that the external signal more efficiently triggers the decay of the off state when its frequency is in resonance with that of the laser field at $t = \bar{t}$, i.e., as soon as amplification becomes possible. It happens that, contrary to the case of a class-A laser, this frequency coincides with the cw-operation frequency of the semiconductor laser. In both cases optimum detection occurs for the resonance between the external signal and the laser field at the time when amplification becomes first possible. For class-A lasers the frequency is constant during the linear regime, while for class-B lasers we have a clear example of transient resonance. Finally, it has to be emphasized that a calculation based on the common linear approximation for the evolution in time of n, as that used previously in [11], yields the completely unphysical result of optimum detection for infinite detuning of the injected signal. The reason is that, in this case, n never reaches an asymptotic steady state, so $A(t)$ diverges for long times.

The frequency selectivity for detection can be determined by defining the detection bandwidth $\Delta\bar{\gamma}$ as the value of $\bar{\alpha}$ that reduces $G(\bar{\alpha})$ to 1/2. In this way we obtain

$$\Delta\bar{\gamma}_2 = \sqrt{0.69\epsilon^2 \mu\lambda(1+\alpha^2)} , \tag{28}$$

which for the parameters considered yields a detection bandwidth of $\sim$ 50-GHz full width at half maximum (FWHM). Instead, an operational definition of the detection bandwidth as the value of γ_2 that produces a 50%

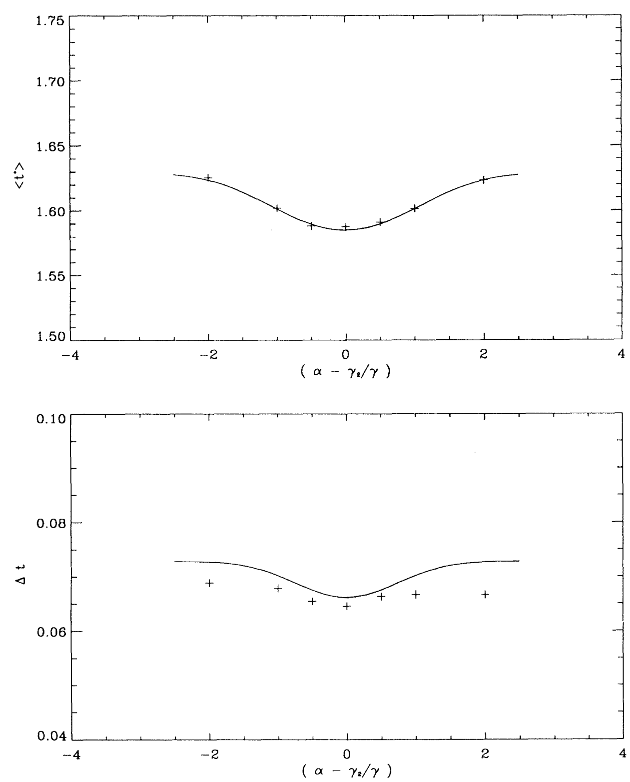

FIG. 1. Upper pannel: Comparison between the theoretical results for the mean PT (solid lines) and the result obtained from the numerical integration (symbols). Lower pannel: Same as before, but for the standard deviation of the PT. Laser parameters used in the numerical integration are the following: $g = 5.6 \times 10^4\ \mathrm{s}^{-1}$, $\gamma = 4 \times 10^{11}\mathrm{s}^{-1}$, $\gamma_e = 5 \times 10^8\mathrm{s}^{-1}$, $C = 14 \times 10^{16}\mathrm{s}^{-1}$, $\beta' = 1.1 \times 10^4\mathrm{s}^{-1}$, and $N_0 = 6.8 \times 10^7$. For the bias value of the injection current C_b we take $C_b = 0.905\ C_{\mathrm{th}} = 3.4 \times 10^{16}\mathrm{s}^{-1}$, $\alpha = 5$, and $s_e^2 = 5 \times 10^{-3}$.

reduction in mean PT or variance yields different results according to the intensity of the external field. However, both definitions coincide for weak external signals.

Finally, it should be noted that a variation of α in ± 1 implies a variation of the optimum frequency in $\sim$ 60 GHz, which is larger than the detection bandwidth. Since the linewidth enhancement factor α is not generally known with better accuracy that ± 1 [16], Eq. (38) gives a possible alternative to better determinations of the α factor [17].

In order to provide evidence of the suitability of the approximations used to obtain our results above, we have performed a numerical integration of Eqs. (5) and (6). We have performed averages over 5000 trajectories for several values of γ_2, and typical values for the different semiconductor laser parameters. The results are shown in Fig. 1. It can be seen that the numerical values of the mean PT [Fig. 1(a)] are in good agreement with the theoretical result, with less than a 1% difference. The results for the standard deviations [Fig. 1(b)] show larger differences (less than 10% in all cases). These differences are mainly due to the fact that the PT distribution has a slowly decaying tail which is not properly represented with our accuracy. A clear signature of this fact is that the discrepancies between theory and numerical integration increase for increasing $|\bar{\alpha}|$, because the larger the $|\bar{\alpha}|$ the larger the PT variance.

In summary, we have seen that measurements of the mean PT or better its variance in the gain switching of a single-mode semiconductor laser can detect a weak injected signal whose frequency lies in a bandwidth of ≈ 50 GHz around the cw frequency of the semiconductor laser. The technique is rather sensitive to the value of the α factor and it could provide us with an alternative method to determine α.

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APPENDIX

The statistical properties of $h(t)$ can be calculated from Eqs. (15) and (16), with $n(t)$ given by (12). For the mean value we have

$$\begin{aligned}\langle h(t)\rangle &= s_e \int_0^{t-\bar{t}} ds\ \exp\left\{\frac{-\mu}{2\epsilon}\left[(1+i\alpha)\lambda\left(s-\frac{1-e^{-\epsilon s}}{\epsilon}\right)+i\frac{\bar{\alpha}}{\mu}s\right]\right\} \\ &= \frac{s_e}{\epsilon}\frac{e^{z_0}}{{z_0}^{z_0+i\theta}}\left\{\gamma(z_0+i\theta,z_0)-\gamma\left[z_0+i\theta,z_0\ e^{-\epsilon(t-\bar{t})}\right]\right\}\end{aligned} \tag{A1}$$

where we have defined $z_0 = \mu\lambda(1+i\alpha)/(2\epsilon^2)$ and $\theta = \bar{\alpha}/(2\epsilon^2)$, and $\gamma(a,z)$ denotes the incomplete γ function of a up to z [14]. In order to make further progress, we expand the incomplete γ functions in a power series [18], yielding

$$\begin{aligned}&\gamma(z_0+i\theta,z_0)-\gamma\left(z_0+i\theta,z_0\ e^{-\epsilon(t-\bar{t})}\right) \\ &\quad = {z_0}^{z_0+i\theta}\sum_{n=0}^{\infty}\frac{(-z_0)^n}{n!}\frac{1-\left[e^{-\epsilon(t-\bar{t})}\right]^{z_0+i\theta+n}}{z_0+i\theta+n}.\end{aligned} \tag{A2}$$

For usual semiconductor laser parameters, $\mathrm{Re}(z_0+i\theta+n)\gg 1$, and then we can neglect the second incomplete γ function in front of the first one. Moreover, since $|z_0|\gg 1$, we approximate [18]

$$\begin{aligned}\gamma(z_0+i\theta,z_0) &\approx \Gamma(z_0+i\theta) \\ &\sim \sqrt{2\pi}e^{-(z_0+i\theta)}(z_0+i\theta)^{z_0+i\theta-1/2},\end{aligned}$$

so we finally obtain

$$\langle h(t)\rangle \approx \frac{s_e}{\epsilon}\sqrt{\frac{2\pi}{z_0}}e^{-i\theta}\left[1+i\frac{\theta}{z_0}\right]^{z_0+i\theta-1/2}. \tag{A3}$$

It must be stressed that the linear approximation to (12) that has been previously used [11] to calculate the PT statistics of a semiconductor laser in the absence of injected signal yields completely wrong results in the present case.

The variance of $h(t)$ can be calculated in a similar way. From (17) we have that

$$\begin{aligned}\Sigma^2(t) &= 4\eta\int_{\bar{t}}^{t} ds\ n(s)\ e^{-2A_1(s)} \\ &= 4\eta\int_{\bar{t}}^{t} ds\left[\frac{2\epsilon}{\mu}\frac{dA_1(s)}{ds}+1\right]e^{-2A_1(s)} \\ &= 4\eta\left\{\frac{\epsilon}{\mu}\left[1-e^{-2A_1(t)}\right]\right. \\ &\quad \left.+\frac{e^a}{\epsilon a^a}\left[\gamma(a,a)-\gamma(a,a\ e^{-\epsilon(t-\bar{t})})\right]\right\},\end{aligned} \tag{A4}$$

where $a=\frac{\mu\lambda}{\epsilon^2}\gg 1$. By exactly the same reasons as in the former case, we finally obtain

$$\Sigma^2(t)\approx 4\eta\left(\frac{\epsilon}{\mu}+\sqrt{\frac{2\pi}{\lambda\mu}}\right). \tag{A5}$$

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