Perfect correlations of three-particle entangled states in cavity QED

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Compared to two-particle states, entangled states of three or four particles can lead to a much stronger test of local realism than is possible with Bell inequalities. It is the purpose of this paper to mention an exactly soluble quantum model in which entanglement of atomic states with cavity photon states leads to a perfectly correlated three-particle state. We demonstrate how to realize this model atom-field system and to create and observe such states, in the context of cavity quantum electrodynamics.

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The high-efficiency detection of optical photons has allowed in the past decade various experiments in which photon coherence functions of higher order have been carefully measured. In these experiments photon antibunching [1], the violation of Bell's inequalities [2], intensity interference effects [3], and the nonlocal character of correlations of parametrically pumped photons in optical down conversion have been observed [4].

Laboratory studies of atoms in cavities have provided additional fundamental insights into the properties of the electromagnetic interaction with atoms. Cavity quantum electrodynamics (CQED) provides a modern context in which to explore (both theoretically and experimentally) many purely quantum properties of the electromagnetic field [5] involving only a very low number of quanta. Due to the relatively loss-free character of high-Q optical and microwave cavities the atom-photon interactions can have anomalously large coupling strengths and time scales compared to standard QED phenomena occurring in "free" space [6].

Various quantum correlations can be tested experimentally by probing the atomic states of the atoms leaving the cavity, or by a measurement of the cavity photons. Correlations of the cavity photons with an atom in the cavity can reveal fundamental quantum-mechanical correlations typical for entangled systems. The entanglement of states, so fundamental in the Einstein-Podolsky-Rosen (EPR) correlations [7], has been tested experimentally in various optical experiments. Violations of Bell's inequalities have been reported in different cases. It has been shown that the local reality assumption implicit in Bell's inequalities is violated in experiments involving atomic photon cascades [2], or in measurements of polarization in down-conversion processes [4]. These tests of Bell's inequalities required measurements of photon correlations for several orientations (four orientations usually) of the polarizers.

Recently it has been shown [8] by Greenberger, Horne, and Zeilinger (GHZ) that special entangled states involving three or four particles lead to a much stronger refutation of local realism. In such many-particle correlations only a *single* set of observations is required in order to demolish the local-reality assumption. One particularly simple entangled GHZ state of three spin-$\frac{1}{2}$ particles, as discussed by Mermin [9], has the following form:

$$|\psi\rangle=\frac{1}{\sqrt{2}}(|+,+,+\rangle-|-,-,-\rangle)\ , \tag{1}$$

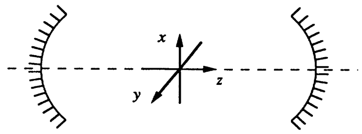
where $|+\rangle$ or $|-\rangle$ specifies spin up or down along the appropriate z axis. This entangled state provides an "always" versus "never" test of local realism. A realistic experimental arrangement permitting a three-particle test would be desirable, but straightforward generalizations of two-particle atomic interferometry or photon pairs emitted in a cascade suffer fairly obvious drawbacks. For example, a three-photon $J=0\rightarrow J=0$ cascade cannot satisfy dipole selection rules. Only a few specific schemes for the generation of GHZ states have been proposed so far [8–10].

It is the purpose of this paper to mention an exactly soluble model of CQED in which entanglement of atomic states with cavity photon states leads to the state given by Eq. (1). In the framework of this CQED model a conceptually straightforward test can be designed to measure three-particle GHZ correlations.

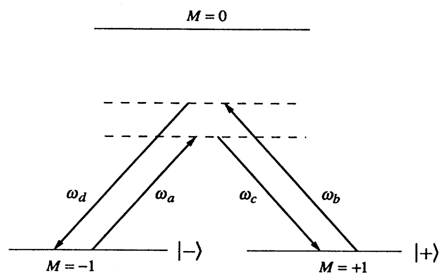
The model we are considering consists of one atom and four radiation modes [11]. The radiation modes are associated with the two transverse-polarization states of each of two longitudinal modes of a cavity, as sketched in Fig. 1(a). The modes are pairwise degenerate in frequency: $\omega_a=\omega_c$ and $\omega_b=\omega_d$. The atom has a $J=1$ ground state and the photons in the cavity modes can induce transitions between the $M=-1$ and $+1$ sublevels of the ground state, via circularly polarized virtual transitions to a far-off-resonant upper $M=0$ level. This is shown in Fig. 1(b). If the $M=0$ ground sublevel is initially unoccupied it will not be active at any later time because only fields polarized along z could cause $M=0$ to $M=0$ transitions. Thus even though there are three sublevels in the $J=1$ state, only two of them are participants in the interaction.

The effective interaction Hamiltonian of such transitions has the following form ($\hbar=1$):

$$H_{\rm int}=\lambda(a_a a_c^\dagger+a_b^\dagger a_d)\sigma^\dagger+{\rm H.c.}\ , \tag{2}$$



(a)



(b)

FIG. 1. (a) The cavity and coordinate system. (b) Transition diagram of our CQED model.

where we have assumed that the coupling constant λ is the same for the two channels involved in the photon transitions [12]. As usual we have denoted by σ and $\sigma^\dagger$ the atomic lowering and raising operators and by a_i and $a_i^\dagger$ $(i=a,b,c,d)$ the four boson annihilation and creation operators of the cavity modes.

The free Hamiltonian of the model is

$$H_0=\sum_i \omega_i a_i^\dagger a_i + E_-\sigma_{--}+E_+\sigma_{++} \ . \tag{3}$$

We adopt the simplest situation and take $E_+=E_-$, in which case the last two terms of H_0 are the unit operator times E_- and can be ignored. The unperturbed states of the free Hamiltonian shall be denoted by $|n_a,n_b,n_c,n_d;\pm\rangle$, where the n_i denote photon numbers and $\pm$ are the atomic indices. We also have assumed that the cavity modes support the resonant-frequency relations $\omega_d-\omega_b=\omega_a-\omega_c=E_+-E_-=0$.

In order to make contact with the spin-$\frac{1}{2}$ EPR or GHZ correlations we shall reformulate the boson interactions in terms of spin variables [13]. The four independent cavity modes can be used as independent boson components, in Schwinger's representation [14], to give the angular-momentum operators $\mathbf{L}_1$ and $\mathbf{L}_2$ as follows:

$$L_{1+}=a_a^\dagger a_c \ , \quad L_{1-}=a_c^\dagger a_a \ , \quad L_{1z}=\tfrac{1}{2}(n_a-n_c) \ , \tag{4a}$$

$$L_{2+}=a_d^\dagger a_b \ , \quad L_{2-}=a_b^\dagger a_d \ , \quad L_{2z}=\tfrac{1}{2}(n_d-n_{b)} \ . \tag{4b}$$

Note that $\mathbf{L}_1$ involves the right-polarized ω_a photons and the left-polarized ω_c photons, while $\mathbf{L}_2$ involves the left-polarized ω_b photons and the right-polarized ω_d photons. Because all these modes are independent of each other the angular-momentum operators $\mathbf{L}_1$ and $\mathbf{L}_2$ commute:

$$[\mathbf{L}_1,\mathbf{L}_2]=0 \ . \tag{5}$$

The eigenvalues of $\mathbf{L}_1^2$ and $\mathbf{L}_2^2$ are $l_1(l_1+1)$ and $l_2(l_2+1)$ where $l_1=(n_a+n_c)/2$ and $l_2=(n_b+n_d)/2$. The $\mathbf{L}_1$ and $\mathbf{L}_2$ systems are spin-$\frac{1}{2}$ "particles" if $n_a+n_c=1$ and $n_b+n_d=1$. The third angular momentum involved in our CQED interaction arises naturally from the atomic transition operators. It is well known that this angular momentum is equivalent to a spin $\frac{1}{2}$:

$$S_+=\sigma^\dagger \ , \quad S_-=\sigma \ , \quad S_z=\tfrac{1}{2}\sigma_z \ . \tag{6}$$

The CQED Hamiltonian (2) expressed in angular-momentum variables describes the interaction of a fictitious spin $\frac{1}{2}$ (the atom) and a system with the total angular momentum $\mathbf{L}=\mathbf{L}_1+\mathbf{L}_2$ (combinations of field modes), and has the form

$$H_{\text{int}}=\lambda(L_-\sigma^\dagger+L_+\sigma) \ . \tag{7}$$

Now we demonstrate how to apply this atom-field system to create and observe spin-$\frac{1}{2}$ GHZ states. Consider spin correlations involving, for example, three independent z components. We will be most interested in the expectation value $\langle L_{1z}L_{2z}S_z\rangle$. This expectation value can be understood as a combination of *four* correlated measurements of two photon-number operators with the atomic inversion operator. From the definitions (4) we obtain that

$$\langle L_{1z}L_{2z}S_z\rangle=\tfrac{1}{8}\langle n_an_d\sigma_z-n_an_b\sigma_z-n_cn_d\sigma_z+n_cn_b\sigma_z\rangle \ . \tag{8}$$

In CQED the photon-number operators can exhibit strong correlations with the atomic state due to the entanglement of the photons with the atom in the cavity. We shall discuss the generation of such highly correlated states in the framework of our CQED model.

The Hamiltonian of our model is fully soluble, i.e., all energy eigenvectors and eigenvalues can be obtained exactly [15]. But for the purpose of this presentation we shall confine our attention only to the lowest nontrivial "sector" of the Hilbert space of the system. This is because we are interested in the realization of the GHZ state in the framework of this CQED model. From the definitions (4) it is clear that the $\mathbf{L}_1$ space that corresponds to $l_1=\frac{1}{2}$ is spanned by states which involve only $(1_a,0_c)$ and $(0_a,1_c)$ ω_a and ω_c photons. For the same reasons the $\mathbf{L}_2$ space that corresponds to $l_2=\frac{1}{2}$ is spanned only by $(1_b,0_d)$ and $(0_b,1_d)$ ω_b and ω_d photons. These CQED states can be denoted by $|m_1,m_2,m_s\rangle$, where the magnetic numbers correspond to spin up or spin down of the angular momenta $\mathbf{L}_1$, $\mathbf{L}_2$, and $\mathbf{S}$. The relation to the bare field-atom states is straightforward; for example, $|+,+,+\rangle=|1_1,0_b,0_c,1_d;+\rangle$.

It is not enough to say that three spin-$\frac{1}{2}$ particles exist. The difficult task is to show how the three particles interact in a physically realizable way and in a way that permits GHZ "always-never" correlations to be observed. This is what we do next, as indicated by Fig. 2.

First we prepare an initial cavity state by depositing one photon in the ω_a mode and one photon in the ω_d mode. This can be achieved, for example, by passing an atom which can undergo a two-photon cascade spontane-

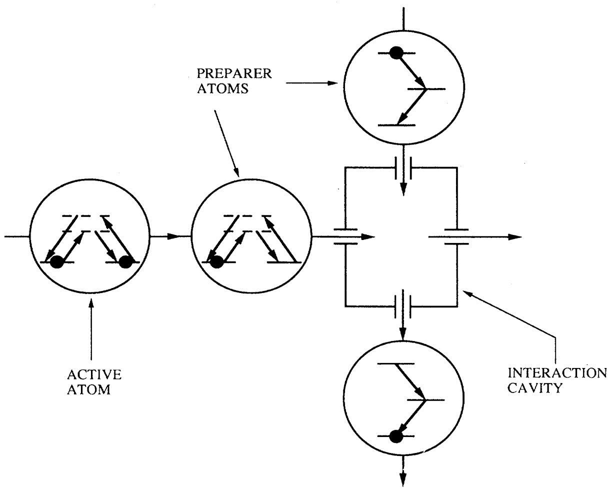


FIG. 2. Atoms used in the preparation of the GHZ state. First, an atom deposits two photons in the cavity with a two-photon cascade spontaneous emission. A preparer atom interacts with the field to give the state described by Eq. (10). Finally, the active atom in a dipole coherent state is passed through the cavity and we have the desired entangled state.

ous emission through the cavity. We can monitor the state of the atom coming out of the cavity to make sure we have the desired two-photon state [16]. Then, an active atom prepared initially in the state $|\text{atom}\rangle = |-\rangle$ is injected into the cavity. As a result of the injected atom, the initial state in the cavity is no longer stable and will evolve in time. As a result of the interaction given by the Hamiltonian (2) the only states $|\text{atom}\rangle \otimes |\text{field}\rangle$ that are dynamically accessible in the cavity form the following chain:

$$|1_a,1_b,0_c,0_d;+\rangle \leftrightarrow |1_a,0_b,0_c,1_d;-\rangle \leftrightarrow |0_a,0_b,1_c,1_d;+\rangle \ , \tag{9a}$$

which, in our spin-$\frac{1}{2}$ notation, is the same as

$$|+,-,+\rangle \leftrightarrow |+,+,-\rangle \leftrightarrow |-,+,+\rangle \ . \tag{9b}$$

This chain of states spans a closed sector in the Hilbert space. It is easy to diagonalize the interaction Hamiltonian in this sector and as a result we obtain the following eigenvalues: $\sqrt{2}\lambda$, 0, and $-\sqrt{2}\lambda$. Due to the interaction only states from the chain (9) will occur in the cavity. These states will oscillate with this "vacuum" Rabi frequency. One can show that after a time $\sqrt{2}\lambda t = \pi/2$ the initial state $|+,+,-\rangle$ in the chain will become

$$\begin{aligned}|\text{field}\rangle \otimes |+\rangle &= -\frac{i}{\sqrt{2}}(|+,-\rangle + |-,+\rangle) \otimes |+\rangle \\ &= -\frac{i}{\sqrt{2}}(|1_a,1_b,0_c,0_d\rangle \\ &\qquad + |0_a,0_b,1_c,1_d\rangle) \otimes |+\rangle \ .\end{aligned} \tag{10}$$

Thus, as a result of the atomic pumping, the field in the cavity is in a superposition state of one photon in each of the ω_a and ω_b modes and one photon in each of the ω_c and ω_d modes.

We suppose that the evolution time $t = \pi/2\sqrt{2}\lambda$ is also the atomic transit time through the cavity. After the atom leaves the cavity, the cavity field in (10) cannot evolve further. Note that the state of the emerging atom can be monitored to verify the field state. A second atom is prepared (e.g., by passage through a Ramsey zone) in a dipole coherent state:

$$|\text{atom}\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle) \tag{11}$$

and then this atom passes through the cavity. As a result of the interaction of the second atom with the prepared state of the field, the chain given by Eq. (9) is supplemented by an additional chain of states given by

$$|1_a,1_b,0_c,0_d;-\rangle \leftrightarrow |0_a,1_b,1_c,0_d;+\rangle \leftrightarrow |0_a,0_b,1_c,1_d;-\rangle \tag{12a}$$

and this is equivalent to

$$|+,-,-\rangle \leftrightarrow |-,-,+\rangle \leftrightarrow |-,+,-\rangle \ . \tag{12b}$$

The state of the field in the cavity evolves during the passage of the second atom. The atomic state evolves as well, and after the atomic transition time the combined $|\text{atom}\rangle \otimes |\text{field}\rangle$ state has become

$$|\text{final}\rangle = \frac{1}{\sqrt{2}}(|+,+,-\rangle - |-,-,+\rangle) \ . \tag{13}$$

The $|\text{final}\rangle$ state is therefore an entangled state and is in fact just the desired GHZ state (1), with a change of the atomic label from $|+\rangle$ to $|-\rangle$. We see that in this CQED scheme we can achieve a fully dynamical generation of perfect GHZ states.

We can perform photon-number measurements involving the following correlations: $[a,d,A]$, $[a,b,A]$, $[c,d,A]$, and $[c,b,A]$, where A refers to the atom. These photon-number measurements correlated with the state of the atom in the cavity provide a three-particle spin measurement of the z components in the GHZ state. However, this is not the full story because to detect GHZ correlations we are supposed to measure correlations of the spin components which correspond to the following expectation values:

$$E(x,y,y) \equiv \langle L_{1x} L_{2y} S_y \rangle \ , \tag{14a}$$

$$E(y,x,y) \equiv \langle L_{1y} L_{2x} S_y \rangle \ , \tag{14b}$$

$$E(y,y,x) \equiv \langle L_{1y} L_{2y} S_x \rangle \ , \tag{14c}$$

and

$$E(x,x,x) \equiv \langle L_{1x} L_{2x} S_x \rangle \ . \tag{15}$$

The perfect correlations of the GHZ state lead to

$$E(x,y,y) = E(y,x,y) = E(y,y,x) = +1 \ , \tag{16}$$

while perfect correlations involving only the x axes lead to

$$E(x,x,x) = -1 \ . \tag{17}$$

These multidirectional correlation functions can be obtained from the original correlation function (8) by a definite procedure.

In this second stage of our experiment (see Fig. 3) we first separate the ω_a and ω_c photons from the ω_b and ω_d photons emerging from the cavity by a proper set of filters and polarizes, as sketched in Fig. 3. We then start the measurement of the angular-momentum components $\mathbf{L}_1\cdot\mathbf{n}(\phi_1)$ and $\mathbf{L}_2\cdot\mathbf{n}(\phi_2)$ for arbitrary unit vectors (orientation angles) ϕ_1 and ϕ_2. These orientation angles can be defined in an abstract x-y plane in the following way. We let the ω_a and the ω_c photons fall on a detector. At the detector the positive-frequency part of the electric field can be expressed by the following formula:

$$a(\phi_1)=\frac{1}{\sqrt{2}}(a_a+a_c e^{-i\phi_1}) , \tag{18}$$

where the device denoted ϕ_1 in Fig. 3 is designed to supply a path delay and the proper rotation of polarization to permit the interference of the ω_a and ω_c modes. The prefactor has been selected in order to preserve the commutation relation $[a(\phi_1), a^\dagger(\phi_1)]=1$. At the first detector D_1 the field-intensity operator is given by the following formula:

$$\hat{I}(\phi_1)=a^\dagger(\phi_1)a(\phi_1)$$
$$=\tfrac{1}{2}(n_a+n_c+a_a^\dagger a_c e^{-i\phi_1}+a_c^\dagger a_a e^{i\phi_1}) , \tag{19}$$

i.e., an interference of the ω_a photons with the ω_c photons takes place [17].

Using the angular-momentum definitions we can rewrite this formula in the following form:

$$\hat{I}(\phi_1)=\tfrac{1}{2}(n_a+n_c)+\mathbf{L}_1\cdot\mathbf{n}(\phi_1) . \tag{20}$$

This formula relates the intensity at detector D_1 (see Fig. 3) with the angular momentum $\mathbf{L}_1$ projected on a unit direction $\mathbf{n}(\phi_1)=(\cos\phi_1,\sin\phi_1,0)$ in the x-y plane. The measurement of the angular momentum $\mathbf{L}_2$ can be achieved in the same way. The corresponding electric-field operator is

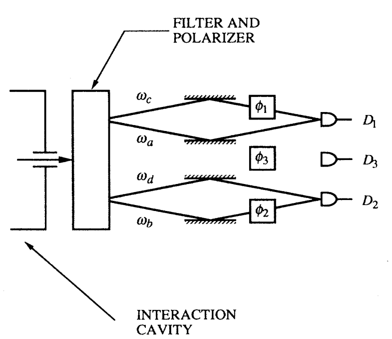


FIG. 3. Schematic description of the three-particle correlations involving detectors D_1, D_2, and D_3. With filter and polarizer, the ω_a and ω_c modes are mixed with an angle ϕ_1 and detected by the detector D_1. The ω_b and the ω_d modes are mixed with an angle ϕ_2 and detected by the detector D_2. The atomic state is rotated through angle ϕ_3 and detected by detector D_3. Joint clicks at the detectors D_1, D_2, and D_3 reveal the perfect correlations of the GHZ states.

$$a(\phi_2)=\frac{1}{\sqrt{2}}(a_d+a_b e^{-i\phi_2}) \tag{21}$$

and at detector D_2 the field-intensity operator is related to the $\mathbf{L}_2$ projection through the similar formula

$$\hat{I}(\phi_2)=\tfrac{1}{2}(n_d+n_b)+\mathbf{L}_2\cdot\mathbf{n}(\phi_2) . \tag{22}$$

These results show that by interference of the outgoing photons from the cavity modes, it is possible to achieve a measurement of the fictitious angular momenta $\mathbf{L}_1$ and $\mathbf{L}_2$ which is completely analogous to a standard spin projection measurement using two analyzers with corresponding directions $\mathbf{n}(\phi_1)$ and $\mathbf{n}(\phi_2)$.

We are left with the problem of the "rotation" of the atomic spin S_z, and this can be achieved by a rotation of the atomic state on its Bloch sphere. The beam is subjected to a $\pi/2$ pulse, and to a rotation by an angle ϕ_3 in the abstract x-y plane (abstract Bloch sphere of the atomic two-level system). As a result of this rotation the excited state of the atom becomes $|+\rangle_{\phi_3}=(1/\sqrt{2})(|+\rangle+e^{i\phi_3}|-\rangle)$ and the ground state becomes $|-\rangle_{\phi_3}=(1/\sqrt{2})(-e^{-i\phi_3}|+\rangle+|-\rangle)$. A procedure that leads to the generation of such states is known in the framework of two-level coherent transients [18].

These rotations of the atomic states on the Bloch sphere are of course equivalent to rotations of the S_z operator by the spherical angles $(\theta=\pi/2,\phi_3)$ and as a result of this procedure we obtain the following atomic observable:

$$S(\phi_3)=S_x\cos\phi_3+S_y\sin\phi_3=\mathbf{S}\cdot\mathbf{n}(\phi_3) . \tag{23}$$

Combining this and the previous results we obtain the CQED photon-atom correlations in the following form:

$$E(\phi_1,\phi_2,\phi_3)=\langle L_1(\phi_1)L_2(\phi_2)S(\phi_3)\rangle , \tag{24}$$

where the expectation value is calculated in the GHZ state. In this correlation function the angles ϕ_1 and ϕ_2 are related to the interference pattern of the cavity photons while the angle ϕ_3 corresponds to the rotation of the atomic population on the Bloch sphere.

Expression (24) is the central result of this paper, because it shows that it is possible to obtain a one-to-one correspondence between the three-particle spin-$\frac{1}{2}$ GHZ correlations and the CQED dynamics. The GHZ state of the CQED system is an entangled state of the cavity photons and the atom. Spin correlations given by Eq. (24) correspond to measurements of the photon interference pattern correlated with the state of the atom in the cavity.

As the last issue of this paper we shall address the problem of what actually has to be observed and measured in order to claim a complete refutation of local realism in the GHZ correlations. Let us notice that the chain states (9) have $n_a+n_c=n_b+n_d=1$ and as a result the intensity operators given by Eqs. (20) and (22) are equivalent to spin-$\frac{1}{2}$ projection operators, i.e, we have $I(\phi_i)=I^2(\phi_i)$, $i=1,2$. The measurements of atomic populations are related to measurements of the spin-$\frac{1}{2}$ projector $P(\phi_3)=\frac{1}{2}[1+\boldsymbol{\sigma}\cdot\mathbf{n}(\phi_3)]$. As a result of this identification the correlation function

$$p(\phi_1,\phi_2,\phi_3)=\langle I(\phi_1)I(\phi_2)P(\phi_3)\rangle \tag{25}$$

gives the *joint* probability for a detection involving two interference patterns characterized by ϕ_1 and ϕ_2 and the atomic spin orientation characterized by ϕ_3. Perfect GHZ correlations lead for $\phi_1+\phi_2+\phi_3=\pi$ to $p=\frac{1}{4}$, while for $\phi_1+\phi_2+\phi_3=0$ the joint probability is equal to zero. This means that for this particular orientation in the x-y plane no joint detection of photons in the interference patterns correlated with the atomic state is possible. This is the essence of the "never" versus "always" refutation of local realities in the GHZ argument.

In summary, we have proposed an experiment in which to observe particularly interesting entangled three-particle (GHZ) states. We used an atom-cavity interaction process that makes use of a two-channel combination of lambda transitions between degenerate sublevels of the ground state of the atom and it involves only conventional dipole-allowed two-photon physics. Proper attention to cavity symmetry and dipole selection rules is sufficient to eliminate, in principle, interfering transitions. By detecting appropriately selected mode pairs, making use of the known cavity-atom dynamical evolution, and choosing conveniently the atomic transition time through the cavity, we have obtained exactly the desired GHZ spin-$\frac{1}{2}$ correlations. We note that our proposed experiment not only suggests a method for realization of these so-far unobserved states, but does so in a distinctly unusual way within the framework of previous tests of the violation of local realism, i.e., by combining five completely distinct physical systems (atom and four field modes) to make the three "spins."

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