Electron scattering by a potential in the presence of a strong single-mode radiation field

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We describe the collisions of an electron with a target atom in the presence of a strong, linearly polarized, monochromatic, oscillating electric field in the Kramers-Henneberger gauge. We show that the resulting set of coupled-channel equations, originally derived by Gavrila and Kaminski [Phys. Rev. Lett. **52**, 613 (1984)], represents electron scattering by a vibrating charge initially in a highly excited state. This formulation leads to a mechanical model of the atom-field interaction from which we can draw better insights into the nature of the process. We solve these coupled equations numerically in the close-coupling approximation by a combination of the linear algebraic and R-matrix propagator approaches. We specifically treat electron-proton collisions in an intense field and investigate the capture-escape resonances over a range of intensities and frequencies. We also investigate multiphoton ionization of atomic hydrogen in various intensity regimes. We extend our calculations for single-photon ionization into the superintense regime ($> 10^{16}$ W/cm^2) and compare our results with results from time-dependent solutions of the Schrödinger equation.

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I. INTRODUCTION

With the development of high-power laser systems, the behavior of atomic systems in strong electromagnetic fields has become a subject of great interest [1]. Such lasers rapidly ionize atoms, usually by a multiphoton absorption process, with the ejected electrons moving in the combined field of the ions and of the laser. Besides multiphoton ionization, electron scattering by atoms and ions in the presence of an intense electromagnetic field has also attracted considerable scientific interest. Simple approximations for electron scattering were introduced early by Kroll and Watson [2] with subsequent developments bringing a myriad of approximations, mainly on the formal level [1]. In a 1984 paper, Gavrila and Kaminski [3] shed considerable physical insight on the field by using the Kramers-Henneberger (KH) transformation [4] along with a classical representation of the electromagnetic field. By this transformation, they reduced the problem to the motion of an electron in a time-dependent potential. In their approach, the classical motion of the electron in the electromagnetic fields is first calculated, then the original potential is described from the "point of view" of the electron, in an accelerating frame. In this manner the motion of the electron is transferred to that of the center of the potential, thus giving rise to a time-dependent force field. Interestingly, the Schrödinger equation holds in this accelerating frame in the same form as in the usual inertial frame [4]. After introducing the Floquet ansatz [1] and performing a Fourier expansion of the time-dependent wave function of the electron, they [3] obtained a system of coupled partial differential equations for the component functions. Gavrila and Kaminski also showed that in the high-frequency and high-intensity limit, this system essentially reduces to a single-channel equation for the zeroth Fourier component function along with perturbative expressions for the other terms in which the coupling potential plays the role of a transition operator. The potential entering the zeroth Fourier component function can easily be interpreted as the time-averaged potential of the oscillating center of force. This, of course, physically means that in the high-frequency limit the scattering electron "sees" this average field. The Gavrila-Kaminski approach [5] has yielded considerable physical insight; however, many aspects of the problem still remained obscure.

The purpose of this paper is to provide an approach to the complete physical interpretation of the Gavrila-Kaminski theory of electron scattering by a potential in the presence of a strong single-mode laser field and give some numerical results for scattering by a Coulomb potential and for multiphoton ionization of hydrogen. We shall trace the physical ideas incorporated into the Gavrila-Kaminski method back to the papers of Bloch and Nordsieck [6], Nordsieck [7], and Pauli and Fierz [8]. These latter authors discussed the problem of infrared radiation accompanying electron scattering by a potential and used the quantized description of the electromagnetic field. Furthermore, they used oscillator wave functions with spatial coordinates for the description of the quantized states of the electromagnetic field. For further reference, we shall refer to these quantized states of the electromagnetic field as quantum-mechanical states of "ethereal oscillators" [9]. Their approach represented a mechanical model for the radiation field and the introduction of approximation schemes "quite analogous to what is done in the theory of molecules" [7]. In this article, first we shall apply the Bloch-Nordsieck method to electron scattering by a potential in the presence of a strong, linearly polarized single-mode laser field. We shall introduce the "quantized-field" version of the Kramers-Henneberger transformation, following Pauli and Fierz [8], which, in turn, was based on an earlier transformation of Bloch and Nordsieck [6]. Introducing

the close-coupling expansion for the mechanical model and taking the "quasiclassical" limit, in which the states of the "ethereal oscillators" are described by large "vibrational" quantum numbers, we obtain the Gavrila-Kaminski system of equations. This lends itself to a physical interpretation of the channel functions and coupling potentials of the Gavrila-Kaminski system as well as opens up possibilities for the introduction of alternative approximation schemes. The above procedure therefore provides a *mechanical model* for electron scattering by a potential in the presence of a strong, single-mode laser field. We conclude with a section on numerical results for electron scattering by a Coulomb potential in the presence of a strong, linearly polarized monochromatic radiation field [10]. As in the case of a circularly polarized radiation field [11], we obtain capture-escape resonances. We also present results of calculations for the multiphoton ionization (MPI) of atomic hydrogen and compare them with results obtained by using numerous other techniques. These techniques include other formulations that employ the Floquet ansatz [12–17] as well as various numerical techniques for solving the time-dependent Schrödinger equation [18–23].

II. FORMALISM

A. Ethereal oscillators

The representation of the electromagnetic field as a system of oscillators originates from Lord Rayleigh [24], who showed that this representation, along with the equipartition theorem, leads immediately to what is now called the Rayleigh-Jeans radiation law. The quantization of these ethereal oscillators was first suggested by Ehrenfest [25] and was actually employed by Debye [26] to obtain Planck's radiation law. In the present work, we apply the ethereal-oscillator approach [9] as formulated by Bloch and Nordsieck [6], Pauli and Fierz [8], and others [27]. We shall show that this representation of the radiation field is advantageous in developing approximation schemes for electron scattering by atoms and ions in the presence of a laser field. While Rosenberg and collaborators [28] have employed the quantized electromagnetic field in the photon-number representation of creation and annihilation operators, we believe that we might gain in physical insight and practicability if we return to the original approach of Bloch and Nordsieck [6], who used the ethereal-oscillator representations of the electromagnetic field along with the *spatial coordinates* of these oscillators [27].

We shall adopt the notation of Bloch and Nordsieck, who introduced the vector-potential operator by the definition

$$\mathbf{A}=2c(\pi\hbar/\Omega)^{1/2}\sum_{s,\lambda}\omega_s^{-1/2}\hat{\boldsymbol{\epsilon}}_{s\lambda}[P_{s\lambda}\cos(\mathbf{k}_s,\mathbf{r})+Q_{s\lambda}\sin(\mathbf{k}_s,\mathbf{r})]\ , \tag{1}$$

where Ω is the quantization volume, $\hbar=h/2\pi$ with h being Planck's constant, c is the speed of light, the summation index s characterizes the direction and circular frequency ω_s of the various waves with the propagation vector $\mathbf{k}_s$, λ refers to their state of polarization, and $\hat{\boldsymbol{\epsilon}}_{s\lambda}$ is a unit vector in the direction of polarization. The dynamical variables $P_{s\lambda}$ and $Q_{s\lambda}$ are related to the photon annihilation and creation operators $a(\lambda,\mathbf{k}_s)$ and $a^\dagger(\lambda,\mathbf{k}_s)$ according to

$$2^{-1/2}(P_{s\lambda}+iQ_{s\lambda})=a(\lambda,\mathbf{k}_s)\ , \tag{2a}$$

$$2^{-1/2}(P_{s\lambda}-iQ_{s\lambda})=a^\dagger(\lambda,\mathbf{k}_s) \tag{2b}$$

and obey the commutation laws

$$[P_{s\lambda},Q_{s'\lambda'}]=-i\delta_{ss'}\delta_{\lambda\lambda'}\ ,\qquad [P_{s\lambda},P_{s'\lambda'}]=[Q_{s\lambda},Q_{s'\lambda'}]=0\ . \tag{3}$$

We shall use the nonrelativistic approximation, following Nordsieck [7], and take for the total Hamiltonian of the electron plus electromagnetic field (radiation field plus electrostatic field) the form

$$\mathcal{H}=\frac{1}{2m}\left[\mathbf{p}-\frac{e}{c}\mathbf{A}\right]^2+V(\mathbf{r})+\tfrac{1}{2}\sum_{s,\lambda}\hbar\omega_{s\lambda}[P_{s\lambda}^2+Q_{s\lambda}^2]\ , \tag{4}$$

with m and e referring to the mass and charge of the electron, respectively. We then write $\mathbf{A}$ in the form

$$\frac{e}{c}\mathbf{A}=\sum_{s,\lambda}\mathbf{a}_{s\lambda}[P_{s\lambda}\cos(\mathbf{k}_s,\mathbf{r})+Q_{s\lambda}\sin(\mathbf{k}_s,\mathbf{r})]\ , \tag{5}$$

with

$$\mathbf{a}_{s\lambda}=2e(\pi\hbar/\Omega\omega_s)^{1/2}\hat{\boldsymbol{\epsilon}}_{s\lambda}\ . \tag{6}$$

We assume that within the collision region $\cos(\mathbf{k}_s,\mathbf{r})$ and $\sin(\mathbf{k}_s,\mathbf{r})$ do not change appreciably [29]. In addition, we introduce the long wavelength or *dipole approximation* by taking $\mathbf{r}$ equal to zero, which gives $\cos(\mathbf{k}_s,\mathbf{r})\equiv 1$ and $\sin(\mathbf{k}_s,\mathbf{r})\equiv 0$. We note that according to Gavrila and Kaminski [3] the dipole approximation is justified even for Coulombic potentials for visible or ultraviolet laser fields. In the dipole approximation we then obtain from Eq. (5)

$$\frac{e}{c}\mathbf{A}=\sum_{s,\lambda}\mathbf{a}_{s\lambda}P_{s\lambda} \tag{7}$$

and from Eq. (4) we then have

$$\mathcal{H}=\frac{1}{2m}\mathbf{p}^2-\frac{1}{m}\sum_{s,\lambda}(\mathbf{p},\mathbf{a}_{s\lambda})P_{s\lambda}+V(\mathbf{r})+\tfrac{1}{2}\sum_{s,\lambda}\hbar\omega_{s\lambda}[P_{s\lambda}^2+Q_{s\lambda}^2]\ . \tag{8}$$

In the *dipole approximation,* the term containing $\mathbf{A}^2$ can be ignored [30], giving

$$\mathcal{H}=\frac{1}{2m}\mathbf{p}^2-\frac{1}{m}\sum_{s,\lambda}(\mathbf{p},\mathbf{a}_{s\lambda})P_{s\lambda}+V(\mathbf{r})+\tfrac{1}{2}\sum_{s,\lambda}\hbar\omega_{s\lambda}[P_{s\lambda}^2+Q_{s\lambda}^2]\ . \tag{9}$$

We now introduce a *canonical transformation* following Pauli and Fierz [8],

$$\mathbf{r}=\mathbf{r}'-\frac{1}{m}\sum_{s,\lambda}\omega_{s\lambda}^{-1}\mathbf{a}_{s\lambda}Q_{s\lambda}\ ,$$

$$\mathbf{p}=\mathbf{p}'\ ,$$

$$Q_{s\lambda}=Q'_{s\lambda}\ ,\quad P_{s\lambda}=P'_{s\lambda}+\frac{1}{m\hbar\omega_{s\lambda}}(\mathbf{a}_{s\lambda},\mathbf{p})\ . \tag{10}$$

Using Eq. (10) in Eq. (9), we obtain

$$\mathcal{H}=\frac{1}{2m}\mathbf{p}'^2+V\left[\mathbf{r}'-\frac{1}{m}\sum_{s,\lambda}\omega_{s\lambda}^{-1}\mathbf{a}_{s\lambda}Q'_{s\lambda}\right]+\tfrac{1}{2}\sum_{s,\lambda}\hbar\omega_{s\lambda}[P'^2_{s\lambda}+Q'^2_{s\lambda}]+\frac{1}{2m^2}\sum_{s,\lambda}\frac{(\mathbf{a}_{s\lambda},\mathbf{p}')^2}{\hbar\omega_{s\lambda}}\ . \tag{11}$$

The last term on the right-hand side of Eq. (11) is proportional to $\mathbf{p}'^2$ and therefore signifies an electron-mass renormalization term [4,8]. If we assume that this effect is already included in m, then we can ignore this term. As a result, we obtain the following form for our Hamiltonian:

$$\mathcal{H}=\frac{1}{2m}\mathbf{p}'^2+V\left[\mathbf{r}'-\frac{1}{m}\sum_{s,\lambda}\omega_{s\lambda}^{-1}\mathbf{a}_{s\lambda}Q'_{s\lambda}\right]+\tfrac{1}{2}\sum_{s,\lambda}\hbar\omega_{s\lambda}[P'^2_{s\lambda}+Q'^2_{s\lambda}]\ . \tag{12}$$

B. Linearly polarized single-mode laser field

Up to this point, we have considered all components of the radiation field, including the empty modes, to have been included in the Hamiltonian. In the following, we shall follow Rosenberg and co-workers [28], who assumed that the electron self-energy effects are already incorporated into the mass of the electron and that in Eq. (12) the s,λ indices refer to the *external radiation field* (the laser field) only. First we shall consider a single-mode linearly polarized laser field such that $\omega_{s\lambda}\to\omega$, $Q'_{s\lambda}\to Q'$, and $\mathbf{a}_{s\lambda}\to\mathbf{a}$ in Eq. (12) and omit the summation sign. We obtain

$$\mathcal{H}=\frac{1}{2m}\mathbf{p}'^2+V\left[\mathbf{r}'-\frac{1}{m\omega}\mathbf{a}Q'\right]+\tfrac{1}{2}\hbar\omega[P'^2+Q'^2]\ . \tag{13}$$

If $V=0$ in Eq. (13), then the nonrelativistic Volkov state [31] is obtained as a solution of the Schrödinger equation

$$\mathcal{H}_0\Psi_0(\mathbf{r}',Q')=E_0\Psi_0(\mathbf{r}',\mathbf{Q}')\ , \tag{14}$$

where

$$\mathcal{H}_0=\frac{1}{2m}\mathbf{p}'^2+\tfrac{1}{2}\hbar\omega[P'^2+Q'^2]\ . \tag{15}$$

The Ψ_0 function is obtained immediately in the form

$$\Psi_0(\mathbf{r}',Q')=e^{i/\hbar(\mathbf{p},\mathbf{r}')}h_n(Q')\ , \tag{16}$$

where $h_n(Q)$ is a Hermite function such that

$$-\frac{d^2h_n}{dQ^2}+Q^2h_n=(2n+1)h_n\ , \tag{17a}$$

with the normalization

$$\int h_n^*(Q)h_n(Q)dQ=1\ . \tag{17b}$$

The energy eigenvalue is obtained in the form

$$E_0=\frac{p^2}{2m}+[n+\tfrac{1}{2}]\hbar\omega\ , \tag{18}$$

where $p=|\mathbf{p}|$. Returning to the original coordinates $\mathbf{r}$ and Q by Eq. (10), we obtain

$$\Psi_0(\mathbf{r},Q)=e^{i[(\mathbf{a},\mathbf{p})/m\hbar\omega]Q}e^{(i/\hbar)(\mathbf{p},\mathbf{r})}h_n(Q)\ , \tag{19}$$

an extremely simple form for the Volkov state [31].

Consequently, we plan to treat scattering with the Hamiltonian given by Eq. (13) and solve the Schrödinger equation

$$\left[\frac{1}{2m}\mathbf{p}'^2+V\left[\mathbf{r}'-\frac{1}{m\omega}\mathbf{a}Q'\right]+\tfrac{1}{2}\hbar\omega[P'^2+Q'^2]\right]\Psi=E\Psi \tag{20}$$

in the energy continuum. We point out immediately that Eq. (20) has a close similarity to the Schrödinger equation of a particle scattering by a vibrator, which has been studied extensively in conjunction with electron-molecule scattering [32]. *Methods utilized for such collisional studies are readily applicable to the present case.* One of the most powerful of these methods is the close-coupling approach [32], in which the wave function is expanded in terms of "target states." In our case, the ethereal oscillator plays the role of the target, yielding a close-coupling expansion,

$$\Psi(\mathbf{r}',Q')=\sum_{n=0}^{\infty}F_n(\mathbf{r}')h_n(Q')\ . \tag{21}$$

Substituting Eq. (21) into Eq. (20), we obtain (omitting the primes from the variables temporarily)

$$-\frac{\hbar^2}{2m}\nabla^2F_m(\mathbf{r})+\sum_{n=0}^{\infty}V_{nm}(\mathbf{r})F_n(\mathbf{r})+\hbar\omega[m+\tfrac{1}{2}]F_m(\mathbf{r})=EF_m(\mathbf{r})\ , \tag{22}$$

where

$$V_{nm}(\mathbf{r})=\int h_m^*(Q)V\left[\mathbf{r}-\frac{1}{m\omega}\mathbf{a}Q\right]h_n(Q)dQ \tag{23}$$

is the coupling potential between channels. The question of practical importance is how many of these channels must be included to obtain numerically accurate results. As a first step, let us assume that the laser field is relatively weak (e.g., assume $I<10^{11}$ W/cm^2) and that the frequency is high to the extent that the average value of $(1/m\omega)\mathbf{a}Q$ is small compared to the range of the potential $V(\mathbf{r})$. In this case, we invoke a Taylor-series expansion for the interaction potential in the form [4]

$$V\left[\mathbf{r}-\frac{1}{m\omega}\mathbf{a}Q\right]\approx V(\mathbf{r})-\frac{1}{m\omega}(\mathbf{a}Q,\mathbf{V}_1(\mathbf{r}))\ , \tag{24}$$

which gives for the coupling potential

$$V_{nm}(\mathbf{r}) \approx V(\mathbf{r})\delta_{nm} - \frac{1}{m\omega}(\mathbf{a},\mathbf{V}_1(\mathbf{r})) \times \int h_m^*(Q) Q h_n(Q) dQ = V(\mathbf{r})\delta_{nm} - \frac{1}{m\omega}(\mathbf{a},\mathbf{V}_1(\mathbf{r})) \left[\left[\frac{m+1}{2} \right]^{1/2} \delta_{n,m+1} + [m/2]^{1/2}\delta_{n,m-1} \right] . \quad (25)$$

We immediately observe that the mth channel is coupled only to the $(m+1)$th and $(m-1)$th channels, and thus we have three coupled channels only. By including higher-order terms into the Taylor-series expansion in Eq. (24), we obtain a coupling potential with nonvanishing matrix elements of $Q^2, Q^3, \ldots$, which leads the coupling of the mth channel to the $m-2, m+2, m-3, m+3, \ldots$ channels.

C. Semiclassical limit

In the preceding section, we have shown that the problem of electron scattering by a potential in the presence of a single-mode linearly polarized monochromatic laser field reduces to that of scattering with the Hamiltonian given by Eq. (13). Our principal task remains the solution of the time-independent Schrödinger equation (20) with the boundary conditions that for $r' \to \infty$, Ψ reduces to linear combinations of solutions of $\mathcal{H}_0$ given by Eq. (14), that can be written in the form of Eq. (16). As suggested in the preceding section, one possible method is the expansion of Ψ in terms of harmonic oscillator states, Eq. (21), which gives rise to a coupled system of partial differential equations, Eq. (22).

In regimes of interest to us here the laser field will be strong, and therefore the n,m values that enter Eqs. (22) and (23) also will be large [33]. For these high n,m quantum numbers, we can safely use the quasiclassical (or WKB) approximation for the $h_n(Q)$ functions [34] in the form

$$h_n(Q) \approx \sqrt{(2/\pi)} \frac{1}{\sqrt{k_n}} \cos\left[\int_{Q_1}^{Q} k_n(Q')dQ' - \frac{\pi}{4} \right] , \quad (26)$$

with $k_n(Q) = [2\epsilon_n - Q^2]^{1/2} = [2n+1-Q^2]^{1/2}$, $\epsilon_n = 2n+1$, and Q_1 is one of the classical turning points.

In order to obtain the coupled-channel system given by Eq. (22) under semiclassical conditions, we must calculate the coupling potential $V_{nm}(\mathbf{r})$ under these assumptions. Migdal [34] showed that a matrix element of the form

$$U_{nm} = \int_{-\infty}^{\infty} h_n(Q) U(Q) h_m(Q) dQ , \quad (27)$$

where $U(Q)$ is an arbitrary function of Q, $h_n(Q)$ is given by Eq. (26), and $(n-m)/n \ll 1$, can be written as

$$U_{nm} = \frac{2}{\tau} \int_0^{\tau/2} U(Q_n(\varphi)) \cos\frac{2\pi(n-m)\varphi}{\tau} d\varphi , \quad (28)$$

where τ is defined by

$$\tau = 2\int_{Q_1}^{Q_2} \frac{1}{k_n(Q)} dQ . \quad (29)$$

In Eq. (29), Q_1 and Q_2 are the classical turning points and the $Q_n(\varphi)$ function in Eq. (28) is the inverse of the $\varphi_n(Q)$ function, which is defined by the integral

$$\varphi_n(Q) \equiv \int_{Q_1}^{Q} \frac{dQ'}{k_n(Q')} . \quad (30)$$

Since the number of channels that couple are relatively small we can safely assume that the condition $(n-m)/n \ll 1$ holds. In our case we have $\tau = 2\pi$ and

$$\varphi_n(Q) = \arcsin\left[\frac{Q}{\sqrt{2\epsilon_n}} + \frac{\pi}{2} \right] . \quad (31)$$

Thus we obtain

$$Q_n(\varphi) = \sqrt{2\epsilon_n}\sin[\varphi - (\pi/2)] = \sqrt{2\epsilon_n}\cos\varphi , \quad (32)$$

which gives, via Eq. (28),

$$U_{nm} = \frac{1}{\pi}\int_0^{\pi} U\left[\sqrt{2\epsilon_n}\cos\varphi\right]\cos[(n-m)\varphi]d\varphi . \quad (33)$$

Using Eq. (34) for the coupling potential $V_{nm}(\mathbf{r})$, we obtain, when the WKB approximation holds,

$$V_{nm}(\mathbf{r}) \approx V_{nm}^{WKB}(\mathbf{r}) \equiv \frac{1}{\pi}\int_0^{\pi} V\left[\mathbf{r} - \frac{1}{m\omega}\mathbf{a}\sqrt{2\epsilon_n}\cos\varphi\right] \times \cos[(n-m)\varphi]d\varphi . \quad (34)$$

We further assume that in the semiclassical description the electromagnetic field is characterized by the vector potential

$$\mathbf{A} = \mathbf{A}_0 \sin\omega t , \quad (35)$$

with the electric field given by $\mathbf{E}(t) = \mathbf{E}_0\cos(\omega t)$ and the magnitude or strength by $E_0 = |A_0\omega/c|$. We employ the usual relationship between the intensity and field strength: I (W/cm^2) $= 3.5\times 10^{16} E_0^2$ (a.u.). The time-averaged energy density has the form

$$\bar{u} = \frac{1}{8\pi}\frac{\omega^2}{c^2}\mathbf{A}_0^2 , \quad (36)$$

or, in the quantum-field description,

$$\bar{u} = \frac{\bar{n}\hbar\omega}{\Omega} , \quad (37)$$

where $\bar{n}$ is the average number of photons in the quantization volume Ω.

Equating the two expressions we find that the argument of V in Eq. (34) becomes

$$\mathbf{r} - \frac{e}{m\omega c}\mathbf{A}_0\cos\varphi . \quad (38)$$

If we introduce the $\varphi = \omega t$ notation, then we can write Eq. (34) as

$$V_{nm}^{\mathrm{WKB}}(\mathbf{r}) = \frac{\omega}{\pi}\int_0^{\pi/\omega} V(\mathbf{r} - \boldsymbol{\alpha}(t))\cos[(n-m)\omega t]dt \equiv \bar{V}_{n-m}(\boldsymbol{\alpha}_0, \mathbf{r}) , \quad (39)$$

where we define $\boldsymbol{\alpha}(t)$ by the equation

$$\boldsymbol{\alpha}(t)=\frac{e}{m\omega c}\mathbf{A}_0\cos\omega t\equiv\boldsymbol{\alpha}_0\cos\omega t\ , \tag{40}$$

with

$$\boldsymbol{\alpha}_0=\frac{e}{m\omega c}\mathbf{A}_0\ . \tag{41}$$

We readily observe that $\boldsymbol{\alpha}(t)$ is exactly the classical radius vector of the electron moving in the radiation field. We now assume that the external field is described classically by a vector potential given in Eq. (35). Then, according to Eq. (37), we can associate a well-defined $\bar{n}$ value with the field and write $V_{nm}^{WKB}(\mathbf{r})$ in the form of Eq. (39). If we assume that both m and n are close to $\bar{n}$ $[(m-\bar{n})/\bar{n}\ll 1,\ (n-\bar{n})/\bar{n}\ll 1]$, then we can ignore in $\boldsymbol{\alpha}_0$ the difference between n, m, and $\bar{n}$. Under these assumptions, the coupled-channel equations given by Eq. (22) can be written as

$$-\frac{\hbar^2}{2m}\nabla^2F_m(\mathbf{r})+\sum_{n\ (\approx\bar{n})}\bar{V}_{n-m}(\boldsymbol{\alpha}_0,\mathbf{r})F_n(\mathbf{r})+\tfrac{1}{2}\hbar\omega(2m+1)F_m(\mathbf{r})=EF_m(\mathbf{r})\ ,$$

$$\text{with } m\approx\bar{n} \text{ and } n\approx\bar{n}\ . \tag{42}$$

This system of coupled equations can be related immediately to those obtained by Gavrila and Kaminski [3,5], which have the form

$$\left[-\frac{\hbar^2}{2m}\nabla+V_0(\boldsymbol{\alpha}_0,\mathbf{r})-(E+n\hbar\omega)\right]\psi_n(\mathbf{r})=-\sum_{m\ (=-\infty)}^{\infty}V_{n-m}(\boldsymbol{\alpha}_0,\mathbf{r})\psi_m(\mathbf{r})\ , \tag{43a}$$

with $V_n(\boldsymbol{\alpha}_0,\mathbf{r})$ defined by the equation

$$V_n(\boldsymbol{\alpha}_0,\mathbf{r})=(\omega/2\pi)\int V(\mathbf{r}-\boldsymbol{\alpha}_0\cos\omega t)\cos(n\omega t)dt\ , \tag{43b}$$

where $\psi_0(\mathbf{r})$ describes "elastic scattering," while the $\psi_n(\mathbf{r})$ functions with $n\neq 0$ describe bremsstrahlung and inverse bremsstrahlung processes. The integration limits extend from $-\omega/\pi$ to ω/π. The following boundary condition are prescribed for the $\psi_n(\mathbf{r})$ functions [3]:

$$\psi_0(\mathbf{r})\xrightarrow[r\to\infty]{}\exp\left[\frac{i}{\hbar}\left[(\mathbf{p},\mathbf{r})+\gamma_0\ln\frac{i}{\hbar}[pr-(\mathbf{p},\mathbf{r})]\right]\right]+\frac{f_0(\Theta,\varphi)\exp\left[\frac{i}{\hbar}\left[pr-\gamma_0\ln 2\frac{pr}{\hbar}\right]\right]}{r}\ , \tag{44a}$$

$$\psi_n(\mathbf{r})\xrightarrow[r\to\infty]{}\frac{f_n(\Theta,\varphi)\exp\left[\frac{i}{\hbar}\left[p_nr-\gamma_n\ln\frac{2pr}{\hbar}\right]\right]}{r}\ , \tag{44b}$$

where

$$\gamma_n=-\frac{ze^2}{p_n\hbar}\ , \tag{45}$$

p_n is defined by the equation

$$\frac{p_n^2}{2m}=E+n\hbar\omega\quad(p_n>0)\ , \tag{46}$$

and z refers to the residual nuclear charge of our atom (or ion), as assumed that

$$V(r)\xrightarrow[r\to\infty]{}\frac{ze^2}{r}\ . \tag{47}$$

For a short-range potential, of course, $z=0$.

As mentioned in the Introduction, while the calculational scheme defined by Eqs. (43) and (44) is well defined, the exact physical meaning of the $\psi_n(\mathbf{r})$ functions and $V_n(\alpha_0,\mathbf{r})$ coupling potentials is not obvious. We can, however, get some physical insight into these quantities and into the whole problem if we compare Eq. (43) with Eq. (42), which we write here in the form

$$-\frac{\hbar^2}{2m}\nabla^2F_i(\mathbf{r})-[\bar{E}-\hbar\omega(i+\tfrac{1}{2})]F_i(\mathbf{r})+\bar{V}_0(\boldsymbol{\alpha}_0,\mathbf{r})F_i(\mathbf{r})=-\sum_{j\ (\neq i)}\bar{V}_{j-i}(\boldsymbol{\alpha}_0,\mathbf{r})F_j(\mathbf{r})\ , \tag{48}$$

where E of Eq. (42) becomes $\bar{E}$ for clarity.

The coupling and channel potentials in Eqs. (43) and (48) are defined exactly the same way. We only need the equation to satisfy

$$\bar{E}-\hbar\omega[i+\tfrac{1}{2}]=E+n\hbar\omega\ . \tag{49}$$

This can be achieved if we choose i and j in the form

$$i=\bar{n}-n \text{ and } j=\bar{n}-m\ . \tag{50}$$

Then we obtain, from Eq. (48),

$$-\frac{\hbar^2}{2m}\nabla^2F_{\bar{n}-n}(\mathbf{r})-[\bar{E}-\hbar\omega(\bar{n}+\tfrac{1}{2})+n\hbar\omega]F_{\bar{n}-n}(\mathbf{r})+V_0(\boldsymbol{\alpha}_0,\mathbf{r})F_{\bar{n}-n}(\mathbf{r})=-\sum_{m\ (\neq n)}\bar{V}_{n-m}(\boldsymbol{\alpha}_0,\mathbf{r})F_{\bar{n}-m}(\mathbf{r})\ . \tag{51}$$

This shows that, if we make the identification

$$F_{\bar{n}-n}(\mathbf{r})\equiv\psi_n(\mathbf{r}) \tag{52}$$

and

$$E=\bar{E}-\hbar\omega[\bar{n}+\tfrac{1}{2}]\ , \tag{53}$$

then Eq. (51) becomes identical to Eq. (43a). Thus the close-coupling equations associated with the Hamiltonian

given by Eq. (13) in the semiclassical limit give the Gavrila-Kaminski system of equations. We have to remember that originally Eq. (51) was obtained for large values and i and j, whereas Eq. (44) is valid for small values of n and m; thus $\bar{n}$ should be a large number.

Equation (52) gives the physical interpretation of $\psi_n(\mathbf{r})$ as *the channel function associated with the quantum number $\bar{n}-n$ of the ethereal oscillator.* Since $\psi_0(\mathbf{r})$ describes elastic scattering, this means that initially the ethereal oscillator was in the $\bar{n}$th excited state ($\bar{n}\gg 1$) and, after interacting with the incident electron, it gained or lost energy, with $F_{\bar{n}-n}(\mathbf{r})$ corresponding to the losing of $n\hbar\omega$ energy (the electron gaining $n\hbar\omega$ energy). In this context, the *the physical significance* of the $V_n(\boldsymbol{\alpha}_0,\mathbf{r})$ potential becomes clear, since via Eq. (50) we have

$$V_{n-m}(\boldsymbol{\alpha}_0,\mathbf{r})=\bar{V}_{j-i}(\boldsymbol{\alpha}_0,\mathbf{r}) \; ; \tag{54}$$

obviously $V_{n-m}(\boldsymbol{\alpha}_0,\mathbf{r})$ signifies the coupling potential connecting the $i=\bar{n}-n$ and $j=\bar{n}-m$ channels for $\bar{n}\gg 1$, and $i\ll\bar{n}$, $j\ll\bar{n}$. Thus we have achieved a clear physical interpretation of the $\psi_n(\mathbf{r})$ Fourier component functions and the $V_n(\boldsymbol{\alpha}_0,\mathbf{r})$ potentials.

In order to examine the dynamics of an electron in an oscillating electric field in the presence of an atomic potential, we must solve Eq. (43) subject to the appropriate boundary conditions [Eqs. (44)–(45)]. To make this exercise more tractable, we invoke a single-center expansion of the spatial function as

$$\psi_n(\mathbf{r})=r^{-1}\sum_{l_n,m_n} f_{nl_n}(r)Y_{l_nm_n}(\hat{\mathbf{r}}) \; , \tag{55}$$

with $\hat{\mathbf{r}}$ representing the angular coordinates (Θ,ϕ), and Θ the angle between $\boldsymbol{\alpha}_0$ and $\mathbf{r}$. Substituting Eq. (55) into (43), multiplying through by

$$Y_{l_n'm_n'}(\hat{\mathbf{r}})^* \; , \tag{56}$$

and integrating over the angle, we derive a set of coupled second-order differential equations

$$\left[\frac{d^2}{dr^2}-\frac{l_{n'}(l_{n'}+1)}{r^2}-k_{n'}^2\right]f_{\Gamma'}(r)=\sum_{\Gamma}U_{\Gamma'\Gamma}(r)f_{\Gamma}(r) \; , \tag{57}$$

where $k_n^2=(E+n\omega)/\hbar$,

$$U_{\Gamma'\Gamma}(r)=\frac{m}{\hbar^2}\sum_{\lambda}\left[\frac{2l_n+1}{2l_{n'}+1}\right]^{1/2}C(l_n\lambda l_{n'}|m_n0m_n)\times C(l_n\lambda l_{n'}|000)v_{\lambda}^{n'n}(r) \; , \tag{58}$$

and

$$v_{\lambda}^{n'n}(r)=\frac{(2\lambda+1)}{\pi}\int_0^{\pi}dv\,P_{\lambda}(v)\times\int_{-1}^{1}du\frac{V(\mathbf{r}+\boldsymbol{\alpha}_0u\,(T_{n'-n}(u)}{[1-u^2]^{1/2}} \; , \tag{59}$$

with $u\equiv\cos(\omega t)$, $v\equiv\cos(\Theta)$, and $T_k(u)$ $[P_k(v)]$ is the Chebyshev [Legendre] polynomial of order k. The Clebsch-Gordan coefficients are given by $C(l_1l_2l_3|m_1m_2m_3)$, and the channels $\Gamma=(n,l_n)$ are labeled by a Fourier-component quantum number n and its associated angular momentum quantum number l_n. We extract the scattering information by matching the asymptotic form of these radial components to the usual K-matrix conditions, from which we also calculate the S and T matrices.

The coupled-channel equations (57) have several important properties: (1) they are block-diagonal in the azimuthal quantum number ($m_n=m_{n'}\equiv m$); (2) only channels of the same parity [$(n+l_n)$ even or odd] couple; and (3) they display a close resemblance to those for electron scattering from a linear ionic diatomic molecule. The first property implies that we may solve the coupled equations independently for each value of m although the Fourier (n) and partial wave (l_n) components still couple. The second condition simplifies the form of the structure of the equations by forcing certain matrix elements to vanish. Finally, the last observation has allowed us to draw upon the vast lore of electron-molecule computations to bring to bear this knowledge upon the intense field interaction. We may carry this analogy between the scattering equations in the KH gauge and those for electron-molecule collisions further. The internuclear distance R between molecular atoms closely corresponds to the displacement α_0 that represents the separation of effective "charges." The more extended this separation, the larger the basis needed to span the region. One further point should be delineated: we recall that the displacement depends on the ratio of the field strength to the square of the frequency. Therefore, for a fixed α_0, we find that the higher the frequency ω, the larger the corresponding intensity that can be handled. We have treated values of α_0 as large as 8.0 bohrs. This constraint would primarily affect low-frequency, high-intensity cases.

D. Computational procedures

For either scattering or ionization, we must numerically solve Eq. (57) subject to the appropriate boundary conditions. To effect this solution, we apply the linear algebraic (LA) method. Since the intricacies of this approach have been discussed elsewhere [35], we present only a brief overview of the technique. We first convert Eq. (57) to a system of coupled integral equations by the usual prescriptions. We in turn employ an R-matrix strategy and divide the radial space into two regions. In the inner region ($r<a$), where the coupling may be strong and, for more complicated species, exchange and correlation effects could be important, we invoke a pure LA approach. By placing the integrals and wave functions on a discrete radial quadrature mesh $[0-a]$, we transform the coupled equations into a system of algebraic ones. We solve this system by standard linear systems techniques, which produce a solution on a discrete mesh in the inner region. We select this mesh of n_p points to reflect the strength of the potential. For example, the potential in Eq. (59) has a distinct singularity at $r=\alpha_0$; we therefore concentrate the preponderance of quadrature points within a zone around α_0. In the outer region ($r>a$), in which the coupling becomes weak and usually multipolar

in form, we employ an R-matrix propagator approach [36] by which the solution at $r=a$ is extended into the asymptotic region. We determine the scattering quantities, such as the T matrix, by matching the propagated solution to the proper asymptotic forms [Eqs. (44a) and (44b)]. From the scattering matrix, we calculate the cross section or determine the eigenphases. For resonances, we fit [37] the eigenphase sum as a function of electron energy to a Breit-Wigner expression in order to extract the resonance position and width. In the case of ionization, we can in many cases relate the width directly to the rate.

In order to construct a solution, we must make the close-coupling (CC) approximation. We labeled the wave function and thus the matrix elements by channels (n,l_n), which are identified by a particular Fourier component n and a specific partial wave l_n. The sum over these channels in Eq. (57) is infinite; however, in order to obtain a realistic solution, we truncate this sum at some finite number of channels $n_c=n_s n_l$. We designate the number of Fourier components included by n_s with the index n running from $-(n_s-1)/2$ to $+(n_s-1)/2$ in integer units; similarly, we label the number of partial waves included *in each Fourier component* by n_l. The actual maximum value of l_n included, of course, depends on the symmetry studied. We then systematically increase the number of channels n_c until the scattering quantities converge to within a prescribed tolerance. In all cases presented, we have assiduously checked the convergence of the CC expansion, determining quantities to within better than 5%. In the KH gauge, we have found that a symmetric choice of the Fourier components about $n=0$ produces the best convergence properties. Other gauges may require different strategies [17].

The programs have been checked in several ways. First, for the elastic scattering $(n-n'=0)$, we have closed all channels and solved for the bound eigenstates of U_{nn}. For a selection of values of α_0, we obtain excellent agreement with the results of other investigators [5] for the ground σ_g state. Second, we have employed the $e^-+H_2^+$ paradigm to examine quantities with earlier molecular-collision programs [35].

III. RESULTS AND DISCUSSION

A. Electron-proton scattering in a field

We illustrate the scattering of an electron by a target atom in an electromagnetic field by investigating certain resonance effects. In the absence of an external field, the cross section for an electron-proton collision displays no structure and monotonically declines with increasing energy. However, when an oscillating electric field is applied, we immediately observe pronounced resonance features in the scattering quantities. These resonances, termed capture-escape, arise from the exchange of virtual photons with the electron-proton system. For particular energies of the electron, the emission of a certain number of photons will place the resulting energy near a bound state of the compound hydrogen atom system. This particularly favorable configuration insures that the electron remains trapped in this state until absorbing an appropriate number of photons to return to the continuum. Calculations have predicted these resonances for both circularly [11] and linearly [10] polarized light. In Fig. 1, we display a representative example for e^-+p^+ scattering for a field of 0.0207 a.u. (1.5×10^{13} W/cm^2) and frequency $\omega=0.27$ a.u. (7.35 eV) by plotting the magnitude of the T matrix as a function of the electron energy for the elastic channel $[(0,2)\rightarrow(0,2)]$. We notice the distinct Fano profiles indicative of such compound resonance states. The lowest resonance arises from the virtual emission of two photons into the ground ($1s$) state of H with the subsequent absorption of the same number of photons. The higher-energy features correspond to single-photon exchanges with the exited states of H ($n=2$, 3, 4, and 5). The two-photon resonance has predominantly d-wave ($l=2$) character, while the lowest single-photon example displays mainly s-wave qualities, upholding the basic selection rules of a perturbative treatment. Of course, since we operate in a coupled-channel mode, other combinations of photon exchanges can also occur, giving rise to bremsstrahlung or inverse bremsstrahlung processes.

To gain a better insight into this mechanism, we focus on the lowest-energy resonance in Fig. 1. We examine the behavior of its width and position as a function of intensity (field strength) and frequency. In Fig. 2(a) we display the resonance position as a function of field strength for a fixed frequency of 0.27 a.u. We performed the calculations for 15 Fourier components ($n_s=15$), 5 partial waves per component ($n_l=5$), and 51 points ($n_p=51$) on a Gauss-Legendre quadrature mesh in the LA region $(0-a)$. Since the box radius varies with the displacement, we usually choose a to lie within 1.0 bohr of α_0. We then propagate the resulting solutions out to 500 bohrs. As the strength increases, the position moves to lower energies. On the other hand, the width, presented in Fig. 2(b), steadily increases with rising intensity. The position clearly tends toward a limit at zero energy. We can better understand this behavior by examining a simple approximate formula for the resonance energy:

$$E_{\rm res}=n\omega-|\varepsilon_b|-E_j\ , \tag{60}$$

where E_0 is the field strength, ω is the frequency of the

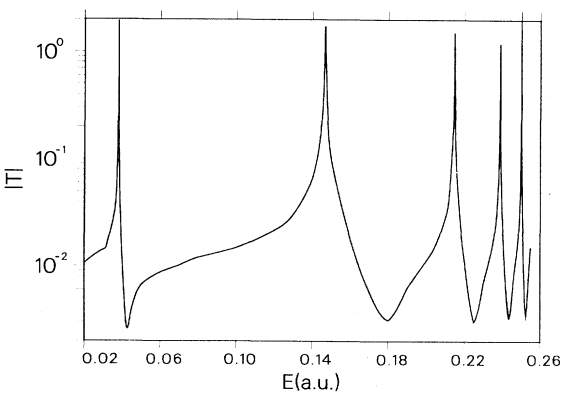

FIG. 1. Capture-escape resonances for e^-+p^+ collisions at $\omega=0.27$ a.u. and a field strength of $E_0=0.027$ a.u.

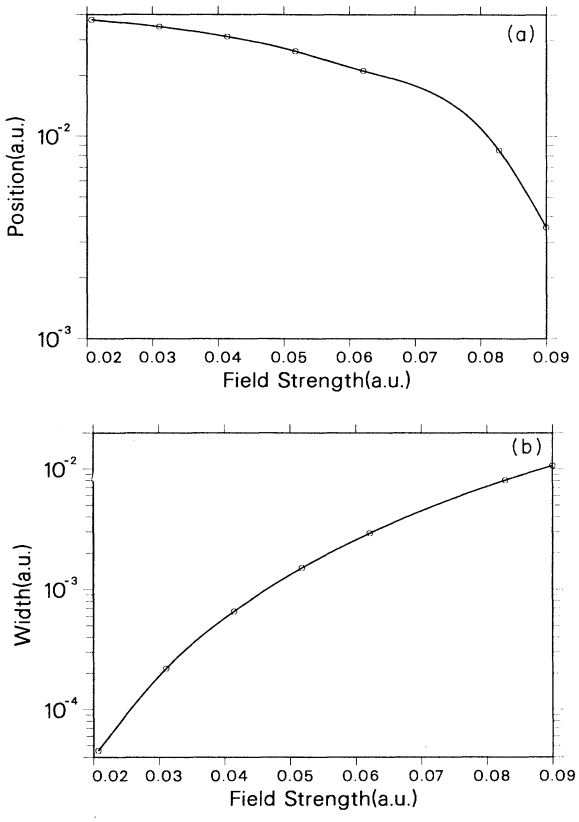

FIG. 2. (a) Position as a function of field strength for the two-photon capture-escape resonance to H ($1s$) for $\omega=0.27$ a.u. (b) Width as a function of field strength for the two-photon capture-escape resonance to H ($1s$) for $\omega=0.27$ a.u.

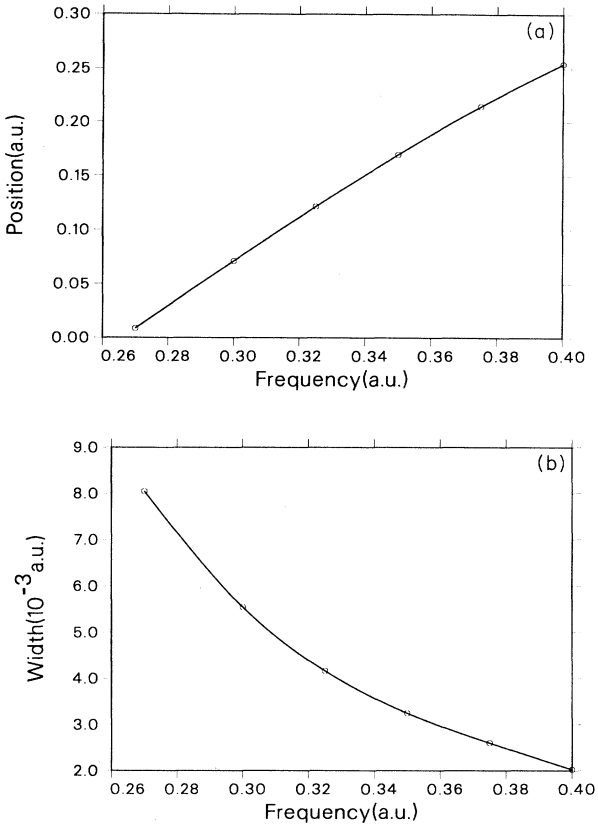

FIG. 3. (a) Position as a function of frequency for the two-photon capture-escape resonance to H ($1s$) for $E_0=0.0808$ a.u. (b) Width as a function of frequency for the two-photon capture-escape resonance to H ($1s$) for $E_0=0.0808$ a.u.

field, and $E_j=E_0^2/(4\omega^2)$, the famous quiver energy. For fixed n and ω, as the intensity increases the resonance position moves to lower energies. We eventually reach a point at which the energy descends into a bound regime and the resonance disappears. To produce a new resonant effect, we must increase the number of photons. Such compound resonances commonly occur in electron scattering from molecules, for example, $e^-+H_2^+$ collisions [38], in which the internuclear separation R plays the analogous role to the displacement α_0. The resonances correspond to doubly excited states of the H_2 system. As R increases, the width broadens and the energy decreases, finally crossing the target H_2^+ curve and becoming a fully bound system. Since we fixed ω, the increase in intensity simply raises α_0, giving comparable behavior. Another interesting effect arises from holding the intensity fixed ($E_0=0.808$ a.u.) and varying the frequency ω. The behavior of the position and width of the lowest-energy resonance is given in Figs. 3(a) and 3(b), respectively. With growing frequency, the position increases and the width declines, seemingly opposite to the effect discussed above. However, our simple formula predicts the trend adequately. For rising frequency, the value of the leading term increases and the quiver term grows smaller, yielding a correspondingly ascending resonant position. These trends seem to hold for a wide variety of the capture-escape resonances, although channel-coupling and interference effects can alter this simple picture. In fact, the play of the resonant poles over the complex plane forms a complicated process that must be treated with great care, especially in those instances where the resonance position moves through a Rydberg series converging on an underlying Fourier component threshold [39]. As we observe, such resonance phenomena have drastic effects on the scattering quantities not only in the immediate neighborhood of the position but also accumulatively along a Rydberg series, as occurs in standard electron-ion collisions [40].

B. Photoionization of hydrogen

In the absence of an oscillating electric field, a bound state of the H atom has zero width and a correspondingly infinite lifetime. However, once the field has been applied, the system has the possibility of ionizing, giving the previously bound level a finite width or lifetime. This lifetime in turn relates, in many cases, directly to the rate at which the system ionizes. We can therefore perform a scattering calculation at quasienergies near the bound level and produce an eigenphase sum that has a distinct

resonance character. By fitting this to a Breit-Wigner form, we extract a width and a shift, and ultimately a rate. In Table I we compare our results with other calculations for the three-photon ionization of the $1s$ state of H by a field at various intensities and a frequency of 0.2 a.u. (5.44 eV). We have performed our calculations for a 75-channel system ($n_s=15$, $n_l=5$, $n_p=60$). The results of Chu and Cooper [12] and Shakeshaft and Tang [13] both invoke the Floquet ansatz but employ different gauges and calculational procedures than ours. The other two results of Kulander [18] and of Pont, Proulx, and Shakeshaft [22] (PPS) come from the direct solution of the time-dependent (TD) Schrödinger equation. At the lower intensity, we obtain very good agreement with the other Floquet approaches as well as one of the TD results (PPS). On the other hand, at the higher field strength we have better accord with the TD solutions of Kulander. Still, all the results lie within 20% of each other, which, given the different procedures, is probably not unreasonable.

We also examine the case of interaction of H with an $E=5$ eV ($\omega=0.183\,75$ a.u.) photon, which again requires a three-step ionization process. In Fig. 4 we compare our results as a function of intensity with those of other practitioners. The agreement between the various techniques is excellent below about 2.0×10^{14} W/cm^2, where we remain still fairly much in the perturbative realm. The region between 2.0×10^{14} and 4.0×10^{14} W/cm^2 has a complex, intricate structure [21], which we shall not examine in any detail. We have simply calculated a few points in the regime and made a smooth transition among them. At the higher intensities, we obtain less favorable agreement. Our results lie between the other two Floquet treatments of Chu and Cooper and of Pindzola and Dörr (Sturmian-Floquet results). At a field of 0.143 a.u. ($\alpha_0=4.23$ bohrs), we differ by about 20% from the latter authors, who in turn agree well with the R-matrix calculation [17,41]. We obtain a width of 2.05×10^{-2} a.u. (8.5×10^{14} s^{-1}) and a shift of -0.1655 a.u., while the R-matrix calculations [17,41] find 1.7×10^{-2} and -0.197 a.u., respectively. The R-matrix width agrees more closely with the Sturmain-Floquet results (1.67×10^{-2} a.u.) of Pindzola and Dörr [21]. We have performed numerous cross checks with the R-matrix calculations, but as yet have not detected the source of the difference. Many candidates suggest themselves. First, numerical problems seem unlikely as both cases appear to have reached a high level of convergence in all the relevant parameters. We have made an extensive convergence check with n_s, n_l, and n_p as large as 19, 6, and 70, respectively, and find that the resonance parameters change very little once we include at least 15 states. We selected a standard mesh of 61 points distributed as 10, 15, 10, 20, and 5 in regions [0,1.0], [1.0,2.75], [2.75,3.8], [3.8,4.8], and [4.8,6.0], respectively. We employed a Gauss-Legendre quadrature and placed an extra point of zero weight at $a=6$ to give the proper matching conditions. In order to examine further this point, we employed an entirely independent computational scheme and computer programs from the LA. We implemented an integral equations propagation (IEP) solution to Eq. (57) based on the Sams-Kouri approach [42]. We performed extensive convergence tests on the mesh since we use the IEP technique to span the entire radial region from $r=0$ to the asymptotic matching radius. For a given number of channels, we find excellent agreement between the LA and IEP results, reinforcing our contention that we have accurately solved the close-coupling equations in the KH gauge. Second, some effect from the gauge utilized may be evincing itself. Our entire calculation remains in the KH gauge from origin to asymptotic regime; the R-matrix procedure employs a gauge transformation from the velocity to the KH gauge. In theory, these two representations should yield the same results, but in practice subtle differences may arise. Third, the methods for extracting the resonance parameters differ: we employ a Breit-Wigner fitting to the T matrix while the R-matrix calculation operates in terms of the Siegert states. Again, these two formulations should yield identical results for well-isolated resonant poles. From calculations with a fitting program [37] that handles overlapping resonances, we feel confident in treating this resonance as isolated, although another resonance lies reasonable close. The final diagnosis may involve a combination of all three of these conditions. We should reiterate, however, that our results at higher intensities are bracketed by the two other Floquet approaches and that differences remain at the 20% level. At these intensities, we have passed into a very complicated region in

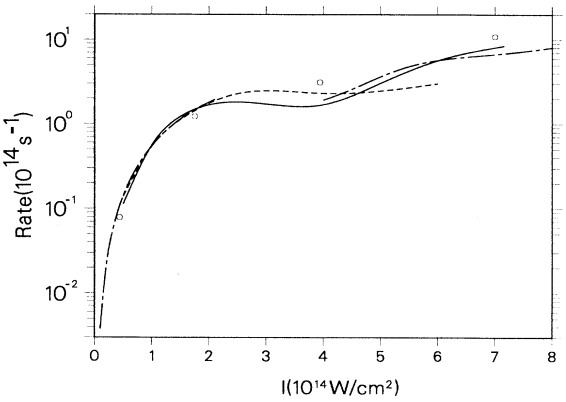

FIG. 4. Photoionization rate of H ($1s$) as a function of intensity at $\omega=0.183\,75$ a.u. Solid line, present results; dashed line, LaGattuta [20]; circles, Chu and Cooper [12]; dot-dashed line, Pindzola and Dörr [21].

TABLE I. Comparison of the photoionization rates for H($1s$) at $\omega=0.20$ a.u. The rates and intensities (I) are in units of 10^{14} s^{-1} and 10^{14} W/cm^2, respectively. The symbols CC, ST, PPS, and K refer to Refs. [12], [13], [22], and [18], respectively.

	Rates				
I	CC	ST	PPS	K	Present
1.75	2.89	2.73	2.7	4.0	2.86
3.94	5.64		6.0	7.0	6.92

which the process we investigate has changed from a three- to a four-photon mechanism and in which the resonance we follow passes through the Rydberg levels underlying the $n=4$ state.

Another interesting regime to explore involves superintense ($I > 10^{16}$ W/cm^2) fields [43]. We enter the realm of the famous stabilization effects by which the ionization rate becomes "suppressed" as the field strength rises. The electron in effect becomes trapped by the oscillating field. As the field increases, the electron becomes confined in regions away from the charge center, thereby decreasing the ionization efficiency since the region around the nucleus is most effective in propelling the electron into the continuum. This phenomenon was predicted by TD calculations in both one and three- dimensions [19,44]. We investigate single-photon ionization ($\omega=1.0$ a.u. $\hbar\omega=27.2$ eV) for these very intense fields and display our calculations in Fig. 5 as a function of intensity. We have excellent agreement over the whole range with Dörr *et al.* [15], who also employed a Floquet approach. We have pushed the calculations to even larger field strengths. We observe that as the field strength increases, we reach a maximum in the ionization rate at around 5×10^{16} W/cm^2. As we further increase the intensity, the rate begins to fall, and a very interesting result ensures: the agreement with the TD solutions of Kulander *et al.* [19], which is not particularly good at the maximum, becomes much better. The origin of this effect may lie in the manner of the TD calculations. For the more intense case, the ramping prescriptions are selected specifically to obtain localization, which gives a closer correspondence to the model we treat in the KH gauge. Also, around 7×10^{16} W/cm^2, an additional photon is required to ionize the system due to the strength of the quiver term. We have seen at lower intensities that this transition region is quite complicated and the agreement among methods becomes poorer. By the very high intensities, we have again reached a cleaner region of the resonance space. We therefore observe that this phenomenon does appear to have an analog in the time-independent results.

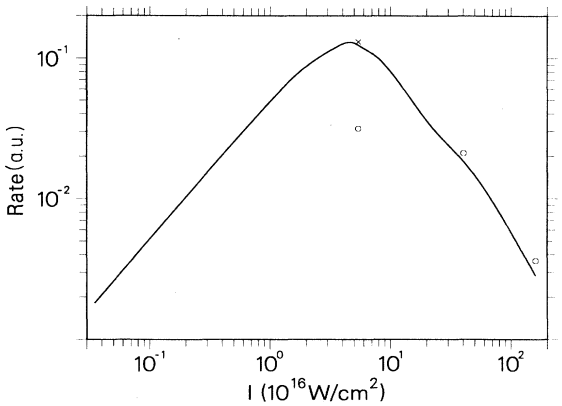

FIG. 5. Photoionization rate of H (1s) as a function of intensity for $\omega=1.0$ a.u. Solid line, present results, cross Dörr *et al.* [15]; circles, Kulander *et al.* [19].

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