Influence of the virtual-photon processes on the squeezing of light in the two-photon Jaynes-Cummings model

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In this paper, we have investigated the influence of the counter-rotating terms on the squeezing of light in the two-photon Jaynes-Cummings model by means of the nonrelativistic QED. We verified that the effect of the virtual-photon field increases the squeezing. The relations between the degree of squeezing and the frequency of field, the mean photon number, and the atom-field coupling constant have also been analyzed.

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I. INTRODUCTION

Substantial interest has centered on the recent experimental observations [1–3] of squeezed field states [4,5] which present a new nonclassical effect in the theory of radiation field and may have potential application in optical communication and gravitational wave detection. Recently, Brune *et al.* have reported the realization of the first two-phonon quantum oscillator by employing Rydberg atoms in a high-Q superconducting microwave cavity [6]. This means that it is possible to detect squeezed light in the two-photon Jaynes-Cummings (JC) model. Scully *et al.* predicted that the two-photon correlated-emission laser can produce the stable squeezed light [7,8]. So it is valuable to investigate the squeezing phenomenon of the light in the two-photon JC model. The previous papers have mentioned the influence on the squeezing due to the different initial atomic state [9–11], the frequency mismatch between the frequency of field and the atomic transition frequency [12], and the cavity dissipation [13]. Although Lais and Steimie have obtained the time evolution of squeezing in the one-photon JC model without the rotating-wave approximation (RWA) using a continued fraction method [14], the influence of the virtual-photon processes (counter-rotating wave terms) [15,16] on the squeezing of the radiation field in the two-photon JC model is not examined. The presently available results on the effect of the virtual-photon field claimed that the virtual-photon field is the source of the Lamb shift [17,18] and can ensure the causality of the atom-field coupling system [19,20], and that the virtual-photon processes, which are the important causes for quantum fluctuations, have also been verified [21–25]. So it is necessary to discuss the influence of the virtual-photon processes on the squeezing of light in the two-photon JC model.

The aim of this paper is to point out the influence of the virtual-photon field on the squeezing of light in the two-photon JC model. In Sec. II, we obtain the time evolution of the density matrix of the atom-field coupling system without the RWA using a perturbation theory [26]. In Sec. III, we verify that the effect of the virtual-photon field increases the squeezing, and the relations between the degree of squeezing and the frequency of field, the mean photon number, and the atom-field coupling constant have also been analyzed.

II. TIME EVOLUTION OF THE DENSITY MATRIX OF THE TWO-PHOTON JC MODEL WITHOUT THE RWA

The Hamiltonian for a system of a two-level atom with the same parity interacting with a single-mode quantized radiation field via a two-photon process without the RWA is

$$H = H_0 + V \ , \qquad (1)$$

where

$$H_0 = \omega a^\dagger a + \omega_0 S_3 \quad (\hbar = 1) \ , \qquad (2)$$

$$V = \epsilon (a^{\dagger 2} + a^2)(S_+ + S_-) \ . \qquad (3)$$

Here $a^\dagger$ and a are the creation and annihilation operators for the field. ϵ is the atom-field coupling constant. ω_0 and ω are the atomic transition and the field frequency, respectively. $S_3, S_\pm$ are the atomic operators that obey

$$[S_3, S_\pm] = \pm S_\pm, \quad [S_+, S_-] = 2S_3 \ .$$

For simplicity, we only consider the field to be resonant with the atomic transition frequency via a two-photon process, i.e., $\omega_0 = 2\omega$. In the interaction picture, the interacting Hamiltonian of the atom-field coupling becomes

$$V^I(t) = \epsilon (a^{\dagger 2} S_- + a^2 S_+ + a^{\dagger 2} S_+ e^{i4\omega t} + a^2 e^{-i4\omega t}) \qquad (4)$$

and the annihilation operator of the field satisfies

$$a^I(t) = a^S \exp(-i\omega t) \ . \qquad (5)$$

If the initial state vector of the atom-field system is

$$|\Psi_{AF}(0)\rangle = \sum_n F_n |b,n\rangle \ , \qquad (6)$$

this means that the atom is in its ground state $|b\rangle$, and the field is in the superposition state $\sum_n F_n|n\rangle$. At time t, it develops into

$$|\Psi^I_{AF}(t)\rangle=\sum_n a_n(t)|a,n\rangle+b_n(t)|b,n\rangle\ . \tag{7}$$

Bringing the Schrödinger equation into the interaction picture, we obtain

$$i\frac{d}{dt}a_n(t)=\epsilon\sqrt{(n+2)(n+1)}b_{n+2}(t)+\epsilon\sqrt{(n-1)n}\,b_{n-2}(t)\exp(i4\omega t)\ , \tag{8}$$

$$i\frac{d}{dt}b_{n+2}(t)=\epsilon\sqrt{(n+2)(n+1)}a_n(t)+\epsilon\sqrt{(n+4)(n+3)}a_{n+4}(t)\exp(-i4\omega t)\ . \tag{9}$$

It is easy to see that the first terms on the right-hand side in Eqs. (8) and (9) represent the contribution of the real-photon processes (the rotating-wave terms) to the probability amplitude $a_n(t),b_{n+2}(t)$, and the second terms imply the influence of the virtual-photon processes (the counter-rotating-wave terms) on $a_n(t),b_{n+2}(t)$.

In order to obtain the solution of $a_n(t),b_{n+2}(t)$, we must mention that the influence of virtual-photon processes on $a_n(t),b_{n+2}(t)$ is smaller than that of the real-photon processes. Using the solution of $a_n(t),b_{n+2}(t)$ with the RWA [10–12],

$$a_n^0(t)=-iF_{n+2}\sin(C_1t)\ , \tag{10}$$

$$b_{n+2}^0(t)=F_{n+2}\cos(C_1t)\ , \tag{11}$$

$$C_1=\epsilon\sqrt{(n+2)(n+1)}\ , \tag{12}$$

and we can regard $a_n^0(t),b_{n+2}^0(t)$ as the solution of the zero-order approximation of Eqs. (8) and (9). Using the perturbation theory [26], we substitute Eqs. (10)–(12) into Eqs. (8) and (9) and only retain terms up to the first order in ϵ/ω; then we obtain

$$a_n(t)=A\left[-iF_{n+2}\sin(C_1t)-\frac{n(n-1)}{2\bar{n}^2}C_1F_{n+2}(\{\exp[i(B_1t-4\varphi)]-\exp(-i4\varphi)\}/B_1+\{\exp[i(B_2t-4\varphi)]-\exp(-i4\varphi)\}/B_2)\right]\ , \tag{13}$$

$$b_{n+2}(t)=A\left[-iF_{n+2}\cos(C_1t)-\frac{\epsilon\bar{n}^2}{2\sqrt{(n+5)(n+6)}}F_{n+2}(\{\exp[-i(A_1t-4\varphi)]-\exp(i4\varphi)\}/A_1-\{\exp[-i(A_2t-4\varphi)]-\exp(i4\varphi)\}/A_2)\right]\ , \tag{14}$$

where

$$A=1+\frac{\epsilon\bar{n}^2}{2\sqrt{(n+5)(n+6)}}\cos(C_1t)\{[\cos(A_1t-4\varphi)-\cos(4\varphi)]/A_1-[\cos(A_2t-4\varphi)-\cos(4\varphi)]/A_2\}-\frac{n(n-1)C_1}{2\bar{n}^2}\sin(C_1t)\{[\sin(B_1t-4\varphi)+\sin(4\varphi)]/B_1+[\sin(B_2t-4\varphi)+\sin(4\varphi)]/B_2\}\ ,$$

$$A_1=4\omega-\epsilon\sqrt{(n+5)(n+6)},\quad A_2=4\omega+\epsilon\sqrt{(n+5)(n+6)}$$

$$B_1=4\omega-\epsilon\sqrt{(n-2)(n-3)},\quad B_2=4\omega+\epsilon\sqrt{(n-2)(n-3)}\ .$$

Here the state vector $|\Psi^I_{AF}(t)\rangle$ has been normalized and the radiation field that is in the coherent state at $t=0$ has been assumed, i.e.,

$$F_n=\exp(-|\alpha|^2)\alpha^n\sqrt{n!}\ , \tag{15}$$

where

$$\alpha=\bar{n}^{1/2}\exp(i\varphi)\ .$$

Here $\bar{n}$ is the mean photon number of the coherent field and φ is the phase angle of α.

From Eqs. (13) and (14), we can easily see that $\epsilon\sqrt{n(n-1)}/B_1$, $\epsilon\sqrt{n(n-1)}/B_2$, $\epsilon\sqrt{(n+3)(n+4)}/A_1$, and $\epsilon\sqrt{(n+3)(n+4)}/A_2$ are not infinitesimal if the radiation field is intensive. In this case the RWA is not reasonable [27]. For simplicity, we only consider that the radiation field is not very intensive so that $\epsilon\sqrt{n(n-1)}/B_1$, $\epsilon\sqrt{n(n-1)}/B_2$, $\epsilon\sqrt{(n+3)(n+4)}/A_1$, and $\epsilon\sqrt{(n+3)(n+4)}/A_2$ are not very large. Then we can substitute $a_n(t),b_{n+2}(t)$ into Eq. (7) using a perturbation theory. We obtain the density matrix $\rho^I(t)$ of the atom-field coupling system

$$\rho^I(t)=|\Psi^I_{AF}(t)\rangle\langle\Psi^I_{AF}(t)|$$
$$=\sum_{n,k}\begin{bmatrix} a_n(t)a_k^*(t)|n\rangle\langle k| & a_n(t)b_{k+2}^*(t)|n\rangle\langle k+2| \\ b_{n+2}(t)a_k^*(t)|n+2\rangle\langle k| & b_{n+2}(t)b_{k+2}^*(t)|n+2\rangle\langle k+2| \end{bmatrix} . \tag{16}$$

III. INCREASE OF THE SQUEEZING OF THE LIGHT DUE TO THE VIRTUAL-PHOTON PROCESSES

In order to analyze the squeezing properties of the radiation field, we introduce two Hermitian quadrature operators d_1 and d_2,

$$d_1=[a\exp(i\omega t)+a^\dagger\exp(-i\omega t)]/2 , \tag{17a}$$

$$d_2=[a\exp(i\omega t)-a^\dagger\exp(-i\omega t)]/2i , \tag{17b}$$

with the commutation $[d_1,d_2]=1/2i$, and the corresponding uncertainty relation $\langle\Delta d_1^2\rangle\langle\Delta d_2^2\rangle\gg 1/16$. If one of the uncertainties $\langle\Delta d_1^2\rangle$ or $\langle\Delta d_2^2\rangle$ of a state satisfies the relation $Q_i=\langle\Delta d_i^2\rangle-1/4<0$ $(i=1,2)$, the state is called the squeezed state [4,5]. It is easy to get that

$$\langle\Delta d_1^2\rangle=[2\langle a^\dagger a\rangle+1+\langle a^2\rangle\exp(i2\omega t)+\langle a^{\dagger 2}\rangle\exp(-i2\omega t)-\langle a\exp(i\omega t)+a^\dagger\exp(-i\omega t)\rangle^2]/4 , \tag{18a}$$

$$\langle\Delta d_2^2\rangle=[2\langle a^\dagger a\rangle+1-\langle a^2\rangle\exp(i2\omega t)-\langle a^{\dagger 2}\rangle\exp(-i2\omega t)+\langle a\exp(i\omega t)-a^\dagger\exp(-i\omega t)\rangle^2]/4 . \tag{18b}$$

Using the density matrix $\rho^I(t)$ and retaining the terms to ϵ/ω, we obtain

$$\begin{aligned}\langle a^\dagger a\rangle=\langle\rho^I(t)(a^\dagger a)^I\rangle=\bar n-1+e^{-\bar n}\sum_{n=-2}^{\infty}\frac{\bar n^{n+2}}{(n+2)!}\cos(2C_1t)\\ -\sin^2(C_1t)\cos(C_1t)\frac{\epsilon\bar n^2}{\sqrt{(n+5)(n+6)}}\left[\frac{\cos A_1t-1}{A_1}-\frac{\cos A_2t-1}{A_2}\right]\\ +\frac{\epsilon n(n-1)}{\bar n^2}\sqrt{(n+1)(n+2)}\left[\frac{\sin B_1t}{B_1}+\frac{\sin B_2t}{B_2}\right] ,\end{aligned} \tag{19}$$

$$\begin{aligned}\langle a\exp(i\omega t)\rangle=e^{-\bar n}\sum_{n=-2}^{\infty}\frac{\bar n^{(2n+5)/2}}{(n+2)!}e^{i\varphi}\Bigg\{\left[\frac{n+1}{n+3}\right]^{1/2}\sin(C_1t)\sin(C_2t)+\cos(C_1t)\cos(C_2t)\\ +\left[\frac{\cos A_1t-1}{A_1}-\frac{\cos A_2t-1}{A_2}\right]\\ \times\frac{\epsilon\bar n^2}{4\sqrt{(n+5)(n+6)}}\left[\left[\frac{n+1}{n+3}\right]^{1/2}\sin(2C_1t)\sin(C_2t)-2\sin^2(C_1t)\cos(C_2t)\right]\\ +\left[\frac{\sin B_1t}{B_1}+\frac{\sin B_2t}{B_2}\right]\\ \times\frac{n(n-1)C_1}{4\bar n^2}\left[\left[\frac{n+1}{n+3}\right]^{1/2}\cos^2(C_1t)\sin(C_2t)-\sin(2C_1t)\cos(C_2t)\right]\Bigg\} ,\end{aligned} \tag{20}$$

$$\langle a^2 \exp(i2\omega t)\rangle = e^{-\bar{n}} \sum_{n=-2}^{\infty} \frac{\bar{n}^{n+3} e^{i2\varphi}}{(n+2)!} \left\{ \left[\frac{(n+1)(n+2)}{(n+3)(n+4)} \right]^{1/2} \sin(C_1 t)\sin(C_3 t) + \cos(C_1 t)\cos(C_3 t) \right.$$
$$+ \left[\frac{\cos A_1 t - 1}{A_1} - \frac{\cos A_2 t - 1}{A_2} \right] \frac{\epsilon \bar{n}^2}{4\sqrt{(n+5)(n+6)}}$$
$$\times \left[\left[\frac{(n+1)(n+2)}{(n+3)(n+4)} \right]^{1/2} \sin(2C_1 t)\sin(C_3 t) - 2\sin^2(C_1 t)\cos(C_3 t) \right]$$
$$+ \left[\frac{\sin B_1 t}{B_1} + \frac{\sin B_2 t}{B_2} \right] \frac{n(n-1)C_1}{4\bar{n}^2} \left[2 \left[\frac{(n+1)(n+2)}{(n+3)(n+4)} \right]^{1/2} \cos^2(C_1 t)\sin(C_3 t) \right.$$
$$\left. \left. - \sin(2C_1 t)\cos(C_3 t) \right] \right\} , \tag{21}$$

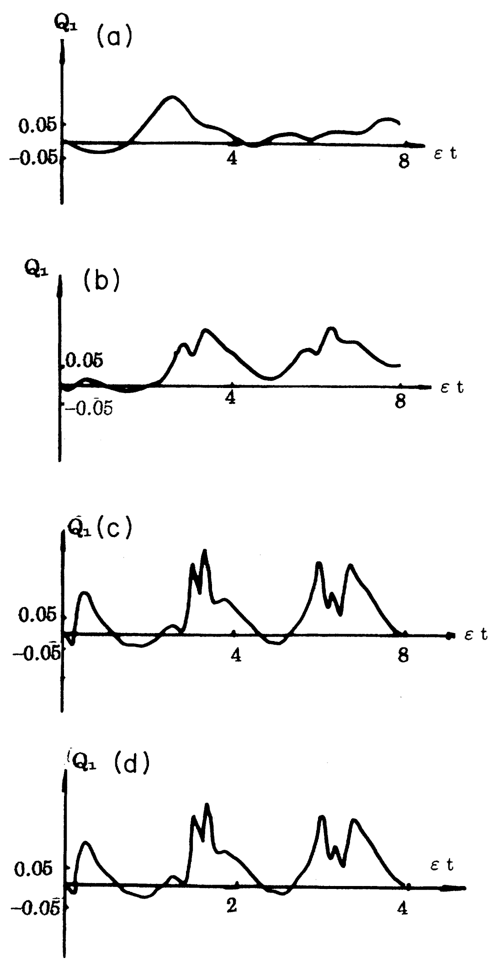

FIG. 1. The time evolution of the parameter Q_1 with the RWA for (a) $\omega=1$, $\epsilon=0.1$, $\bar{n}=1$; (b) $\omega=1$, $\epsilon=0.1$, $\bar{n}=4$; (c) $\omega=1$, $\epsilon=0.1$, $\bar{n}=6$; (d) $\omega=1$, $\epsilon=0.05$, $\bar{n}=6$.

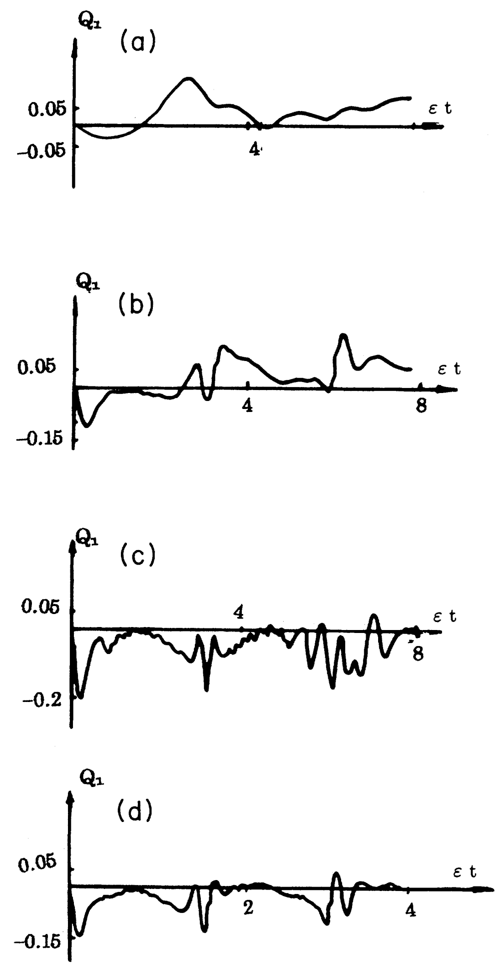

FIG. 2. The time evolution of the parameter Q_1 without the RWA for (a) $\omega=1$, $\epsilon=0.1$, $\bar{n}=1$; (b) $\omega=1$, $\epsilon=0.1$, $\bar{n}=4$; (c) $\omega=1$, $\epsilon=0.1$, $\bar{n}=6$; (d) $\omega=1$, $\epsilon=0.05$, $\bar{n}=6$.

where

$$C_2=\epsilon\sqrt{(n+2)(n+3)}, \quad C_3=\epsilon\sqrt{(n+3)(n+4)} \ .$$

Substituting Eqs. (19)–(21) into Eq. (18), we can discuss the squeezing properties of the radiation field with the aid of the numerical method.

In Fig. 2, we show the time evolution of Q_1 in the case $\varphi=0$, $n=1$, 4, 6 and $\epsilon=0.1$, 0.05 without the RWA. Here the value of the mean photon number $\bar{n}$ and the coupling constant ϵ ensure that the amplitudes of

$$\epsilon\sqrt{(n+3)(n+4)}/A_1, \quad \epsilon\sqrt{(n+3)(n+4)}/A_2 \ ,$$
$$\epsilon\sqrt{(n-1)n}\ /B_1, \quad \epsilon\sqrt{(n-1)n}\ /B_2$$

are not very large. In Fig. 1, we plot the time evolution of Q_1 with the RWA, which was predicted early [10–12]. Comparing Fig. 1 with Fig. 2, we find that the squeezing degree of the component $\langle \Delta d_1^2 \rangle$ of the field without the RWA increases by comparison with the result in the RWA, and the time of the continuous squeezing phenomenon is lengthened due to the virtual-photon processes. We also note a particular time interval of squeezing that is absent in the two-photon JC model with the RWA because of the influence of the virtual-photon processes. This result is in agreement with that of Lais *et al.* on the one-photon JC model [14]. The cause inducing this result is the interference between the virtual-photon processes and the real-photon processes in the atom-field coupling system, which reflect the appearance of terms containing the first order of ϵ/ω in $\langle \Delta d_1^2 \rangle$ [23–25].

Comparing Fig. 2 with Fig. 1 we can also see that the change of Q_1 is not evident where $\bar{n}$ is small ($\bar{n}=1$) due to the virtual-photon processes. But with an increase in the mean photon number $\bar{n}$ ($\bar{n}=4,6$), the changes in the degree of squeezing and squeezing time region are clear. This means that the rotation-wave approximation is legitimate only when the radiation field is weak [27]. From Figs. 2(b)–2(d), it is easily seen that the appearance of squeezing is periodic. The periodic property is the same as that described by Figs. 1(c) and 1(d) [10–12]. The reason is that we only discuss the weak field here. So we can neglect the influence on oscillating periodic time due to the virtual-photon processes [24]. If the field is intensive, the rotating-wave approximation is unsuitable, and the oscillating periodic property can markedly change [19,28].

Furthermore, from Figs. 2(c) and 2(d), we can find that not only the oscillating frequency of Q_1 but also the amplitude of Q_1 changes markedly in the two-photon JC model without the RWA in different ϵ. But in the RWA as shown in Figs. 1(c) and 1(d), the degree of squeezing predicted in Refs. [10–12], only changes with $\bar{n}$ and φ, and is not related to ϵ and ω. Here, our present result shows that the influence of the virtual-photon processes on the degree of squeezing is related to not only the intensity of the field but also the optical frequency ω and the atom-field coupling constant ϵ. The reason is that there appear the first-order terms in ϵ/ω in Eqs. (19)–(21) due to the interference between the real-photon processes and the virtual-photon processes. So, considering the effect of the virtual-photon field, the different optical frequency and the coupling constant will induce a different degree of squeezing of the field in the atom-field coupling system, even if the intensity of the field is the same. The degree of squeezing is completely decided by the character of the atom-field coupling system. Thus, if the effect of the virtual-photon field in the atom-field coupling system is taken into account, the atomic behavior [21,23,24], and the phase properties of the field [25] as well as the nonclassical properties of the field are all dependent on the optical frequency ω, the field intensity $\bar{n}$, and the coupling constant ϵ.

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