Intermittent chaos in Hamiltonian systems: The three-dimensional hydrogen atom in magnetic fields

P. Schmelcher and L. S. Cederbaum
Theoretische Chemie, Physikalisch—Chemisches Institut, Universität Heidelberg, Im Neuenheimer Feld 253, W-6900 Heidelberg, Federal Republic of Germany
(Received 27 May 1992; revised manuscript received 11 September 1992)

The trajectories of the highly excited hydrogen atom with nonvanishing pseudomomentum **K** are investigated. As a characteristic feature, we observe the intermittent behavior of the internal as well as of the center-of-mass motions which consist of alternating phases of strongly chaotic and quasiregular motion. During these phases, interesting physical phenomena appear. These phenomena as well as the intermittent behavior of the trajectories can be explained by the observation that, due to the finite nuclear mass, an additional confining potential appears. This confining potential has strong impact on the ionization of the hydrogen atom in general. We also investigate the correlation between the values of the internal angular momentum L_z and the internal coordinate ρ. In fact, we are able to control intermittency by choosing different values of the energy, pseudomomentum, and magnetic-field strength.

PACS number(s): 31.15.+q, 05.45.+b, 32.60.+i

I. INTRODUCTION

The hydrogen atom in a strong homogeneous magnetic field is one of the simplest physical systems which exhibits a transition from regularity to chaos. Classically this was shown by looking at Poincaré sections and Lyapunov exponents and quantum mechanically by studying the energy-level statistics of the atom [1]. A large number of electronic and energy levels even up to the field-free ionization threshold are nowadays known experimentally [2] as well as theoretically [3,4] to a high degree of accuracy. However, almost all theoretical investigations on the hydrogen atom in a magnetic field deal only with a special case: they neglect the influence of the collective on the internal electronic motion of the atom.

The Hamiltonian of a neutral atom in a homogeneous magnetic field possesses a constant of motion, the pseudomomentum [5] **K**, which is closely related to the collective motion of the atom. By introducing the center-of-mass coordinate and the pseudomomentum as a canonically conjugated pair of variables it is possible to eliminate the center-of-mass coordinate from the Hamiltonian and perform a so-called pseudoseparation of the center-of-mass motion. For a vanishing magnetic field the hydrogen atom is essentially a one-dimensional radial problem and cannot exhibit chaos. At a finite field strength and $\mathbf{K}=0$, which is the case extensively studied in the literature, we encounter a two-dimensional problem exhibiting chaos. For $\mathbf{K}\neq 0$, the symmetry is further reduced, leading to a truly three-dimensional problem related to the relevant situation of a hydrogen atom in crossed electric and magnetic fields. This leads to new phenomena, in particular, as we shall see, to the appearance of intermittency in the classical mechanics of the system. Intermittency in Hamiltonian systems or discrete area-preserving maps has so far been investigated only for a few cases (see, for example, Refs. [6] and [7], respectively).

The investigation of intermittency in the internal motion of the hydrogen atom and its illuminating relation to the motion of the center of mass of the atom is the central issue of this work. After the pseudoseparation the center of mass and internal motion of the atom remain intimately coupled and this coupling leads to interesting physical phenomena. In Refs. [8] and [9] the classical center-of-mass motion of the hydrogen atom in a magnetic field has been investigated for the special case $\mathbf{K}=0$ and angular momentum $L_z \geq 0$. One of the most striking results of these investigations was the fact that the center-of-mass motion undergoes a transition from confinement, i.e., motion in a bounded range of coordinate space, to deconfinement, i.e., unbounded motion, if the internal motion passes from regularity to chaos. In particular, it was shown that the chaotic center-of-mass motion exhibits many properties of a random motion and obeys a linear diffusion law. In the present paper we investigate the classical internal relative—and center-of-mass motion for the general case $\mathbf{K}\neq 0$ of the hydrogen atom, which has not been studied so far in the literature.

II. HAMILTONIAN AND THE INFLUENCE OF THE FINITE NUCLEAR MASS

The Hamiltonian which usually appears in the literature on the hydrogen atom in a magnetic field reads as follows:

$$H_1=\frac{1}{2\mu}\left[\mathbf{p}-\frac{e}{2}\mathbf{B}\times\mathbf{r}\right]^2-\frac{e^2}{|\mathbf{r}|}\ , \qquad (1)$$

where $(\mathbf{r},\mathbf{p})$ is the canonical conjugated pair of variables for the internal relative motion. **B** is the magnetic-field vector, which is in the following assumed to point along the z direction. μ was in the literature chosen to be either the reduced mass of the electron and the nucleus or the electron mass. For both cases the Hamiltonian H_1 does not correctly take into account the effects due to the finite nuclear mass and is, therefore, not able to describe the correct two-body behavior of the hydrogen atom in a magnetic field. The exact Hamiltonian which is derived by the canonical pseudoseparation of the center-of-mass motion (see [Refs. [5] and [9]) takes on the following appearance:

$$H=\frac{1}{2M}(\mathbf{K}-e\mathbf{B}\times\mathbf{r})^2+\frac{1}{2\mu}\left[\mathbf{p}-\frac{e}{2}\frac{\mu}{\mu'}\mathbf{B}\times\mathbf{r}\right]^2-\frac{e^2}{|\mathbf{r}|}\,, \tag{2}$$

where $\mathbf{K}$ is the constant pseudomomentum. $\mu=(mM_0/M)$ and $\mu'=[mM_0/(M_0-m)]$ are different reduced masses and m, M_0, and M are the mass of the electron, the nucleus, and the total mass, respectively. It is only for the special case of vanishing pseudomomentum $\mathbf{K}$ and vanishing internal angular momentum $L_z=0$ that the two Hamiltonians H_1 and H are identical [9].

The coupling term between the collective and internal relative motion of the hydrogen atom is given by the motional Stark term $(e/M)(\mathbf{B}\times\mathbf{K})\cdot\mathbf{r}$. Due to its collective motion the hydrogen atom experiences an additional constant electric field which is oriented perpendicular to the magnetic field. As already mentioned in the Introduction the hydrogen atom with $\mathbf{K}\neq\mathbf{0}$ is, therefore, closely related to the relevant situation of the H atom in crossed electric and magnetic fields. The Hamiltonian H_2 used in the literature to describe the hydrogen atom in crossed fields can be obtained from the Hamiltonian H_1 by simply adding the Stark term $H_2=H_1+e\mathbf{E}\cdot\mathbf{r}$, where $\mathbf{E}$ is the electric field.

There exists now an important difference between the Hamiltonian H_2 in crossed fields, which does not correctly take into account the effects due to the finite nuclear mass, and the exact two-body Hamiltonian H. In H_2, the potential for the internal motion is $V_2=-e^2/|\mathbf{r}|+e\mathbf{E}\cdot\mathbf{r}$ and the corresponding kinetic energy is given by the first term on the right-hand side (rhs) of Eq. (1) [10]. In H the kinetic energy of the internal motion has a slightly different appearance [second term on the rhs of Eq. (2)], but more importantly the acting potential $V=-e^2/|\mathbf{r}|+e\mathbf{E}\cdot\mathbf{r}+(e^2/2M)(\mathbf{B}\times\mathbf{r})^2+\mathbf{K}^2/2M$, where $\mathbf{E}=(1/M)(\mathbf{B}\times\mathbf{K})$, has an additional potential term $(e^2/2M)(\mathbf{B}\times\mathbf{r})^2$ which arises from the first quadratic term on the rhs of Eq. (2). This potential does not appear in the Hamiltonian H_2 and vanishes for infinite nuclear mass. Taking into account the effects due to the finite nuclear mass, therefore, changes the potential for the atomic motion and is, as we shall see below, the origin of the intermittent behavior of the trajectories.

In Fig. 1 we have illustrated the potential energies for the two Hamiltonians H_2 and H. Figure 1(a) shows an intersection of the potential energy V_2 of the Hamiltonian H_2 along the direction of the external electric field. First we observe the well-known fact [10,11] that this potential possesses a so-called Stark saddle point. Below the saddle-point energy the relative motion of the system is bounded and above this threshold energy the system is unbounded and can, at least in principle, ionize. Figure 1(b) shows the exact potential curve V for the hydrogen atom with $\mathbf{K}\neq\mathbf{0}$ in a magnetic field which includes all effects due to the finite nuclear mass. In addition to the Stark and Coulomb potential [see Fig. 1(a)] a diamagnetic potential term $(e^2/2M)(\mathbf{B}\times\mathbf{r})^2$ is now present. This leads to the fact that for energies above the new diamagnetic Stark saddle-point energy the relative motion is still bounded, i.e., confined to some finite coordinate range. The hydrogen atom in a magnetic field, therefore, cannot ionize in the direction perpendicular to the magnetic field. We shall see later on that only for energies above the new diamagnetic Stark saddle-point energy the intermittent behavior of the trajectories of the atom can be observed. We mention that the saddle point of the potential shown in Fig. 1(b) does not persist for all values of the pseudomomentum, magnetic-field strength, and energies. By increasing the field strength it is always possible to destroy the saddle-point structure of the potential. Also then intermittency may occur because of the available extended range of the coordinates perpendicular to the magnetic field.

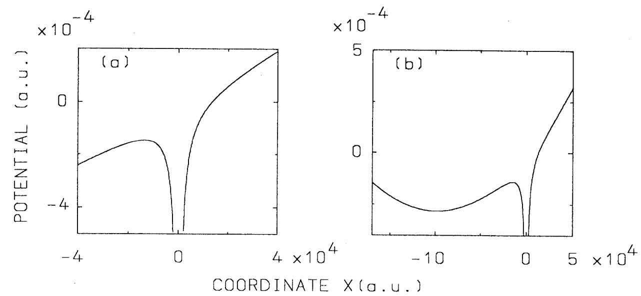

FIG. 1. (a) Combined potential V_2 for a Coulomb and external electric field shown along the direction of the electric field. For the field strength of the external field we have taken the same value and direction as for the motional Stark field of (b), i.e., $\mathbf{E}=(1/M)(\mathbf{K}\times\mathbf{B})$. (b) Combined potential V of the Coulomb, the motional Stark field, and the diamagnetic potential term $(e^2/2M)(\mathbf{B}\times\mathbf{r})^2$. Values of the pseudomomentum and field strength are $\mathbf{K}=(0.0,1.0,0.0)$ and $B=10^{-5}$ a.u. (The constant term $\mathbf{K}^2/2M$ is not included.)

The impact of the finite nuclear mass is drastic. As discussed above, the electron can only escape (ionize) in the direction of the magnetic field due to the finite mass of the nucleus. At infinite separation the Coulomb term disappears and the total energy is semipositive definite [see Eq. (2)]. For $\mathbf{K}\neq\mathbf{0}$, the ionization threshold of the hydrogen atom, i.e., the minimal energy to achieve an infinite separation of the electron and the nucleus, is at $E=0$. This obviously corresponds to $z\rightarrow\infty$ and finite x and y [see Eq. (2); $\mathbf{B}$ is parallel to the z axis]. Finally we present the equations of motion belonging to the Hamiltonian H

$$\dot{\mathbf{R}}_S=\frac{1}{M}\mathbf{K}-\frac{e}{M}(\mathbf{B}\times\mathbf{r})\,, \tag{3a}$$

$$\dot{\mathbf{r}}=\frac{1}{\mu}\mathbf{p}-\frac{e}{2\mu'}(\mathbf{B}\times\mathbf{r})\,, \tag{3b}$$

$$\dot{\mathbf{p}}=-\frac{e}{M}(\mathbf{B}\times\mathbf{K})-\frac{e}{2\mu'}(\mathbf{B}\times\mathbf{p})+\frac{e^2}{4\mu}\mathbf{B}\times(\mathbf{B}\times\mathbf{r})-e^2\frac{\mathbf{r}}{|\mathbf{r}|^3}\,, \tag{3c}$$

where $\mathbf{R}_S$ is the center-of-mass coordinate. Equation (3a) clearly shows that, apart from a purely translational term $(\mathbf{K}/M)t$, the center-of-mass motion is completely determined by the internal relative coordinate $\mathbf{r}$. The only influence of the center-of-mass motion on the internal motion is given by the constant inhomogeneous Stark term $-(e/M)(\mathbf{B}\times\mathbf{K})$ in Eq. (3c). However, as we shall see, the presence of this term changes the classical dynamics of the internal and consequently of the center-of-mass degrees of freedom drastically.

III. RESULTS

The typical new phenomenon for the trajectories of the highly excited hydrogen atom with nonvanishing pseudomomentum $\mathbf{K}\neq\mathbf{0}$ is their intermittent behavior. Intermittency means that the trajectory alternately shows both quasiregular and chaotic phases. During the quasiregular phases the trajectory stays near tori in phase space and looks very regular in comparison with the chaotic phases of the motion. Figure 2 shows for a *typical* trajectory (we have run hundreds of trajectories in order to obtain an overview of their behavior in different regions of phase space) the projection of the internal motion on a plane perpendicular to the magnetic-field axis. One immediately realizes that there exist two alternating types of motion. During one phase of motion the electron and the nucleus are in the x,y plane close together and this shows up through the black bubble on the very rhs of Fig. 2. During this phase of motion the Coulomb and diamagnetic interactions are of comparable order of magnitude and the trajectory is, therefore, chaotic. Within the concept of the determination of local Lyapunov exponents [12,13], this means that we obtain a nonvanishing local Lyapunov exponent. During the other regular-looking phase the electron and the nucleus move far apart from each other. The relative motion in the x,y plane then approximately takes place on a circle with a large radius. The Coulomb energy provides here only a small correction to the strongly dominating magnetic interaction. This phase of motion has an essentially vanishing local Lyapunov exponent.

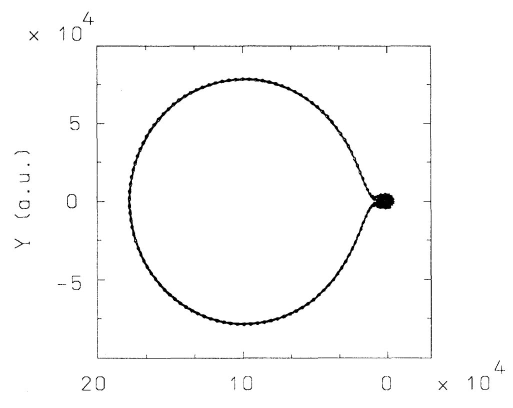

FIG. 2. Typical intermittent trajectory of the internal relative motion for nonvanishing pseudomomentum (projection onto the x,y plane, the magnetic field is oriented along the z axis). Field strength, energy, and pseudomomentum are $B=10^{-5}$, $E=1.722\times10^{-4}$, $\mathbf{K}=(0.0,1.0,0.0)$. Initial conditions are $\mathbf{r}=(0.7,0.8,0.4)$ and $\mathbf{p}=(0.7,0.8,p_z)$ where P_z is adapted to the energy shell. All values in atomic units.

The radius of the circle of the regular-looking phase of the internal x,y-motion of an intermittent trajectory can be understood in terms of the pseudomomentum $\mathbf{K}$. For two oppositely charged *free* particles in a magnetic field the total pseudomomentum is closely related to the guiding centers [5] $\mathbf{r}_{L_1},\mathbf{r}_{L_2}$, i.e., the centers of the classical Landau orbits of the individual particles. In the laboratory (Cartesian) coordinate system we obtain

$$\mathbf{K}=e\mathbf{B}\times[\mathbf{r}_{L_1}-\mathbf{r}_{L_2}]\ . \tag{4}$$

This means that the total pseudomomentum is proportional to the cross product of the magnetic-field vector and the distance vector between the two guiding centers of the free particles. Since the magnetic strongly dominates the Coulomb interaction during the quasiregular circular motion shown in Fig. 2, Eq. (4) can be applied. The radius r of the circle is then approximately given by

$$r=-\frac{1}{eB^2}|\mathbf{B}\times\mathbf{K}|\ , \tag{5}$$

i.e., it is completely determined by the magnetic-field vector and the pseudomomentum.

On the other hand, we obtain a completely different interpretation of the pseudomomentum if the electron and the nucleus are very close together. In the latter case the Coulomb dominates the magnetic interaction and hence the pseudomomentum is approximately the linear kinetic momentum of the translational center-of-mass motion [the second term on the rhs of Eq. (3a) is then small].

Figure 3 shows the center-of-mass motion for the trajectory whose internal motion is given in Fig. 2. It consists of alternating phases of purely translational and circular motions. As already mentioned, the electron and the nucleus are strongly bounded, i.e., close together,

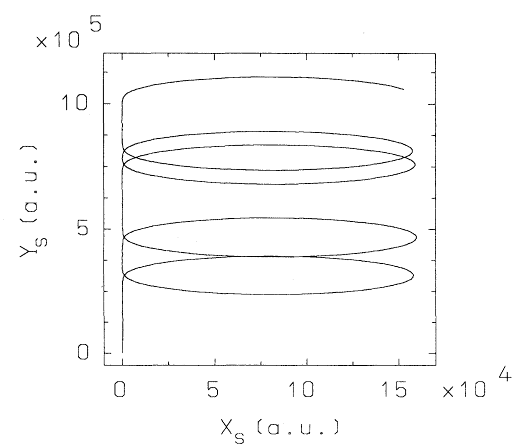

FIG. 3. Center-of-mass motion belonging to the internal motion shown in Fig. 1. Initial conditions are as in Fig. 1 and $\mathbf{R}_S=(0.0,0.0,0.0)$. All values in atomic units.

during the time interval of chaotic internal motion. This is precisely the time period during which the center of mass performs a purely translational motion. The time periods of quasiregular circular internal motion correspond to the periods of circular center-of-mass motion. Intermittency, therefore, shows up in the center-of-mass motion by alternating phases of more or less straight-lined and circular motion.

Let us now look at the motion of the electron and the nucleus in a plane perpendicular to the magnetic field in the *laboratory coordinate system*. During the quasiregular phases of motion the electron is localized in a small range of coordinate space whereas the nucleus performs the large amplitude motion on the circle shown in Fig. 2. At first glance this statement seems to contradict the traditional physical picture in which the light electron moves around the heavy nucleus while the latter performs only tiny vibrations. However, since the Coulomb energy provides only a small perturbation to the magnetic interaction during the quasiregular phase of motion we expect the nucleus and the electron to perform their individual cyclotron motions which are more or less perturbed by the Coulomb interaction. The radius of the cyclotron motion for the nucleus is due to its bigger mass, much larger than that of the electron. The strong localization of the electron and the large amplitude motion of the nucleus is, therefore, a characteristic feature for the case of a strongly dominating magnetic field. Before entering the investigation of the intermittent motion we briefly discuss some additional properties of the intermittent trajectories.

According to our extensive numerical study intermittent trajectories occur over a wide range of values of the energy and the pseudomomentum and can be obtained for the highly excited bounded hydrogen atom, i.e., for $E<0$, as well as for unbounded energies of this system, i.e., for $E>0$. Intermittency is, therefore, not a property of the trajectories of the strongly bound hydrogen atom but a phenomenon which occurs slightly below or above the ionization threshold which lies at $E=0$. According to the potential picture discussed in Sec. II the intermittent behavior of trajectories can only be observed for energies larger than the diamagnetic Stark saddle-point energy. For energies below this point the motion is confined to a small range of relative coordinates where the Coulomb and magnetic forces are both relevant. Above this threshold the allowed coordinate space perpendicular to the magnetic field [see Fig. 1(b)] is greatly extended and the particles can depart from each other reducing thereby their mutual interaction. This coincides with our observation of the occurrence of the quasiregular phase of motion. Because of the confinement potential V, the particles are bound to return to each other and this gives rise to the observed chaotic phase of motion. It is possible to predict whether intermittency will occur or not depending on the values of the energy, the pseudomomentum, and the magnetic-field strength.

In the following we would like to discuss the properties of the intermittent trajectories from the point of view of the behavior of the internal relative angular momentum L_z. For nonvanishing pseudomomentum only the *total* angular momentum component $\mathcal{L}_z$, but not the relative angular momentum L_z, is a conserved quantity. The angular momentum L_z, therefore, varies with time and can, in particular, take on negative and positive values. The phase space which is available for the trajectories, however, depends now strongly on the value of the angular momentum L_z. In order to understand this point and the above-mentioned "reversed roles" of the electron and the nucleus let us begin our discussion by considering a *free* electron in a magnetic field. Then L_z is a conserved quantity. The minimal (kinetic) energy the electron can have depends on the sign of the angular momentum L_z: for $L_z>0$ it is $E_{\min}=(|e|B/m)L_z$, whereas for $L_z<0$ we obtain $E_{\min}=0$ (m is the electron mass). $E_{\min}=0$ means that the electron is fixed in phase space. Therefore, in a magnetic field the electron can possess no kinetic energy at all and still have a finite negative angular momentum $L_z<0$. This already shows that negative values of the angular momentum are distinct for the electron.

As a next step we consider the ionization thresholds for the hydrogen atom with $\mathbf{K}=\mathbf{0}$. For positive values of the conserved relative angular momentum we obtain $E_{\text{th}}=(|e|B/m)L_z$, whereas for negative values we obtain $E_{\text{th}}=-(|e|B/M_0)L_z$, where M_0 is the mass of the nucleus. If we compare these results with the above-discussed minimal kinetic energy of a free electron (nucleus) in a magnetic field we immediately realize that for positive relative angular momentum L_z the threshold kinetic energy E_{th} is due to the motion of the electron and the nucleus stands still whereas for negative values of L_z the energy E_{th} is due to the motion of the nucleus and the electron is frozen.

More formally this result can also be derived by inspecting the equation of motion of the azimuthal angle φ which is the canonical coordinate belonging to the momentum L_z

$$\dot{\varphi}=-\frac{eB}{2\mu'}+\frac{L_z}{\mu\rho^2}\ , \tag{6}$$

where $\rho=(x^2+y^2)^{1/2}$. At the ionization thresholds we have $\rho=\rho_{\min}=(2|L_z|/|e|B)^{1/2}$ and consequently

$$\dot{\varphi}=\begin{cases}-\dfrac{|e|B}{M_0}<0 & \text{for } L_z<0\\[2mm] +\dfrac{|e|B}{m}>0 & \text{for } L_z>0\ .\end{cases} \tag{7}$$

Negative values of the sense of rotation can only be obtained by the rotation of a positive charge, i.e., the nucleus around the magnetic-field axis, whereas positive values of $\dot{\varphi}$ have their origin in the motion of the electron. We therefore conclude that for negative angular momentum values $L_z<0$ and for energies near the ionization threshold the nucleus of the atom performs the motion and the electron is localized in space. This is in strong contrast to the traditional physical picture of the hydrogen atom which tells us that the electron moves around the nucleus. This picture is, as we have seen, in general only correct for positive values of the angular momentum L_z.

We return to our discussion of the intermittent trajectories for the hydrogen atom with $\mathbf{K}\neq 0$. We prove now that the large amplitude motion of the quasiregular phases of the internal motion (see Fig. 2) is always associated with large negative values of the angular momentum L_z. According to the above discussion, the large amplitude motion can then only be performed by the nucleus whereas the electron is localized in space. As a first step we transform the underlying Hamiltonian from Cartesian to cylindrical coordinates (ρ,φ,z). The Hamiltonian is then a function of the three canonical pairs of internal relative variables $(\{\rho,p_\rho\},\{\varphi,L_z\},\{z,p_z\})$ and depends on two parameters: the pseudomomentum and the magnetic-field strength. In order to obtain the allowed range of angular momentum values L_z for fixed energy, pseudomomentum, and magnetic-field strength we express the angular momentum as $L_z = L_z(\rho,\varphi,z;p_\rho,p_z;E,\mathbf{B},\mathbf{K})$ using the Hamiltonian function. The extremal values L_z with respect to $(\varphi,z;p_\rho,p_z)$ are then obtained by setting the corresponding partial derivatives equal to zero. The result is $p_\rho = p_z = 0$, $z=0$, and $\varphi = n\cdot\pi$, where n is an integer. L_z remains a function of the distance ρ and takes on the following appearance:

$$L_z=\left[-\frac{B\mu}{2\mu'}\rho^2\pm\rho\left[-\frac{\mu}{M}K_y^2+2\mu E-\frac{\mu}{M}B^2\rho^2+\frac{2\mu}{\rho}+\frac{2\mu}{M}BK_y\rho\right]^{1/2}\right], \tag{8}$$

where we have, without loss of generality, assumed that the pseudomomentum points along the positive y direction, i.e., $\mathbf{K}=(0,K_y,0)$ and $K_y>0$. In order to obtain a real value for the angular momentum L_z the argument of the square root in Eq. (8) must be positive and this criterion gives us the allowed range of (L_z,ρ) pair values on the energy shell. In Fig. 4 we show this range, which has the form of a club exemplary for the energy $E=-5\times10^{-5}$, and the pseudomomentum $K_y=0.3$. [These values are different from those of the trajectory shown in Figs. 2, 3, and 5. However, the shape of the (L_z,ρ) phase-space diagram is similar for both cases but can be illustrated more easily for $K_y=0.3$ and $E=-5\times10^{-5}$.] We immediately realize that large ρ values are always associated with large negative values of the angular momentum L_z, which extend in our example of Fig. 4 down to approximately -1750. In contrast to this, positive values of the angular momentum L_z are restricted to a small range of relatively small values of ρ.

The quasiregular and chaotic phases of the intermittent trajectories (see Fig. 2) can now also be discussed in the context of our phase diagram of Fig. 4. During the quasiregular phases of the trajectory, i.e., the large amplitude motion in the coordinate ρ, we have always large negative values of the angular momentum L_z and the magnetic interaction dominates strongly over the Coulomb energy. According to our above arguments on the threshold behavior of the hydrogen atom for positive and negative values of the angular momentum, this large amplitude motion in the x,y plane of the laboratory coordinate system is performed by the nucleus, whereas the electron is strongly localized. During the chaotic phase of the intermittent trajectory the electron and the nucleus are close together and interact strongly via the Coulomb potential. According to the shape of the phase-space diagram in Fig. 4 both small negative and small positive values of the angular momentum L_z are then allowed. The trajectory will show chaotic behavior until it finds its "way out" to regions of phase space with large negative values of L_z, where it behaves quasiregular.

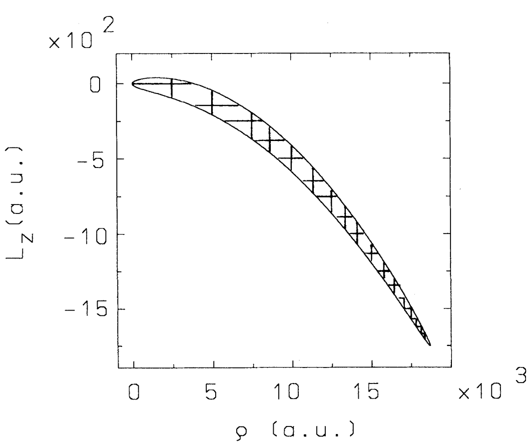

FIG. 4. Range of (L_z,ρ) values on the energy shell for the energy value $E=-5\times10^{-5}$. Pseudomomentum and magnetic-field strength are $K_y=0.3$ and $B=10^{-5}$. All values in atomic units.

With the help of Eq. (8) and the resulting (L_z,ρ) plot or, equivalently, with the aid of the acting potential V [note that $E-V$ is essentially what appears in the square root in Eq. (8)] it is therefore in general, i.e., for given values of the energy, pseudomomentum, and magnetic-field strength, possible to predict whether a trajectory of the highly excited hydrogen atom has phases of quasiregular motion and, in particular, to what extent a large amplitude motion in the coordinate ρ is allowed. Or, in other words, by choosing appropriate values of the pseudomomentum, energy, and magnetic-field strength, we can control the intermittent behavior of our trajectory.

To complete our picture of intermittent trajectories we present in Figs. 5(a) and 5(b) the internal relative z coordinate and L_z as a function of time for the same typical trajectory whose internal x,y motion and center-of-mass motion are shown in Figs. 2 and 3. During the chaotic phases of the trajectory the z coordinate is relatively small which corresponds to the picture that the electron and the nucleus are close together. During the quasiregular phases, i.e., the phases for which the internal x,y

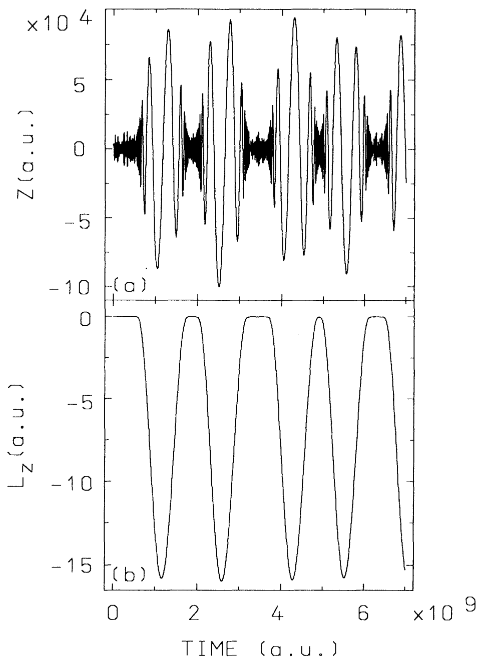

FIG. 5. (a) Motion of the internal relative z coordinate as a function of time. Initial conditions and parameter values as given in Fig. 1. (b) Relative angular momentum component L_z along the magnetic-field axis as a function of time for the trajectory shown in Figs. 1, 2, and 4(a). All values in atomic units.

motion shows an approximately circular motion with a large radius, the z coordinate shows regular-looking oscillations with large amplitude. In the laboratory coordinate system this large z amplitude motion is performed by the electron.

In the time dependency of the angular momentum L_z shown in Fig. 5(b) we clearly observe that the quasiregular phases of motion correspond to large negative values of the angular momentum L_z, whereas the chaotic phases of motion correspond to small values of L_z in correspondence with the above general considerations [see Eq. (8) and below].

IV. BRIEF CONCLUSIONS

We have investigated the trajectories of the highly excited hydrogen atom with nonvanishing pseudomomentum $\mathbf{K}$ in a homogeneous magnetic field. As a typical new phenomenon which does not or only marginally occur for $\mathbf{K}=0$, we have established the intermittent behavior of the internal as well as center-of-mass motion. The intermittent behavior of the trajectories could be illuminated by considering the potential terms in the exact Hamiltonian for the hydrogen atom in a magnetic field. This potential V consists of a Coulomb term, an electric field, and a diamagnetic energy term which appears because the nuclear mass is finite. This latter potential term is relevant for the phenomena observed here and markedly influences the ionization pattern of the hydrogen atom in general crossed electric and magnetic fields.

By inspecting the values of the angular momentum L_z and the coordinate ρ on the energy shell we could show that the quasiregular phases of the intermittent motion are always characterized by large negative values of the angular momentum L_z. Motion with large negative values of L_z, however, can, in the laboratory coordinate system, only be performed by the nucleus. We therefore end up with the interesting physical picture that the approximately circular motion of the quasiregular phases of the intermittent trajectory is performed by the nucleus.

More precisely, in the laboratory coordinate system the electron is localized in the x,y plane and performs a regular large amplitude motion in the z direction whereas the nucleus approximately is localized in the z direction and performs an almost circular motion with a large radius in the x,y plane. Whenever the nucleus returns to the waiting electron a chaotic phase does take place. During the chaotic phases of motion the electron and the nucleus are close together and perform a purely translational center-of-mass motion. Moreover, we are able to control the intermittent behavior of the trajectory in the sense that we can systematically choose those values of the energy, pseudomomentum, and magnetic-field strength for which an extended range of (L_z,ρ) values on the energy shell is available (or, equivalently, the energy is above the saddle point of the acting potential V) making the quasiregular phases of motion possible.

ACKNOWLEDGMENTS

We thank H.-D. Meyer for helpful discussions. The Deutsche Forschungsgemeinschaft is gratefully acknowledged for financial support.

[1] H. Friedrich and D. Wintgen, Phys. Rep. **183**, 37 (1989), and references therein.
[2] A. Holle, G. Wiebusch, J. Main, B. Hager, H. Rottke, and K. H. Welge, Phys. Rev. Lett. **56**, 2594 (1986); A. Holle, G. Wiebusch, J. Main, K. H. Welge, G. Zeller, G. Wunner, T. Ertl, and H. Ruder, Z. Phys. D **5**, 279 (1987).
[3] D. Delande and J. C. Gay, Phys. Rev. Lett. **57**, 2006 (1986).
[4] G. Wunner, U. Woelk, I. Zech, G. Zeller, T. Ertl, F. Geyer, W. Schweitzer, and H. Ruder, Phys. Rev. Lett. **57**, 3261 (1986).
[5] J. E. Avron, I. W. Herbst, and B. Simon, Ann. Phys. (NY) **114**, 431 (1978).
[6] G. Stolovitzky and J. A. Hernando, Phys. Rev. A **43**, 2774 (1991).
[7] A. B. Zisook, Phys. Rev. A **25**, 2289 (1982).
[8] P. Schmelcher and L. S. Cederbaum, Phys. Lett. A **164**, 305 (1992).
[9] P. Schmelcher and L. S. Cederbaum, Z. Phys. D **24**, 311 (1992).
[10] C. W. Clark, E. Korevaar, and M. G. Littman, Phys. Rev. Lett. **54**, 320 (1985).
[11] J. Main and G. Wunner, Phys. Rev. Lett. **69**, 586 (1992).
[12] H. Fujisaka, Prog. Theor. Phys. **70**, 1264 (1983).
[13] P. Grassberger and I. Procaccia, Physica (Amsterdam) **13D**, 34 (1984).