Scheme for measuring a Berry phase in an atom interferometer

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We present a concept for measuring a Berry phase using an atom interferometer in an optical Ramsey arrangement. The adiabatic cyclic process needed for the appearance of a Berry phase consists of the adiabatic change of population in one of the interferometer arms. A closed circuit in parameter space is realized by the time-dependent Doppler detuning of two additional strongly focused Gaussian beams, which couple selectively to one of the interferometer arms. It is shown that the resulting additional atomic phase consists of a Berry phase corresponding to the phase difference of the two Gaussian beams and a dynamical phase caused by the dynamical Stark effect. By recombining the two interferometer arms this total phase gives rise to a shift of the interference pattern. In case of a Ramsey arrangement with magnesium atoms the parameters of an experimental realization have been calculated. Further improvements using laser-cooled and trapped atoms are discussed.

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I. INTRODUCTION

The recent realization of atom interferometers has opened fields in matter wave interferometry, stimulating theoretical and experimental investigations of fundamental quantum-mechanical questions. This development is based as well on "classic" atom interferometers formed by microfabricated matter gratings [1] or double slits [2] as on arrangements with laser beams as atomic beam splitters. The geometry of an optical four-beam Ramsey arrangement [3–5], which has been interpreted by Bordé as an atom interferometer [6] shows promising properties and has lead to interesting investigations [7,8]. Here we examine its possibility for measuring a Berry phase [9].

Figure 1 shows schematically the coherent splitting and recombination of the atomic wave in a Ramsey atom interferometer. In the terminology of interferometry the first laser beam acts as a beam splitter, coherently dividing the atomic wave function into two components. These components differ in their external states, caused by the transferred photon momentum, as well as in their internal quantum states due to the resonant absorption or stimulated emission processes. The arrangement of the following three lasers facilitates the recombination of the two interferometer arms. Because of their different energies the two wave components travel with different dynamical phases which interfere after passing through the fourth interaction zone. Compared to interferometers using mechanical beam splitters the Ramsey interferometer has the advantage that the two arms are labeled by different internal atomic states. This enables the selective interaction with one arm even without the need for a spatial restriction of the interaction zones to this arm. Based on these ideas we report on a concept for measuring a Berry phase [9], i.e., a special geometrical phase in quantum mechanics [10].

According to its definition the appearance of the Berry phase requires an adiabatic cyclic process in parameter space: We thus consider a quantum system described by a Hamiltonian $H(\mathbf{R})$ which depends on a set of external parameters $\mathbf{R}$ varying slowly in time such that the adiabatic theorem is satisfied. Berry has shown that under these conditions the phase of the wave function, satisfying the time-dependent Schrödinger equation, does not only consist of the dynamical but also of a geometrical part. If the adiabatic parameter $\mathbf{R}$ describes a closed curve C in parameter space $(C:[t_0,t_e]\rightarrow\mathbb{R}^3, t\mapsto\mathbf{R}(t))$, this phase will be independent of the phase chosen for the stationary eigenstates. This geometrical phase γ is given by [9]

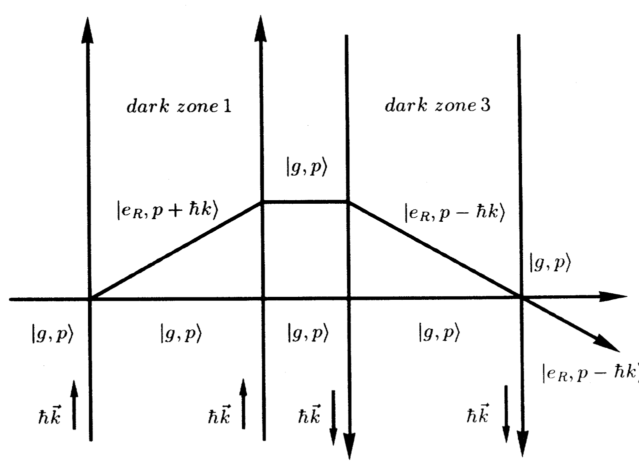

FIG. 1. Ramsey atom interferometer. Only those atomic trajectories corresponding to the high-frequency (blue) component of the Ramsey fringes are shown.

$$\gamma = i \oint_C \mathbf{A}_n(\mathbf{R}) d\mathbf{R} , \tag{1}$$

with a pseudovector potential

$$\mathbf{A}_n(\mathbf{R}) \equiv i \langle n(\mathbf{R}) | \nabla_{\mathbf{R}} | n(\mathbf{R}) \rangle . \tag{2}$$

Here $|n(\mathbf{R})\rangle$ denotes the adiabatic state obeying the stationary Schrödinger equation for constant $\mathbf{R}$:

$$H(\mathbf{R}) | n(\mathbf{R}) \rangle = E_n(\mathbf{R}) | n(\mathbf{R}) \rangle . \tag{3}$$

In the example described here the cyclic process consists of an adiabatic change of population in one of the interferometer arms. This is realized by the adiabatic following in a strong laser field which connects only the ground state with a third state. We illustrate this model in Sec. II, whereas Sec. III summarizes the theoretical framework. For the case of a Ramsey experiment with Mg atoms the parameters for an experimental realization have been calculated quantitatively. The results are given in Sec. IV. It will be shown that the crucial point for the observation of a geometrical phase is the width and the velocity distribution of the atomic beam. Improvements utilizing laser cooled and trapped atoms are discussed as well.

II. BASIC CONCEPT

During the interaction of an atom with a laser field, a closed cycle corresponding to an adiabatic change of population can be achieved by the time-dependent Doppler detuning in the curved wave fronts of two additional Gaussian beams B_1 and B_2 as sketched in Fig. 2. The wave vector $\mathbf{k}$ of the optical field changes its direction continuously, leading to the time-dependent Doppler detuning [11,12]. Placing the additional beams in the first or third dark zone of the Ramsey interferometer (Fig. 1) allows the selective interaction with one arm of the interferometer.

The frequency of the additional laser beams is chosen to be resonant with a transition between the ground state $|g\rangle$ and a third state $|e_B\rangle$. The lifetime of the state $|e_B\rangle$ is supposed to be long compared to the time of the adiabatic evolution; thus spontaneous emission can be neglected. The other excited state $|e_R\rangle$ which couples to $|g\rangle$ by the Ramsey beam splitters (see Fig. 1) evolves freely and serves in this way as a reference state for the interference experiment. Treating the coupling between $|g\rangle$ and $|e_B\rangle$ in the rotating frame and neglecting counter-rotating terms (rotating-wave approximation), the adiabatic parameter $\mathbf{R}$ is given by

$$\mathbf{R} = (\Omega_+(t), \Omega_-(t), \Delta(t)) ,$$

with

$$\Omega_+(t) = \Omega(t)\cos\phi(t), \quad \Omega_-(t) = \Omega(t)\sin\phi(t) ,$$

where ϕ denotes the field phase, Ω denotes the Rabi frequency [cf. Eq. (13)], and Δ denotes the detuning between the transition frequency and the frequency of the additional laser.

Figure 3 shows the closed curve C in parameter space described by $\mathbf{R}$ during the interaction of the atom with the Gaussian beams B_1 and B_2. The field phases $\phi_{1,2}$ of each Gaussian beam are chosen to be constant in time. Entering the first strongly focused Gaussian light field B_1, the atom experiences a large negative detuning which forces $\mathbf{R}$ to begin its movement at the $-\hat{3}$ axes. Due to the Gaussian intensity distribution and the change of the detuning, $\mathbf{R}$ rotates into the $(\hat{1}\text{-}\hat{2})$ plane and arrives at the

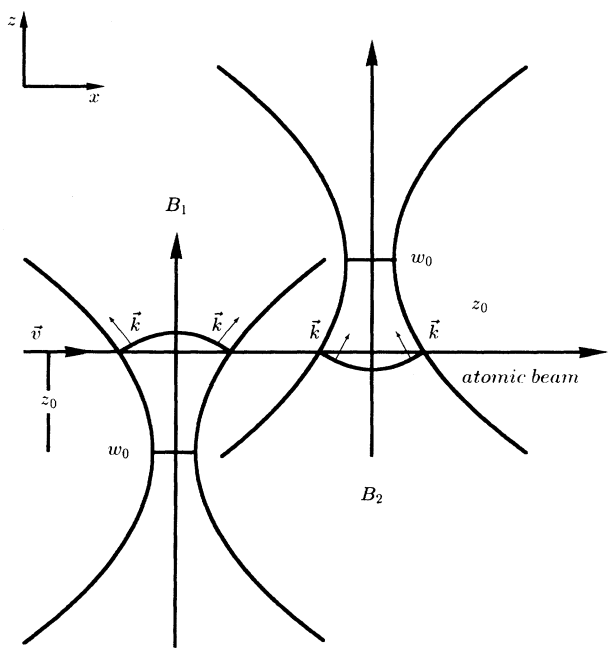

FIG. 2. Configuration of two Gaussian beams B_1 and B_2 crossing the atomic beam perpendicularly to induce an adiabatic cyclic population transfer between two atomic states.

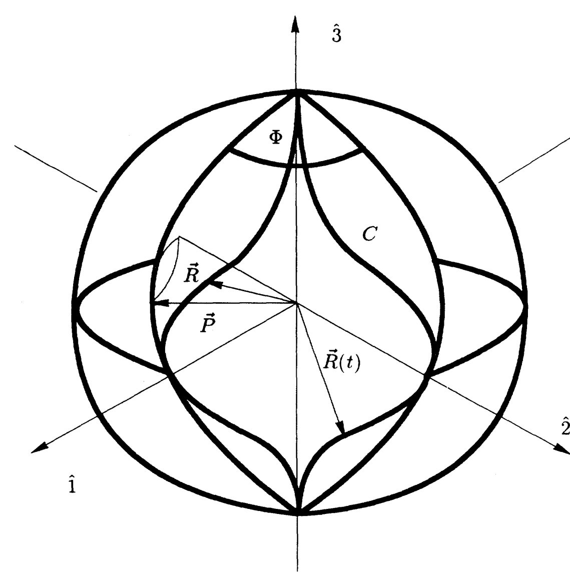

FIG. 3. Closed curve C in parameter space described by the pseudo-field-vector $\mathbf{R}$ in the configuration of Fig. 2. In case of adiabicity the Bloch vector $\mathbf{P}$ remains nearly parallel to the field vector $\mathbf{R}$.

positive $\hat{3}$ axes when the atom leaves the first Gaussian beam. With the help of the second Gaussian field B_2, as shown in Fig. 2, the adiabatic parameter is continuously carried back to its starting position. The curve described by $\mathbf{R}$ encloses a nonvanishing area which is determined by the phase difference $\phi_2-\phi_1$ between the Gaussian beams B_1 and B_2.

The parameter $\mathbf{R}$ corresponds to the pseudo-field-vector in the Bloch picture [13]. During the adiabatic process the precessing Bloch vector remains nearly parallel to the pseudo-field-vector $\mathbf{R}$. Starting in the ground state the interaction with the beam B_1 leads to an adiabatic inversion, while the second light field B_2 turns the excited-state population back into the ground state [11,12].

Summing up, the dynamic evolution results in an additional change of the atomic phase in one arm only. By recombining the two arms of the interferometer the total phase change gives rise to a shift of the interference pattern. By changing the phase difference of the Gaussian beams, this Berry phase can be distinguished from the dynamical phase.

III. THEORETICAL FRAMEWORK

The atomic beam consists of three level atoms with the states $|g\rangle$ and $|e_R\rangle$, coupled by the Ramsey beam splitters, and the state $|e_B\rangle$, which is connected to $|g\rangle$ during the interaction with the Gaussian light fields B_1,B_2. The laser fields are supposed to be classical fields. The Hamiltonian H describing the interaction with a monochromatic light field reads:

$$H=H_A+V_{L-A}\ , \tag{4}$$

with

$$H_A=\hbar\omega_{0R}|e_R\rangle\langle e_R|+\hbar\omega_{0B}|e_B\rangle\langle e_B|\ , \tag{5}$$

$$V_{L-A}=-\boldsymbol{\mu}\cdot\mathbf{E}\ , \tag{6}$$

Here ω_{0R} and ω_{0B} denote the atomic frequencies concerning the transitions $|e_R\rangle\to|g\rangle$ and $|e_B\rangle\to|g\rangle$, respectively, and $\boldsymbol{\mu}$ denotes the dipole operator.

As the Gaussian beams B_1 and B_2 do not overlap, the Hamiltonian H includes only the interaction with a single Gaussian beam B_1 or B_2, respectively, at one time. The electric field $\mathbf{E}$ of a Gaussian beam with polarization $\boldsymbol{\epsilon}$, frequency ω_B, and field phase ϕ, propagating in z direction, reads:

$$\mathbf{E}(t,x,z)=\tfrac{1}{2}\boldsymbol{\epsilon}E(x,z)e^{-i[\omega_B t-\phi-\chi(x)]}+\text{c.c.} \tag{7}$$

The field amplitude is given by

$$E(x,z)=\left[\frac{1}{\epsilon_0 c\pi}\right]^{1/2}\left[\frac{2P}{w^2(z)}\right]^{1/2}\exp\left[-\frac{x^2}{w^2(z)}\right]\ , \tag{8}$$

with the beam radius

$$w(z)=w_0[1+(z/z_R)^2]^{1/2} \tag{9}$$

and the phase χ:

$$\chi(x)=\frac{z_R}{r(z)w_0^2}x^2\ . \tag{10}$$

Here P denotes the total power, z_R the Rayleigh length, w_0 the waist, and $r(z)$ the radius of curvature [11]. If we suppose the atom to cross the x axis at time $t=0$ with x component v_x of the atomic velocity, the phase χ at the atom's position reads

$$\chi(t)=\frac{z_R}{r(z)w_0^2}(v_x t)^2\ . \tag{11}$$

The time derivative of the phase χ is the momentary frequency $\mathbf{kv}$. Writing the Schrödinger equation in matrix form and using the rotating-wave approximation and the transformation

$$U=\begin{bmatrix}1 & 0 & 0\\ 0 & \exp[-i\chi(t)] & 0\\ 0 & 0 & 1\end{bmatrix}\ , \tag{12}$$

which reveals the detuning caused by the Doppler effect, leads to the Hamiltonian

$$\tilde{H}_{\text{static}}=\hbar\begin{bmatrix}\Delta_R-\dfrac{\Delta_{\text{Dop}}}{2} & 0 & 0\\ 0 & \dfrac{\Delta_{\text{Dop}}}{2} & \dfrac{\Omega}{2}e^{i\phi}\\ 0 & \dfrac{\Omega}{2}e^{-i\phi} & -\dfrac{\Delta_{\text{Dop}}}{2}\end{bmatrix}\ , \tag{13}$$

where $\Delta_R:=\omega_{0R}-\omega_R$ and $\Delta_B:=\omega_{0B}-\omega_B$ denote the detunings and $\Omega:=-(1/\hbar)E(x,z)\langle e_B|\boldsymbol{\epsilon}\cdot\boldsymbol{\mu}|g\rangle$ the Rabi frequency; Δ_{Dop} is given by $\Delta_{\text{Dop}}=\Delta_B+\dot{\chi}$. In Eq. (13) we have chosen the origin of the energy scale in the center between the states $|e_B\rangle$ and $|g\rangle$.

As the state $|e_R\rangle$ does not couple to the other states it accumulates during the evolution a phase factor

$$\exp(-\Delta_R t)\exp\left[i\int_{t_0}^{t}\frac{\Delta_{\text{Dop}}(\tau)}{2}d\tau\right]\ . \tag{14}$$

Therefore we will work in the following with the submatrix H_{static}, involving only the states $|e_B\rangle$ and $|g\rangle$:

$$H_{\text{static}}=\frac{\hbar}{2}\begin{bmatrix}\Delta_{\text{Dop}} & \Omega e^{i\phi}\\ \Omega e^{-i\phi} & -\Delta_{\text{Dop}}\end{bmatrix}\ . \tag{15}$$

In case of adiabaticity, the time evolution of a state $|\Psi(t)\rangle$ is given by [9]

$$|\Psi(t)\rangle=e^{i\gamma_n(t)}\exp\left[-\frac{i}{\hbar}\int_{t_0}^{t}E_n(\mathbf{R}(t))dt\right]|n(\mathbf{R}(t))\rangle\ . \tag{16}$$

The first phase factor denotes the Berry phase, the second the dynamical phase, and $|n(\mathbf{R}(t))\rangle$ satisfies the time-independent Schrödinger equation [Eq. (3)], with $H=H_{\text{static}}$. The "dressed" states [14] read:

$$|+\rangle=\cos(\Theta)|g\rangle+\sin(\Theta)e^{i\phi}|e_B\rangle\ , \tag{17}$$

$$|-\rangle=\cos(\Theta)|e_B\rangle-\sin(\Theta)e^{-i\phi}|g\rangle\ , \tag{18}$$

with

$$\cos(2\Theta) \equiv \frac{-\Delta_{\mathrm{Dop}}}{(\Delta_{\mathrm{Dop}}^2+\Omega^2)^{1/2}} , \tag{19}$$

$$\sin(2\Theta) \equiv \frac{\Omega}{(\Delta_{\mathrm{Dop}}^2+\Omega^2)^{1/2}} , \tag{20}$$

and the eigenvalues

$$E_{\pm} = \pm\frac{\hbar}{2}(\Delta_{\mathrm{Dop}}^2+\Omega^2)^{1/2} . \tag{21}$$

Figure 4 shows the energy-time diagram corresponding to the configuration presented in Fig. 2. Depending on the sign of the detuning at the starting point t_0 the atom will remain in the state $|+\rangle$ [$\Delta_{\mathrm{Dop}}(t_0)<0$, see Fig. 4] or in the state $|-\rangle$ [$\Delta_{\mathrm{Dop}}(t_0)>0$] during the adiabatic process.

Let ϕ_1 and ϕ_2 denote the constant phases of the first (B_1) and of the second (B_2) Gaussian beam. Between the two beams, where Ω is approaching zero, the phase can be thought to change continuously from ϕ_1 to ϕ_2 and from ϕ_2 to ϕ_1 behind the second beam (see Fig. 2). By this way **R** and the eigenstates depend continuously on the time t during the entire cycle.

The Berry phase [Eq. (1)] for a complete cycle C can be calculated in the form [15]

$$\gamma_{\pm}(C) = \mp \oint_C \{\sin^2[\Theta(t)]\dot{\phi}(t)\}dt , \tag{22}$$

where the indices $\pm$ correspond to the adiabatic states $|+\rangle$ and $|-\rangle$. Equation (22) leads to

$$\gamma_{+}(C) = -(\phi_2-\phi_1) , \tag{23}$$

$$\gamma_{-}(C) = +(\phi_1-\phi_2) . \tag{24}$$

Thus the Berry phase is given by the phase difference of the Gaussian beams (see Fig. 3).

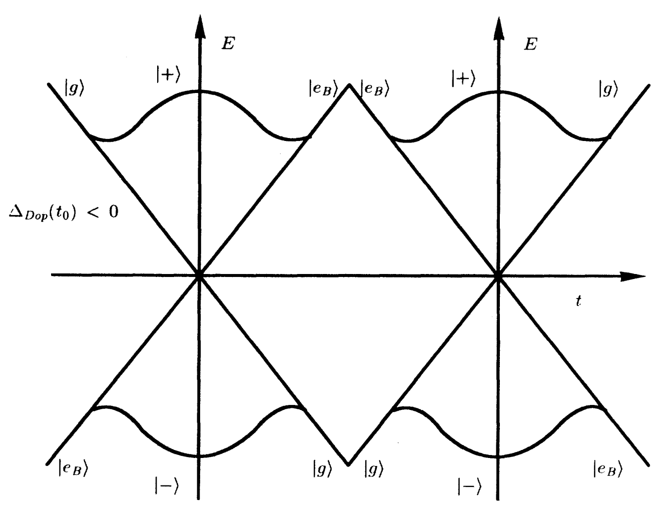

FIG. 4. Energy-time diagram corresponding to the configuration in Fig. 2.

IV. ASPECTS OF AN EXPERIMENTAL REALIZATION

With regard to a Ramsey interferometer realized with the magnesium 457-nm intercombination line ${}^1S_0 \rightarrow {}^3P_1$, the relevant experimental parameters have been calculated. Splitting the excited state ${}^3P_1$ into its magnetic sublevels with quantum numbers $m=-1,0,1$ two of them may serve as the states $|e_R\rangle$ and $|e_B\rangle$, for instance: $|e_R\rangle \equiv {}^3P_{1,m=0}$, $|e_B\rangle \equiv {}^3P_{1,m=-1}$ (Fig. 5). The state $|e_B\rangle$ is coupled selectively to the ground state $|g\rangle \equiv {}^1S_0$ by the interaction with σ_- polarized light.

The conditions for adiabaticity have been investigated by solving the time-dependent Schrödinger equation

$$i\hbar|\dot{\Psi}(t)_{\mathrm{num}}\rangle = H_{\mathrm{static}}|\Psi(t)_{\mathrm{num}}\rangle , \tag{25}$$

for different values of the experimental parameters P, ω_0, Δ_B, v_x, v_z, and the crossing point z_0 between the atomic trajectory and the z axes of the Gaussian beam (see Fig. 2).

In order to force **R** to begin its motion at the $-\hat{3}$ axes with only moderate optical power available, a strongly focused Gaussian beam is necessary. For a waist of $w_0=2$ μm and a laser power of $P=20$ mW an adiabatic process requires low atomic velocities of about $v_x=100$ m/s. The maximum velocity, which will still guarantee an adiabatic passage, increases with increasing power and decreasing z_0. Experimental parameters leading to the adiabatic conditions are discussed in [11,12]. Compared to Rabi oscillations an adiabatic process is relatively insensitive on a variation of the experimental parameters [12]. If for a given power P_0 the state $|\Psi(t)\rangle$ follows adiabatically, it will remain in the adiabatic domain by increasing P. For very large values of P is it possible to produce a coherent superposition in the $\hat{1}-\hat{2}$ plane for a certain time.

In case of adiabaticity the wave function $|\Psi(t)\rangle$ will be given after the interaction with both Gaussian beams at a time t_e by

$$|\Psi(t_e)\rangle = e^{i\Phi}\exp\left[-i\int_{t_0}^{t_e}(\Delta^2+\Omega^2)^{1/2}dt\right]|g\rangle . \tag{26}$$

The second factor in Eq. (26), the dynamical phase, is a sensitive function of the experimental parameters. In order to avoid destructive interference the dynamical phases of atoms which contribute to the Ramsey fringes in a real experiment have to differ by less than π. This restricts the spatial width Δz as well as the velocity distribution Δv of the atomic beam.

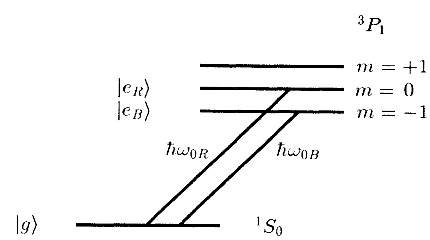

FIG. 5. Part of the magnesium level scheme. The transition $|g\rangle-|e_R\rangle$ is used by the Ramsey interferometer and the transition $|g\rangle-|e_B\rangle$ for the cyclic evolution leading to a Berry phase.

The following example clarifies the experimental difficulties due to this condition. The dynamical phase Φ_{dyn} is calculated for an atomic beam with mean velocity $\bar{v}_x = 50$ m/s, $\bar{v}_z = 0$ m/s, mean crossing point $\bar{z}_0 = 200$ μm, and two strongly focused Gaussian beams with waist $w_0 = 2$ μm, distance $D = 64$ μm, total power $P = 10$ mW, and detuning $\Delta_B = 0$. For this set of parameters, we have calculated from Eq. (25) a probability for nonadiabatic transitions of 0.04%. The dependence of the dynamical phase on z_0, v_z, v_x is shown in Table I. Thus a very narrow spatial width of $\Delta z \sim 3$ μm and a velocity spread of $\Delta v < 1$ m/s is required.

Due to the momentum transfer during the interaction with the two Gaussian beams, this arm of the interferometer is displaced by $\delta z = D\hbar k / m v_x$. In the example given above the displacement reads $\delta z = 46$ nm. In order to maintain a good fringe contrast the transversal coherence length i.e., width of the atomic wave packets $L_c = \hbar / \Delta p_z$ has to be larger than this displacement. This requires a width of the transverse velocity Δv_z of less than 5 cm/s.

A possibility to circumvent these strong experimental demands is offered by a pulsed Ramsey interference experiment on atoms captured in a magneto-optical trap [16]: Here the spatial sequence of the four Ramsey laser beams, shown in Fig. 1, is replaced by a sequence of two pairs of counterpropagating laser pulses in time, applied to a dense laser-cooled ensemble of trapped atoms. The coupling between the states $|g\rangle$ and $|e_B\rangle$, shown in Fig. 3, can be realized by one or two additional pulses, which are applied between the first and the second Ramsey laser pulse. By chirping their frequency an atomic evolution analog to the adiabatic following in the Gaussian beams B_1 and B_2 can be achieved. With the results given above scaled to a trap experiment, a rapid adiabatic passage can be produced by a chirp rate $d\Delta/dt$ of 0.5 GHz/μs, a pulse length of 3 μs, and an intensity of 20 W/cm^2. These values are well within experimental reach; the fast frequency chirp can be achieved with, e.g., an electro-optical modulator [17]. As these parameters lead to a probability for nonadiabatic transitions of 0.04%, setting less stringent conditions on adiabaticity would also simplify the experiments both on an atomic beam as on trapped atoms.

TABLE I. Dependence of the dynamical phase on variations of the experimental parameters. The values refer to the example given in the text.

$\bar{z}_0$	v_x	v_z	Φ_{dyn}
$\bar{z}_0$	$\bar{v}_x$	$\bar{v}_z$	$\bar{\Phi}_{dyn} = 38.1\pi$
$\bar{z}_0 + 3$ μm	$\bar{v}_x$	$\bar{v}_z$	$\bar{\Phi}_{dyn} - 0.54\pi$
$\bar{z}_0$	$\bar{v}_x + 1$ m/s	$\bar{v}_z$	$\bar{\Phi}_{dyn} - 0.5\pi$
$\bar{z}_0 + 3$ μm	$\bar{v}_x + 1$ m/s	$\bar{v}_z$	$\bar{\Phi}_{dyn} - 1.04\pi$
$\bar{z}_0$	$\bar{v}_x$	$\bar{v}_z - 1$ m/s	$\bar{\Phi}_{dyn} + 0.4\pi$
$\bar{z}_0 + 3$ μm	$\bar{v}_x$	$\bar{v}_z - 1$ m/s	$\bar{\Phi}_{dyn} - 0.14\pi$
$\bar{z}_0 + 3$ μm	$\bar{v}_x + 1$ m/s	$\bar{v}_z - 1$ m/s	$\bar{\Phi}_{dyn} - 0.64\pi$

In conclusion we have presented an idea for measuring a Berry phase by a Ramsey atom interferometer. In case of an atomic beam arrangement an experimental realization is very demanding on handling the atomic parameters, especially the spatial width of the beam. The experimental situation can be substantially improved by a pulsed Ramsey interferometer using laser-cooled and trapped atoms.

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