Reply to "Comment on 'Intrinsic decoherence in quantum mechanics'"

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Finkelstein's criticism [Phys. Rev. A **47**, 2412 (1993)] of my paper [Phys. Rev. A **44**, 5401 (1991)] presupposes time measurements of an accuracy explicitly ruled out in the paper.

PACS number(s): 0.3.65.Bz, 0.3.65.Ca

Finkelstein's essential point in his Comment [1] is that the stochastic Hilbert space phase jumps of the model [2] have no observational consequence, as all systems will undergo the same number of phase jumps, thus remaining on the same stochastic branch. This is only true if time measurements are made to a very high degree of accuracy, far higher than the clock model of Finkelstein would suggest. In the model, such ultra-accurate time measurements were ruled out by assumption. Thus two systems, monitored by identical clocks, could undergo a different number of phase jumps even though both clocks are read out at the "same" time. To be more specific let N represent the dimension of the clock Hilbert space.

The eigenvalues of the clock Hamiltonian will be taken as $E_n = \hbar n \omega_c$ where $n \in (1, 2, \ldots, N)$. For the initial clock state take

$$\psi(\phi, 0) = \cos^N \frac{\phi}{2}. \tag{1}$$

This state is both periodic in ϕ and for $N \gg 1$ sharply peaked at $\phi = 2\pi n$ with $n \in (0, 1, 2, \ldots)$. This satisfies Finkelstein's requirements for the clock states. It is important that in Finkelstein's model time is made essentially equivalent to a phase, an equivalence that is probably necessary in any clock model, thus reducing all time measurements to phase measurements. This is an important point to which I shall return. Under free evolution the initial clock state evolves as

$$\psi(\phi, t) = \psi(\phi - \omega_c t), \tag{2}$$

where ω_c is the clock frequency. If this clock state is used to follow the two-level system of Finkelstein, the joint probability for finding the particle in $|L\rangle$ and the clock state in ϕ at time t is

$$P(L, \phi, t) = \cos^{2N} \left(\frac{\phi - \omega_c t}{2} \right) \cos^2(\omega t), \tag{3}$$

where ω is the frequency of oscillation in the two-state system.

Finkelstein now proposes integrating t over one period. This means we are regarding t as a stochastic variable uniformly distributed over one clock period. In physical terms we are supposing that the clock variable ϕ is read-out at a random time over one period. Introducing the scaled time parameter $\tau = \omega_c t$ the conditional probability for the system to be in state L given a clock readout of ϕ is given by

$$P(L|\phi) = \frac{\int_0^{2\pi} d\tau \cos^{2N} \left(\frac{\phi - \tau}{2} \right) \cos^2(\frac{\omega \tau}{\omega_c})}{\int_0^{2\pi} d\tau \cos^{2N} \left(\frac{\phi - \tau}{2} \right)}. \tag{4}$$

Now if

$$N \gg \left(\frac{\omega}{\omega_c} \right)^2 \approx 1 \tag{5}$$

the second term in the integrand of the numerator varies slowly on the scale in which the first term varies, thus we can write

$$P(L|\phi) \approx \cos^2 \left(\frac{\omega \phi}{\omega_c} \right) \tag{6}$$

which is Finkelstein's result in Eq. (5) of his Comment.

Now let us see what happens for the stochastic time model. In this case the joint probability is given by

$$P(L, \phi, t) = \sum_{k=0}^{\infty} P_k(t) \cos^{2N} \left[\frac{1}{2} \left(\phi - \frac{\omega_c k}{\gamma} \right) \right] \cos^2 \left(\frac{\omega k}{\gamma} \right), \tag{7}$$

where

$$P_k(t) = \frac{(\gamma t)^k}{k!} e^{-\gamma t} \tag{8}$$

is the probability for k Hilbert space phase jumps in laboratory time t and γ is the rate of such phase jumps. In this result we see the universal character of the phase jumps as each term in the sum is the product of two terms with the same index k. That is to say, if the clock suffers k phase jumps the system suffers k phase jumps as well. This is a point emphasized by Finkelstein.

Now if N is sufficiently large the clock can indeed resolve a particular stochastic fine-grained history, i.e., the exact times at which a phase jump occurs. The condition that N must satisfy for this to be possible is

$$N > \left(\frac{\gamma}{\omega_c} \right)^2. \tag{9}$$

Now one expects that $\gamma \gg \omega_c$, i.e., the rate of phase jumps in Hilbert space is much greater than any system at laboratory scales. Thus the condition on N in Eq. (9) is much more demanding than that in Eq. (5). The clock

must be so constructed as to permit very accurate phase measurements indeed if the individual phase jumps are to be resolved. In the model of Ref. [2], I proposed that this is in principle (or at the very least in practice) impossible for all systems at laboratory scales. A clock cannot be constructed to resolve a fine-grained history of Hilbert space phase jumps and thus one needs a stochastic time evolution of the sort proposed, when time is the parameter read by ordinary clocks of finite accuracy.

Perhaps a few words as to the motivation behind the model of Ref. [2] are in order. Time is regarded as a parameter by which states are distinguished. If two states differ, then they differ by a rotation in Hilbert space, as the statistical distance between states is determined by the inner product in Hilbert space. The rotation of a state is generated by a unitary transformation. In the case of states distinguished by a time parameter the generator of this unitary transformation is in fact the Hamiltonian. Thus phase changes generated by the Hamilto-

nian are the fundamental fact; time is an inferred parameter to describe the distinguishability of states which differ by such a unitary transformation. The value of such an inferred parameter is limited in accuracy by the statistical nature of quantum mechanics. In fact to distinguish two such states one needs to measure phase, which we know cannot be measured arbitrarily accurately [3]. The model of Ref. [2] was an attempt to build these fundamental limits into the basic description of quantum dynamics. Perhaps it will be of interest to some readers to note that a very similar universal stochastic dynamics was proposed in the classical domain some time ago by Lewis [4]. However in that work the origin of the stochasticity was a little obscure. In quantum mechanics we have an underlying structure of Hilbert space in which to locate the stochasticity; perhaps as random phase jumps of fixed size. Time is an inferred parameter to distinguish states which have suffered different numbers of phase jumps.

[1] J. Finkelstein, preceding paper, Phys. Rev. A **47**, 2412 (1993).

[2] G.J. Milburn, Phys. Rev. A **44**, 5401 (1991).

[3] Michael J. Hall, Quantum Opt. **3**, 7 (1991).

[4] R.M. Lewis, J. Math Phys. **8**, 1449 (1967).