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Atomic Compton-scattering cross sections for small momentum transfer

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An examination is presented of the adequacy of the frequently used incoherent scattering function $S(x,Z)$ for an explanation of available semiconductor detector results concerning atomic Compton-scattering cross sections at small momentum transfer. The conclusion is that the underlying simple theoretical treatment of electron-binding effects has not been accurately tested so far on account of insufficient energy resolution of the reported experiments.

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I. INTRODUCTION

The good, but not always adequate, energy resolution of germanium detectors has been utilized in several experiments [1–9] to determine whole-atom single-differential cross sections per unit solid angle for Compton scattering of γ rays through small angles. The ratio of the atomic cross section to the Klein-Nishina prediction for an initially free and stationary electron is represented in an approximate quantum-mechanical treatment by the incoherent scattering function $S(x,Z)$, where $x=\sin(\theta/2)/\lambda$, θ is the scattering angle, λ is the wavelength of the incident radiation, and Z is the number of electrons in an atom. Extensive descriptions and tabulations [10–12] of $S(x,Z)$ are available. The calculated values of $S(x,Z)/Z$ decrease from unity to zero with decreasing x, and decrease at a given x with increasing Z. The electron binding in atoms is responsible for these variations. The extent to which the different experimental results agree with the tabulations is examined in this Brief Report.

Unless the low-energy threshold for photon detection is specified, the single-differential cross section for Compton scattering is not really defined on account of the soft-photon (or "infrared") divergence. Since the Compton scattering peak occurs at an energy close to the incident photon energy in small-angle scattering experiments, this important issue is not discussed here further.

Theoretical calculations, experimental details, and conclusions are briefly described in Secs. II, III, and IV, respectively.

II. THEORETICAL CALCULATIONS

The tabulations of $S(x,Z)$ mentioned in the Introduction rely on the retention of only the $(e^2/2mc^2)A^2$ term in the nonrelativistic interaction Hamiltonian, on the assumed impulsive nature of the collision process, on the assumption of an independent-particle model for electrons in isolated atoms, and on additional subsidiary approximations. Here, e and m are electron charge and mass, c is the velocity of light, and $\mathbf{A}$ is the vector potential of the radiation. A different formulation [13] based on the impulse approximation has also been extended to include relativistic considerations [14–17]. The detailed ideas and assumptions underlying the different treatments are summarized in a recent review [18] concerned with inelastic scattering of x rays mainly by inner-shell electrons. It is important to note that the validity of the assumptions for small values of x or of energy transfer has not been justified in detail so far.

III. EXPERIMENTAL DETAILS

Some of the experiments (Refs. [3–6, and 9] were performed in a standard scattering geometry, whereas a cylindrically symmetric arrangement employing a ring scatterer was used in the other cases. The direct beam was absorbed by a double shadow cone in Refs. [1,2], and by a uniform cylindrical bar in Refs. [7,8]. The smallest angle of 1.02° was adopted in Ref. [7]. The smallest incident photon energy, namely, 245 keV, was utilized in Ref. [9]. Further details regarding the experiments are summarized in Table I.

The width of a pulse-height spectrum peak due to Compton scattering arises from a combination of intrinsic detector width, and spreads in scattered photon energy owing to the finite angular acceptance and the initial momentum distribution of electrons. The width of the Compton peak is much larger than that of the elastic-scattering component on account of the last two factors. As a result, it was not possible to separate the elastic and the Compton components at scattering angles less than about 15° in the case of 245 and 279.2 keV, and about 6° in the case of 661.6 keV. Thus it is difficult to achieve high accuracy in measurements of $S(x,Z)$ for x values smaller than about 3 Å^{-1}, i.e., in the range of greatest interest for the verification of calculations of electron binding effects on the cross sections. When the elastic and the Compton components are not separated in the pulse-height spectra, the elastic component is calculated according to the modified relativistic form-factor approximation [19] or the relativistic second-order S-matrix treatment [20] and then subtracted [7,8] from the measured scattered intensity in order to determine the Compton component alone. Alternatively, as in Refs. [6,9], a deconvolution of the broad pulse-height distribution into its components is attempted. Under these circumstances, the systematic errors in the experimental values of $S(x,Z)$ are expected to be rather large.

Results in Refs. [5–7] were obtained with multienergy γ-ray sources. It is more difficult in such cases to obtain accurate estimates of the pulse-height continuum due to γ rays of higher energy underlying a given peak of interest [2]. Further, in some of the reports [6,9], the experimental values of $S(x,Z)/Z$ for targets with smaller Z, namely, aluminum and copper, are systematically and somewhat surprisingly smaller than the tabulated values, which are in fact close to the free electron value of unity for $x > 6$ Å^{-1}.

IV. CONCLUSIONS

The inner-shell electrons with binding energies larger than 5 keV constitute less than 18% of the atomic electrons. Even in these cases of relatively strong binding, shell-specific calculations are seen [18] to differ from corresponding experimental results by less than 50%. Therefore, corrections to $S(x,Z)$ are expected to be less than about 10%. Thus if such corrections are to be determined experimentally, it is necessary to ensure experimental errors smaller than a few percent, and consequently to separate the pulse height distributions into

TABLE I. Details concerning γ-ray scattering experiments with semiconductor detectors at small angles θ. $x = \sin(\theta/2)/\lambda$, λ is the incident wavelength, and $h\nu$ is the incident photon energy. The words agreement, lower, or larger in the last column refer to experimental results in comparison with $S(x,Z)$ tabulations of Hubbell *et al.* in Ref. [12]. See also the discussions in Secs. III and IV.

$h\nu$ (keV)	θ (deg)	x range (Å^{-1})	Targets	References and reported errors in $S(x,Z)$	Comments
245	5, 7, 10	0.86–1.72	Al, Cu, Mo, Ta, Pb	[9] ±2 for Al, ~±10% for Cu, ±(6–45)% for Ta	Ta values larger but Al and Cu values significantly lower
279.26	20, 30, 45 60, 80 100, 115	3.91–19.00	Zr, Sn, Ta, Pb. U	[4] ~±5%	Agreement; but eight values ~4% lower when $x < 6$
	8.2, 10.4, 12.3, 15.1	1.61–2.96	Cu	[8] ±10%	Agreement
344, 779, 964, 1086, 1408	5, 7, 10, 15	1.21–14.83	Al, Cu, Mo, Sn, Ta, Pb	[5,6] ~±3% for $x > 2.7$	Agreement except in a few cases.
465, 878, 952, 1189 1302, 1950 2123, 2554 2842	1.02	0.33–2.04	Cu	[7] ~±28% for $x < 1$, ~±9% for $x \sim 2.0$	Agreement but six values ~10% lower for $x < 1.85$
661.6	5, 10, 20, 30, 40, 50, 60	2.33–26.70	Cu, Zn, Cd, Sn, W, Pt, Pb, U	[3] ~±10%	Agreement
	6.4, 8.2, 10.4, 12.3, 15.1	2.98–7.01	Sn, Pb	[8] ±10%	Agreement
1173, 1332	4.51, 5.33, 6.22, 7.11, 8.00, 9.95, 12.05	3.72–11.28	Cu, Sn, Pb	[1,2] ~±4%	Agreement but for $x < 4.5$, 8 values ~6% lower

elastic and Compton contributions. For $x < 6$ Å^{-1}, i.e., in the regime of significant electron binding effects, there are 27 cases from Refs. [2,4,7,9] with experimental values of $S(x,Z)$ marginally but systematically smaller than the tabulated values, though usually within only 1–3 times the stated error. On the other hand, some of the listed experiments show agreement with $S(x,Z)$ in the same regime. It is clear that definitive tests of the $S(x,Z)$ formalism have not been possible mainly on account of inadequate energy resolution.

Order of magnitude improvements in energy resolution and counting rates have been achieved recently with bent-crystal spectrometers and synchrotron x-ray beams, as, for example, in the detailed study [21] of the Compton profile shape of an aluminum single crystal by scattering of 29.5-keV x-rays through 160° ($x = 2.34$ Å^{-1}). If similar high-resolution and high-accuracy techniques are developed to determine absolute values of scattering cross sections, it will be possible to remove ambiguities in experimental values of $S(x,Z)$ even at small values of x. In the same context, it will also be of interest to know the extent to which accurate but laborious calculations based on the relativistic second-order S-matrix approach, or on treatments going beyond the independent-particle and isolated-atom approximations differ from tabulations of $S(x,Z)$.

Note added in proof. New measurements with ~10% error of lead cross sections for the scattering of neutron capture γ rays through 1.02° and 1.8° ($0.3 < x < 2.2$ Å^{-1}) have been reported recently [S. Kahane, R. Moreh, and O. Shahal, Phys. Rev. A **46**, 2489 (1992)]. Since the elastic and the Compton scattering components were not resolved, the Compton scattering cross sections estimated on the basis of $S(x,Z)$ tabulations were subtracted in order to obtain experimental values of elastic scattering cross sections that turned out to be approximately 5% smaller than the modified relativistic form factor (MRFF) predictions. If an allowance is made for possible deviations from $S(x,Z)$ values in the regime of small x, better agreement with MRFF predictions will result in such cases.

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