Fabry-Pérot interferometer for atoms

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We investigate the prospects of a Fabry-Pérot interferometer for atomic matter waves.
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The Fabry-Pérot interferometer is arguably the most important type of optical interferometer, and is ubiquitous in spectroscopy and in the design of laser resonators. In this article, we investigate the possibility of building a similar Fabry-Pérot device, but for the matter waves (de Broglie waves) of neutral atoms. This study is motivated by recent progress in atom optics, where Mach-Zehnder-type atom interferometers have been demonstrated [1–4] as well as by developments in atomic cooling and trapping [5], where it is now possible to localize atoms in the ground state of one-dimensional optical molasses [6–8]. As compared to these advances, a Fabry-Pérot device for atoms would present two immediate advantages: (a) in contrast to the situation in a Mach-Zehnder interferometer, it would produce a localization of the de Broglie wave between the cavity "mirrors"; and (b) in contrast to optical molasses, it would act as a *coherent* atomic trap, the Fabry-Pérot resonances resulting from constructive interferences between the de Broglie waves bouncing back and forth inside the cavity.

An atomic cavity with light-induced mirrors has previously been proposed by Balykin and Lethokov [9], using evanescent waves as mirrors. The present system operates in the quantum regime, where the center-of-mass wave function is delocalized between the two mirrors. In contrast, Ref. [9] considers the classical regime where the size of the center-of-mass wave function is small compared to the distance between mirrors. In this ballistic regime, the confinement of the atom does not result from constructive interferences between partial atomic wave functions. Rather, it is akin to the classical confinement of a ball on a billiard table. From this point of view, our system is more closely related to the recent proposal [10] of a gravitational cavity operating in the quantum regime. Note, however, that the gravitational cavity lengths that are likely to be achieved in practice are also of the order of millimeters, corresponding to unresolvable mode spacings. Our proposed geometry offers the considerable advantage of micrometer-size resonators and resolvable longitudinal modes.

In contrast to one-dimensional optical molasses, the proposed interferometer presents the advantage that the atomic wave function would largely be localized in a field-free region, with a number of obvious advantages. In potential applications of the atom Fabry-Pérot interferometer, the spectral selectivity of the interferometer could be used to prepare atomic beams of excessively high monochromaticity (see Ref. [11] for an alternative approach also using a cavity). More interestingly, its coherent storage capability may be used, e.g., to build "designer atoms" out of neutral atoms, the center-of-mass wave function playing the role of the electronic wave function in normal atoms.

We consider light-induced mirrors deriving from the spatially varying dipole interaction energy between an atom and a near-resonant optical field in regions of high field gradient. We restrict our discussion to one spatial dimension x perpendicular to the optical axis of the fields that act as end mirrors of the interferometer; see Fig. 1. One way to produce high field gradients is to use focused laser beams. Although they invariably lead to unstable resonator configurations, the recent one-dimensional (1D) molasses results show that one can still obtain longitudinal modes with low dissipation. Furthermore, unstable resonators are advantageous in producing the cavity modes of large transverse dimensions required for single longitudinal mode operation, as discussed later on. Alternatively, one might consider using evanescent waves at concave surfaces [9], with the drawback that the resonator could then not operate in transmission. However, the details of the cavity geometry are not essential for the present discussion, which is mostly concerned with a proof of principle.

We describe the atom in a two-level approximation involving two electronic states $|e\rangle$ and $|g\rangle$ separated by a transition frequency ω_a close to the field frequency ω_l. Transforming away the fast oscillations at the field fre-

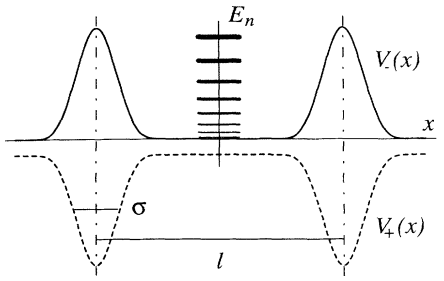

FIG. 1. Possible geometry of a Fabry-Pérot device for atoms, illustrating the potentials V_+ and V_- due to the optical fields, and sketching the resonant modes for the dressed state $|-\rangle$.

quency, the dynamics of the internal state of the atom is governed by the Hamiltonian

$$H(x) = -\hbar\delta\sigma_+\sigma_- - \tfrac{1}{2}\hbar\mathcal{R}(x)\sigma_1 \tag{1}$$

where $\sigma_i, i = 1,2,3,+,-$ are usual pseudospin operators, $\delta \equiv \omega_l - \omega_a$ is the detuning between the field and the atomic frequencies, and $\mathcal{R}(x)$ is the (bare) Rabi frequency of the atom-field interaction. Its x dependence accounts for the varying intensity across the two field profiles.

The local eigenvalues of $H(x)$ are given by $V_-(x) = (\hbar/2)[\Omega(x) - \delta]$ and $V_+(x) = -(\hbar/2)[\Omega(x) + \delta]$, respectively, where $\Omega(x) = \sqrt{\delta^2 + \mathcal{R}(x)^2}$ is the local generalized Rabi frequency. The corresponding local eigenstates are given by

$$|-\rangle(x) = S(x)|g\rangle = \cos\frac{\vartheta(x)}{2}|g\rangle - \sin\frac{\vartheta(x)}{2}|e\rangle \tag{2}$$

and

$$|+\rangle(x) = S(x)|e\rangle = \sin\frac{\vartheta(x)}{2}|g\rangle + \cos\frac{\vartheta(x)}{2}|e\rangle, \tag{3}$$

respectively, where $S(x) = \exp[-i\vartheta(x)\sigma_2/2]$ and $\vartheta(x)$ is the Stückelberg angle, defined by $\delta = \Omega(x)\cos\vartheta(x)$, $\mathcal{R}(x) = \Omega(x)\sin\vartheta(x)$.

From the definition of the Rabi frequency $\Omega(x)$, the eigenvalue $V_-(x)$ is positive and can provide a repulsive potential for the atomic center of mass; in contrast, $V_+(x)$ corresponds to an attractive potential which tends to drag the atom through the optical field profiles. Assuming for now that the electronic states adiabatically respond to the dressing provided by the field as the atom moves along the Fabry-Pérot axis, we conclude that the atom in the dressed state $|-\rangle$ behaves effectively as a scalar particle in the repulsive potential $V(x) \equiv V_-(x)$, its dynamics being governed by the effective Hamiltonian

$$H = \frac{p^2}{2M} + V(x), \tag{4}$$

where p is the atomic center-of-mass momentum and M its mass. Because of the need to reduce the effects of spontaneous emission as much as possible, it is desirable that the dressed state $|-\rangle$ evolves adiabatically into the bare atomic ground state $|g\rangle$ in the field-free region. The defining relation of the Stückelberg angle and of the dressed state $|-\rangle$ shows that this is accomplished for positive values of the atom-field detuning.

We restrict the following analysis to the sub-barrier motion for which the center-of-mass energy E of the atom is such that $E < V_{\max} \equiv \frac{\hbar}{2}(\sqrt{\mathcal{R}_0^2 + \delta^2} - \delta)$. This is the energy range where the light fields act most efficiently as reflectors and the Fabry-Pérot device should operate best as a coherent storage device. Table I shows some values for $V_{\max}$ and peak Rabi frequencies $\mathcal{R}_0$ which may be obtained from the focal intensity of a diffraction limited laser beam ($\sigma = \lambda$) of spectral power 10 mW/γ, $\gamma \equiv \tau^{-1}$ being the natural linewidth. Our estimates for the peak Rabi frequencies are based on a relation $I = I_s(\mathcal{R}_0/\gamma)^2$ between Rabi frequency $\mathcal{R}_0 = \wp\mathcal{E}/\hbar$ and intensity $I = \epsilon_0 c\mathcal{E}^2/2$, which is obtained by expressing the transition matrix element $\wp$ by means of the Eintein A coefficient $\gamma \equiv \tau^{-1} = 4\omega^3\wp^2(4\pi\epsilon_0 3\hbar c^3)^{-1}$—i.e., $I_s = 2\pi ch/(3\lambda^3\tau)$.

As is the case for conventional Fabry-Pérot devices, the double-humped potential $V(x)$ effectively imposes boundary conditions on the solutions of the Schrödinger equation, thereby selecting a quasidiscrete set of "resonant longitudinal modes" where the corresponding center-of-mass wave function is large between the two "mirrors," and small outside.

To estimate the energies of the cavity resonances, we temporarily replace the two barriers by impenetrable walls. This is a reasonable approximation for center-of-mass energies $E < V_{\max}$ and barriers separated by a distance $l \gg \lambda$, where λ is the field wavelength. In an infinite square well approximation, the modes are sinelike, with nodes at the mirrors. The wave numbers of the corresponding wave functions are integral multiples of π/l. The resonance energies are $E_n = (n\hbar\pi/l)^2/(2M)$, n integer, or in terms of the recoil frequency $\omega_{\rm rec} = \pi h/M\lambda^2$, $E_n = \hbar\omega_{\rm rec}(n\lambda/2l)^2$—see Table II for some specific numbers. The number N of resonances in a well of finite depth $V_{\max}$, $N \simeq \sqrt{V_{\max}/E_{\rm rec}}\,2l/\lambda$ is displayed in Table I.

In potential applications of the interferometer as a velocity filter for atomic beams, it is useful to express the locations of the resonances in terms of the atomic velocity. With $v = p/M$ and $p = n\hbar\pi/l$, we readily find that the resonances are equally spaced with $v_n = nv_{\rm rec}(\lambda/2l)$,

TABLE I. Relevant parameters for three possible atoms. The saturation intensity $I_s 2\pi ch/(3\lambda^3\tau)$ and $\mathcal{R}_0$ corresponds to the intensity at focus of a diffraction limited laser beam of spectral power 10 mW/γ. The number of cavity resonances, N, is for a mirror distance $l = 10\lambda$.

	λ (nm)	τ	I_s (W/cm^2)	$\mathcal{R}_0/2\pi$ (GHz)	$\mathcal{R}_0/\delta$	$V_{\max}$ (10^{-4} eV)	N (10^3)
Na[a]	589	16.3 ns	1.25×10^{-2}	151	1.0	129	227
					0.1	16	80
He[b]	1083	100 ns	3.28×10^{-4}	81	1.0	69	123
					0.1	8	42
Mg[c]	457	4.6 ms	0.95×10^{-7}	0.025	1.0	0.02	2.2
					0.1	0.002	0.8

[a] D_2-line $3\,^2S_{1/2} \leftrightarrow 3\,^2P_{3/2}$; see, e.g., Kibble *et al.*, Phys. Rev. **153**, 9 (1967).
[b] Metastable helium $2\,^3S_1 \leftrightarrow 2\,^3P_2$ transition; see, e.g., Sleator *et al.*, Appl. Phys. B **54**, 375 (1992).
[c] $^1S_0 \leftrightarrow {}^3P_1$ intercombination transition; see, e.g., Sterr *et al.*, Appl. Phys. B **54**, 341 (1992).

TABLE II. Recoil values.

	$\nu_{\rm rec}$ (kHz)	$E_{\rm rec}$ (10^{-10}eV)	$v_{\rm rec}$ (cm/s)	$T_{\rm rec}$ (10^{-6}K)
Na (589 nm)	25.0	1.0	2.9	1.2
He (1083 nm)	42.5	1.8	9.2	2.0
Mg (457 nm)	39.3	1.6	3.6	1.9

where $v_{\rm rec} = (2E_{\rm rec}/M)^{1/2}$ is the recoil velocity of the atom—see Table II. We note that although the velocity spacing is smaller by a factor $\lambda/l \ll 1$ than the recoil velocity, such velocity resolutions should be experimentally accessible.

It is important to realize that the small velocity spacing required for single longitudinal mode operation implies rather large transverse mode dimensions. Assuming for simplicity that the longitudinal and transverse atomic temperatures are the same, the separation of the longitudinal modes also implies, e.g., for the transverse direction $\hat{z}$, $\Delta p_z = \hbar\pi/l$, or, with $\Delta z \Delta p_z > \hbar/2$, $\Delta z > l/2\pi$. For $l = 10\lambda$, this gives $\Delta z > 1.6\lambda$. A mode of such large transverse dimensions is best maintained in an unstable resonator geometry. We can obtain an estimate of the number of "bounces" that the atom will undergo before escaping the resonator from the well-known Gaussian beam expansion relation $w^2(x) = w_0^2(1 + x^2/x_0^2)$, where $x_0 \equiv \pi w_0^2/\lambda_{\rm dB}$, w_0 is the Gaussian beam waist, and $\lambda_{\rm dB}$ is the de Broglie wavelength of the atom. (Note that this equation remains valid for unstable Gaussian modes despite the fact that their waist is outside the cavity.) With $w_0 \simeq l/2\pi$ and $\lambda_{\rm dB} = 2l/n$ for the nth longitudinal mode, we find that the transverse beam area doubles after a distance $z \simeq nl/8\pi$. For effective mirror radii of a few optical wavelengths, this shows that hundreds of bounces can be achieved, thus allowing the resonator to operate as a true Fabry-Pérot interferometer.

Consider now quantum tunneling due to the finite width of the potential walls. One can estimate its rate Γ_n quasiclassically by noting that an atom in the nth Fabry-Pérot mode bounces at frequency τ_n^{-1} between the confining light walls, where $\tau_n = l/v_n$ is the time to travel from one mirror to the other. At each bounce, it may tunnel out with probability $P_n^{\rm tu}$ such that $\Gamma_n = \tau_n^{-1} P_n^{\rm tu}$. When compared to the frequency separation $\Delta E_n/(\pi\hbar) = \tau_n^{-1}$, this yields a quality factor $Q \equiv \Delta E_n/\Gamma_n^{\rm tu} = P_n^{\rm tu}$.

The quasiclassical tunnel probability may be expressed as $P_n^{\rm tu} \simeq \exp\{-\int \kappa_n(x)dx\}$, where the "tunnel integral" extends over the classically forbidden region and $\kappa_n(x) = \{2M[V_-(x) - E_n]/\hbar^2\}^{1/2}$. For resonances far below the potential maxima, $E_n \ll V_{\rm max}$, the tunnel integral can be estimated as $[\mathcal{R}_0/\omega_{\rm rec}]^{1/2}$ where we have assumed that the thickness of the classically forbidden region is of the order of the diffraction limit λ of the field profiles. (This thickness would be somewhat smaller for evanescent waves.) Typical laser powers correspond to $\mathcal{R}_0 \gg \omega_{\rm rec}$, so that the tunneling probability is exceedingly small, and the Fabry-Pérot device is mostly suited as a coherent trap rather than as a filter.

Beside ordinary tunneling, coherent trapping inside the Fabry-Pérot interferometer is threatened by nonadiabatic (Landau-Zener) transitions between the quasibound level $|-\rangle$ and the unbound level $|+\rangle$. Such transitions appear if the atom cannot adiabatically follow the local changes in the Rabi frequency $\mathcal{R}(x)$ as it bounces between the mirrors. This situation is met most likely in the region where the field driving of the atom takes over from its free evolution, $\mathcal{R}(x) \approx \delta$. Assuming a Gaussian beam profile, $\mathcal{R}(x) = \mathcal{R}_0 \exp\{-x^2/2\sigma^2\}$, nonadiabatic transitions occur predominantly in an interval $\Delta x^{\rm LZ} \approx \sigma/r$ centered around $x^{\rm LZ} = \sigma r^{1/2}$, where $r = -2\ln(\delta/\mathcal{R}_0)$. Nonadiabatic transitions will be strongly suppressed if the time of flight of the atom through this region, $\Delta\tau_n^{\rm LZ} = \Delta x^{\rm LZ}/v_n$, is longer than a Rabi period $1/\mathcal{R}(x^{\rm LZ}) \approx 1/\delta$. For $\delta \gg \mathcal{R}_0$ this is generally the case. For $\delta \ll \mathcal{R}_0$ and $v/\sigma \ll \mathcal{R}_0$, the equality $\Delta\tau_n^{\rm LZ} = 1/\delta$ gives an upper bound of the critical Landau-Zener detuning as $\delta_n^{\rm LZ} \approx [\mathcal{R}_0 v_n/\sigma]^{1/2}$. Combined with the diffraction limit $\sigma > \lambda$, the maximum trapping velocity v_N then yields $\delta^{\rm LZ} \approx \omega_{\rm rec}^{1/4}\mathcal{R}_0^{3/4}$, or $\delta^{\rm LZ}/\mathcal{R}_0 = [\omega_{\rm rec}/\mathcal{R}_0]^{1/4}$. This and a quick glance over the numbers in the tables confirms that it is generally sufficient to work with detunings of at least one-tenth of the Rabi frequency to suppress nonadiabatic transitions.

We finally turn to the ultimate threat to coherent atom optics, spontaneous emission. Treating the atomic center-of-mass motion classically, the probability to undergo a spontaneous emission act during a time interval dt while the atom is at $x = x(t)$ is given by $dP_E^{\rm SE} = \gamma dt \sin^2\theta(x)/2$, where γ is the natural linewidth and $\sin^2\theta(x)/2 = V(x)/[\hbar\Omega(x)]$ is the excited state population. Using $dt = dx/v(x)$ with $v(x) = \pm\{2[E - V(x)]/M\}^{1/2}$ being the local velocity of the atom, we obtain for the probability to undergo spontaneous emission during one roundtrip

$$P_E^{\rm SE} = \gamma \oint dx \frac{V(x)}{\hbar\Omega(x)} \sqrt{\frac{M}{2[E - V(x)]}}, \tag{5}$$

where the integral extends from the left to the right turning point and back. The results of a numerical evaluation of this integral are displayed in Table III for various values of the atomic energy and detuning. For radiatively strong transitions such as the D_2 line of Na, the suppression of spontaneous emission requires a detuning of the order of 10 Rabi frequencies. In contrast, for weak transitions such as the intercombination transition of Mg, no significant detuning is required. Table III also shows that the low lying states are most robust against spontaneous emission. This is because at larger energies the atom penetrates deeper into the reflecting laser fields and requires more Rabi flops to reverse its momentum.

In summary, we have demonstrated that a Fabry-Pérot device for atoms may indeed be feasible. Such a device

TABLE III. Spontaneous emission probability per round-trip, P_E^{SE}, for various values of the atomic energy and detuning. The mirror distance $l = 10\lambda$ and the Rabi frequencies as displayed in Table I.

	$\mathcal{R}_0/\delta$	$P^{\mathrm{SE}}_{E=0.5V_{\max}}$	$P^{\mathrm{SE}}_{E=0.1V_{\max}}$
Na (589 nm)	1.0		
	0.1	0.08	0.02
He (1083 nm)	1.0	0.19	0.06
	0.1	0.013	0.004
Mg (457 nm)	1.0	2.4×10^{-5}	7.3×10^{-6}
	0.1	1.7×10^{-6}	4.7×10^{-7}

could be used as a filter for atomic beams, as an ordinary Fabry-Pérot device is a frequency filter for light. However, the small tunneling rate through the light mirrors practically limits this application to atomic energies close to the maximum of the potential barriers.

More interestingly, the resonant one-particle states describe atomic wave functions which are coherently delocalized over the macroscopic cavity volume. As such, they may be considered in much the same way as electronic wave functions in atoms, which are coherently spread throughout an orbital. Carrying out this analogy further, we may envision the possibility of building truly macroscopic "designer atoms" by effectively treating the Fabry-Pérot interferometer as the "nucleus" of the "atom." Since atoms come in two species —fermionic or bosonic—an atom Fabry-Pérot would even be isotope selective [12]: A particular resonance mode may be filled with an unlimited number of atoms if the atomic spin is integer (like ^{4}He), while it may be filled with at most $2s + 1$ atoms if the atomic spin is half-integer (like ^{3}He). It is evident that a proper analysis of isotope sensitive many-atom effects requires taking into account the mutual interaction of the atoms in the cavity.

The present discussion omits a number of effects that need to be considered when designing a practical device. They include a detailed discussion of transverse effects, in particular for unstable resonator configurations. Another important effect is gravity, which is important at the atomic velocities considered here. These, and a detailed presentation of our calculations, are planned to be discussed in the future.

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