Resonant-second-harmonic generation of laser radiation in a semiconductor

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A high-power laser radiation, with frequency ω_1 and wave vector $k_1\hat{\mathbf{z}}$, propagating through a semiconductor produces a second-harmonic longitudinal current at $2\omega_1$ and $2k_1\hat{\mathbf{z}}$. When a wiggler magnetic field $(0, k_0\hat{\mathbf{z}})$ is simultaneously present in the system, the $\mathbf{J}\times\mathbf{B}$ force on the electrons produces a transverse second harmonic current at $2\omega_1$ and $(2k_1+k_0)\hat{\mathbf{z}}$, driving second-harmonic electromagnetic radiation. For a specific value of wiggler wave number $k_0=k_{0c}$ the phase-matching conditions for the process are satisfied, leading to resonant enhancement in the efficiency of energy conversion. k_{0c} decreases with the frequency ω_1 of the laser.

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I. INTRODUCTION

Harmonic generation of electromagnetic waves [1–10] is a well-known nonlinear process in dielectrics, semiconductors, and plasmas, with wide-ranging applications. In this process, two (or more) photons of energy $\hbar\omega_1$ and momentum $\hbar\mathbf{k}_1$ each combine to generate a photon of second- (or higher) harmonic radiation of energy $\hbar\omega_2$ and momentum $\hbar\mathbf{k}_2$, where $\omega_1,\mathbf{k}_1$ and $\omega_2,\mathbf{k}_2$ satisfy the linear dispersion relation for electromagnetic waves. The energy and momentum conservation in a second-harmonic process demand

$$\omega_2=2\omega_1,\quad \mathbf{k}_2=2\mathbf{k}_1\ . \tag{1}$$

These conditions are quite restrictive. Semiconductors and plasmas, in particular, are dispersive media; hence Eqs. (1) are not satisfied; i.e., under usual conditions, one cannot have efficient generation of harmonics. In a plasma, the efficiency of second-harmonic generation is seen to be enhanced significantly when the plasma has a density gradient and the laser undergoes significant linear mode conversion into a Langmuir wave. The mode-converted Langmuir wave couples with the laser to produce efficient second-harmonic generation. Such a scheme may not work in a semiconductor, where linear mode conversion into Langmuir waves has not yet been reported.

In this paper, we propose a scheme for generating the resonant second harmonic of laser radiation in a semiconductor by applying a transverse wiggler magnetic field to it. The latter can be produced by placing bar magnets of alternate polarity over the semiconductor. The wiggler can be viewed as a photon $(0, k_0\hat{\mathbf{z}})$ of zero energy and momentum $\hbar\mathbf{k}_0$ that can compensate for the unbalanced momentum between the second harmonic and fundamental photons:

$$\hbar\mathbf{k}_2-2\hbar\mathbf{k}_1=\hbar\mathbf{k}_0\ .$$

The physics of the second-harmonic generation process can be understood as follows.

In the presence of a laser wave

$$\mathbf{E}_1=\hat{\mathbf{x}}A_1\exp[-i(\omega_1 t-k_1 z)]\ ,$$
$$\mathbf{B}_1=\frac{c\mathbf{k}_1\times\mathbf{E}_1}{\omega_1}\|\hat{\mathbf{y}}\ ,$$

the electrons acquire an oscillatory velocity along $\mathbf{E}_1$. The $\mathbf{V}\times\mathbf{B}$ force on them produces a longitudinal electron velocity parallel to the z axis at $(2\omega_1, 2\mathbf{k}_1)$. When a wiggler magnetic field $\mathbf{B}_w=\hat{\mathbf{y}}B_0e^{ik_0z}$ is simultaneously present in the system, the $\mathbf{V}\times\mathbf{B}$ force (due to the beat of the longitudinal electron velocity and the wiggler magnetic field) produces a transverse second-harmonic current along $\hat{\mathbf{x}}$ at $(2\omega_1, 2\mathbf{k}_1+\mathbf{k}_0)$, generating the second-harmonic electromagnetic radiation.

In Sec. II, we obtain an expression for the second-harmonic transverse current density, including the effect of a self-consistent field. In Sec. III, we solve the wave equation to obtain the second-harmonic field. A discussion of results is given in Sec. IV.

II. NONLINEAR CURRENT DENSITY

Consider the propagation of a laser beam

$$\mathbf{E}_1=\hat{\mathbf{x}}A_1\exp[-i(\omega_1 t-k_1 z)]\ ,\qquad \mathbf{B}_1=\frac{c\mathbf{k}_1\times\mathbf{E}_1}{\omega_1}\|\hat{\mathbf{y}} \tag{2}$$

in an n-type semiconductor (cf. Fig. 1) in the presence of a wiggler magnetic field

$$\mathbf{B}_w=\hat{\mathbf{y}}B_0e^{ik_0z}\ . \tag{3}$$

The nonlinear interaction of the laser with electrons in the presence of the wiggler produces a second harmonic whose self-consistent electric vector can be written as

$$\mathbf{E}_2=\hat{\mathbf{x}}A_2\exp[-i(\omega_2 t-k_2 z)]\ , \tag{4}$$

where $\omega_2=2\omega_1$.

The fundamental and second-harmonic electromagnetic waves obey the linear dispersion relation (cf. Fig. 2)

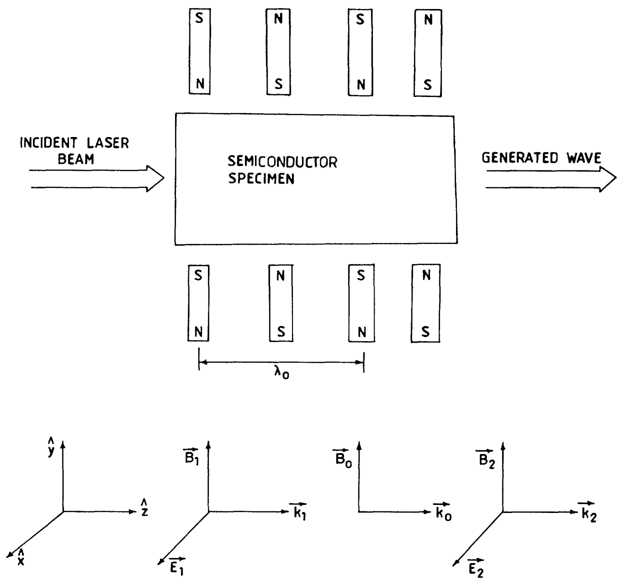

FIG. 1. Schematic of a second-harmonic generator.

$$k^2 \approx \frac{\omega^2}{c^2}(\epsilon_L - \omega_P^2/\omega^2) , \tag{5}$$

where ϵ_L is the lattice permittivity, $\omega_P = (4\pi n_0 e^2/m)^{1/2}$ is the plasma frequency, c is the velocity of light in a vacuum, n_0 is the carrier (electron) concentration, and $-e$ and m are the charge and effective mass of an electron. The wave vector k increases more than linearly with frequency ω; hence,

$$k(2\omega_1) > 2k(\omega_1) ,$$

or

$$k_2 > 2k_1 ;$$

i.e., the sum of the momenta of two pump photons $2\hbar\mathbf{k}_1$ is less than the momentum $\hbar\mathbf{k}_2$ of a second-harmonic photon (cf. Fig. 3). The difference of momentum can be provided to the second-harmonic photon by the wiggler, i.e.,

$$\mathbf{k}_2 = 2\mathbf{k}_1 + \mathbf{k}_0 .$$

Taking all the k's parallel to each other and employing Eq. (5), one obtains

$$k_0 \approx \frac{2\omega_1}{c}[(\epsilon_L - \omega_P^2/4\omega_1^2)^{1/2} - (\epsilon_L - \omega_P^2/\omega_1^2)^{1/2}] . \tag{6}$$

For $\omega_P/\omega_1 \ll \epsilon_L^{1/2}$,

$$k_0 \approx \frac{3}{4}\frac{\omega_P}{c\epsilon_L^{1/2}}\frac{\omega_P}{\omega_1} .$$

The response of electrons to the electromagnetic fields is governed by the equations of motion and continuity [11]

$$m\frac{dv}{dt} = -e\mathbf{E} - \frac{e}{c}\mathbf{v}\times\mathbf{B} - m\nu\mathbf{v} ,$$
$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}) = 0 , \tag{7}$$

where ν is the collision frequency of electrons. At low temperatures, ν is predominantly due to the ionized impurity scattering, whereas at higher temperature it is due to the acoustic phonon scattering. We expand,

$$\mathbf{v} = \mathbf{v}_1 e^{-i(\omega_1 t - k_1 z)} + \mathbf{v}_1' e^{-i[\omega_1 t - (k_1 + k_0)z]}$$
$$+ \mathbf{v}_2 e^{-i(2\omega_1 t - 2k_1 z)} + \mathbf{v}_2' e^{-i[2\omega_1 t - (2k_1 + k_0)z]} , \tag{8}$$
$$n = n_0 + n_1 e^{-i[\omega_1 t - (k_1 + k_0)z]} .$$

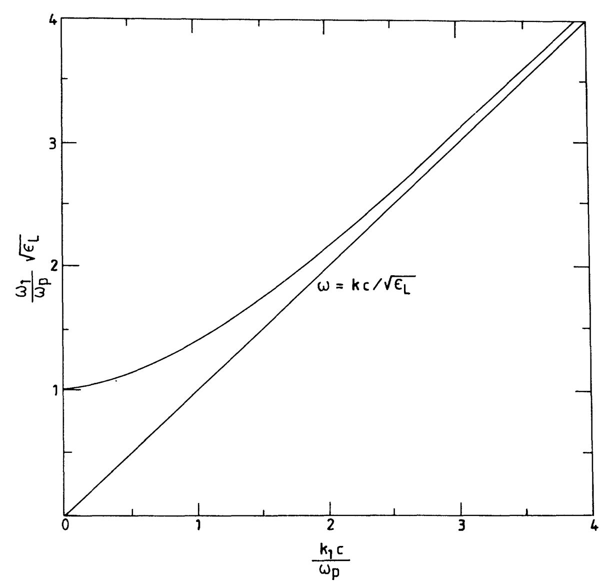

FIG. 2. Dispersion relation for electromagnetic waves in a semiconductor with lattice permittivity $\epsilon_L = 14$.

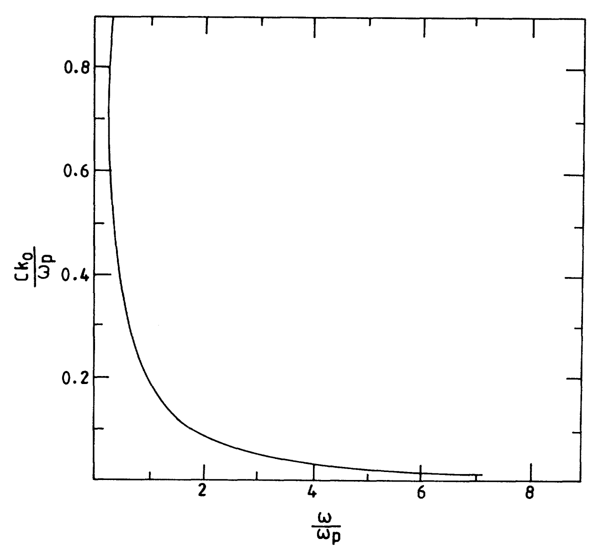

FIG. 3. Optimum wiggler wave number versus the frequency of the laser for resonant second-harmonic generation in a semiconductor with $\epsilon_L = 14$.

$\mathbf{v}_1'$ and $\mathbf{v}_2'$ are longitudinal velocity components that give rise to density perturbations at $\omega_1, \mathbf{k}_1+\mathbf{k}_0$ and $2\omega_1, 2\mathbf{k}_1$. However, for $\omega_1 \gg \omega_P$, these two modes have frequencies much higher than the natural frequency of plasma oscillation; hence the amplitudes of the density oscillations, and the space-charge fields thus generated, are not large, and we have ignored them.

From the equation of motion, we get the linear velocity

$$\mathbf{v}_1 = \hat{\mathbf{x}} \frac{eA_1 e^{-i(\omega_1 t - k_1 z)}}{mi(\omega_1 + i\nu)} . \tag{9}$$

The ponderomotive force [12] on electrons at $(\omega_1, \mathbf{k}_1+\mathbf{k}_0)$ turns out to be

$$\mathbf{F}_1 = -\frac{e}{2c}\mathbf{v}_1 \times \mathbf{B}_w .$$

It gives rise to oscillatory velocity $\mathbf{v}_1'$ and density perturbation η_1:

$$\mathbf{v}_1' = \frac{-e^2 A_1 B_0 e^{-i[\omega_1 t - (k_1+k_0)z]}}{2c\omega_1 m^2(\omega_1+i\nu)} \mathbf{z} ,$$
$$n_1 = \frac{(k_1+k_0) n_0 \mathbf{v}_1'}{\omega_1} . \tag{10}$$

$\mathbf{v}_1$ and $\mathbf{B}_1$ also beat to exert a ponderomotive force on electrons at $(2\omega_1, 2\mathbf{k}_1)$

$$\mathbf{F}_2 = \frac{-e}{2c}\mathbf{v}_1 \times \mathbf{B}_1 ,$$

producing

$$\mathbf{v}_2 = \frac{-e^2 A_1^2 k_1 e^{-i(2\omega_1 t - 2k_1 z)}}{4m^2\omega_1^2(\omega_1+i\nu)} \hat{\mathbf{z}} . \tag{11}$$

$\mathbf{v}_1'$ and $\mathbf{v}_2$ beat with $\mathbf{B}_1$ and $\mathbf{B}_w$, respectively, to produce a transverse second-harmonic ponderomotive force at $(2\omega_1, 2\mathbf{k}_1+\mathbf{k}_0)$:

$$\mathbf{F}_2' = \frac{-e}{2c}\mathbf{v}_1' \times \mathbf{B}_1 - \frac{e}{2c}\mathbf{v}_2 \times \mathbf{B}_w , \tag{12}$$

which yields an oscillatory velocity

$$\mathbf{v}_2'^{\mathrm{NL}} = -\hat{\mathbf{x}} \frac{3e^3 B_0 k_1 A_1^2 e^{-i[2\omega_1 t - (2k_1+k_0)z]}}{16ci\omega_1^3 m^3(\omega_1+i\nu)} . \tag{13}$$

The self-consistent field $\mathbf{E}_2$ also produces an oscillatory velocity

$$\mathbf{v}_2'^{\mathrm{L}} = \frac{e\mathbf{E}_2}{mi(2\omega_1+i\nu)} . \tag{14}$$

The total velocity at $(2\omega_1, 2k_1+k_0)$ is $\mathbf{v}_2' = \mathbf{v}_2'^{\mathrm{L}} + \mathbf{v}_2'^{\mathrm{NL}}$. The second-harmonic nonlinear current density can now be written as

$$\mathbf{J}_2 = -n_0 e \mathbf{v}_2' - \tfrac{1}{2} n_1 e \mathbf{v}_1$$
$$= \frac{-n_0 e^2 \mathbf{E}_2}{mi(2\omega_1+i\nu)} + \frac{\eta_0 e^4 A_1^2 B_0 e^{-i[2\omega_1 t - (2k_1+k_0)z]}}{4ic\omega_1^2 m^3(\omega_1+i\nu)} \times \left[\frac{3k_1}{4\omega_1} + \frac{(k_1+k_0)}{(\omega_1+i\nu)} \right] . \tag{15}$$

III. SECOND-HARMONIC FIELD

The wave equation for the second-harmonic field is written as [13]

$$\nabla^2 \mathbf{E}_2 = \frac{4\pi}{c^2}\frac{\partial \mathbf{J}_2}{\partial t} + \frac{\epsilon_L}{c^2}\frac{\partial^2 \mathbf{E}_2}{\partial t^2} . \tag{16}$$

Using the expression for current density from Eq. (15) in Eq. (16), one gets

$$\mathbf{E}_2 \approx \frac{e^4 B_0 k_1 2\pi n_0 A_1^2}{4\omega_1 c m^3(\omega_1+i\nu)} \left[\frac{3}{4\omega_1} + \frac{1+k_0/k_1}{(\omega_1+i\nu)} \right] \times \frac{e^{-i[2\omega_1 t - (2k_1+k_0)z]}}{\left[4\omega_1^2\epsilon_L - c^2(2k_1+k_0)^2 - \omega_P^2 \left[1 - \frac{i\nu}{2\omega_1} \right] \right]} . \tag{17}$$

For resonance, the denominator in Eq. (17) must vanish. However, it is complex with a small imaginary part, arising due to the collisional damping of the second-harmonic wave. We set the real part of the denominator to zero, corresponding to phase-matching conditions

$$4\omega_1^2\epsilon_L - c^2(2k_1+k_0)^2 - \omega_P^2 \approx 0 . \tag{18}$$

Under this condition,

$$\left| \frac{E_2}{E_1} \right| \approx \frac{eB_0}{mc}\frac{eE_1}{m\omega_1}\frac{k_1}{\omega_1}\frac{\omega_1}{\nu} \left[\frac{3}{4\omega_1} + \frac{1}{\omega_1} \right]$$
$$\approx \frac{7}{4}\frac{\omega_c}{\omega_1}\frac{|v_1| k}{\omega_1}\frac{\omega_1}{\nu} ,$$

where $\omega_c = eB_0/mc$, and we have assumed $\nu/\omega_1 \ll 1$, $k_0/k_1 < 1$. For a typical case of an n-type germanium semiconductor with electron concentration 10^{17} cm^{-3}, $\epsilon_L = 14$, $\nu \approx 2\times10^{11}$ sec^{-1}, $m = 0.3 m_0$ (m_0 being the electron mass in free space), wiggler magnetic field $B_0 = 100$ kG, irradiated by a 10.6-μm CO_2 laser ($\omega = 1.8\times10^{14}$ rad sec^{-1}), one obtains $|E_2/E_1| \approx 0.34$, giving a power conversion efficiency of a few percent. Here we have neglected the insertion loss of the incident laser power, which can be minimized by placing an inhomogeneous dielectric of suitable refractive index profile at the entry port.

IV. DISCUSSION

A wiggler magnetic field aids the generation of a second harmonic in two ways. First, it produces a transverse second-harmonic current $J_{2\omega_1}$ from a longitudinal current J_{2z} through the $\mathbf{J}\times\mathbf{B}$ force. Second, it provides momentum $\hbar k_0$ to the second-harmonic photon, making harmonic generation a resonant process. It could be effective in generating higher harmonics also.

The wiggler wave number required for perfect phase matching in the second-harmonic generation process decreases with the frequency of the laser. For a CO_2 laser propagating through a typical n-type Ge semiconductor with a doping level $n_0 \sim 10^{17}$ cm^{-3}, the wiggler period $(2\pi k_0^{-1})$ turns out to be ~ 0.2 cm, which is technically feasible. Wiggler strength $\gtrsim 10$ kG is required to achieve high efficiency. In addition to the wiggler field, if one applies a strong guide magnetic field, one could improve the efficiency of harmonic generation a great deal through the cyclotron resonance. For short laser pulses, where ν can be taken to be constant, the second-harmonic power scales directly as the square of the power of the fundamental wave. However, for longer pulses of pulse duration exceeding the energy relaxation time, the collision frequency could be significantly changed due to the ohmic heating of electrons, affecting the output power.

The technique could also be applied to study second-harmonic generation in semiconductors where carrier mass is energy dependent, e.g., in n-type InSb. In gaseous plasmas, one could employ higher-power densities while collision frequency is much smaller, hence one may achieve higher efficiency of harmonic generation.

ACKNOWLEDGMENT

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[1] N. Bloembergen, *Nonlinear Optics* (Benjamin, New York, 1965).

[2] M. S. Sodha and P. K. Kaw, Adv. Electron. Electron Phys. **27**, 187 (1969).

[3] D. P. Tewari and V. K. Tripathi, Phys. Rev. **21**, 1698 (1980).

[4] R. Dragila, J. Appl. Phys. **53**, 865 (1982).

[5] C. Liao, P. Bundman, and G. I. Stegeman, J. Appl. Phys. **54**, 6213 (1983).

[6] B. H. Ahn, William W. Clark III, Richard R. Schurtz II, and Calvin D. Bates, J. Appl. Phys. **54**, 1251 (1983).

[7] Xu Zhizhan *et al.*, J. Appl. Phys. **54**, 4902 (1983); C. Grebogi, V. K. Tripathi, and H. H. Chen, Phys. Fluids **26**, 1904 (1983).

[8] T. K. Pramanik and D. P. Bhattacharya, Solid State Commun. **74**, (6), 539 (1990); S. S. Paul and D. P. Bhattacharya, J. Appl. Phys. **64**, 4554 (1988).

[9] S. H. Koenig, R. D. Brown, and W. Schillinger, Phys. Rev. **128**, 1668 (1962).

[10] H. Yamamoto and H. Iwasawa, Proc. IEEE **61**, 504 (1973).

[11] E. M. Conwell, Solid State Phys. Suppl. **9**, 197 (1967).

[12] M. S. Sodha, A. K. Ghatak, and V. K. Tripathi, Prog. Opt. **13**, 169 (1976).

[13] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1962).