Laser cooling of trapped ions in a squeezed vacuum

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Laser cooling of a trapped ion damped by an electromagnetic reservoir in a squeezed-vacuum state is investigated. The cooling rate and the final temperature are given for the case in which the ion is located at the node of a laser standing wave, on resonance with its internal transition. In particular, we find that the ion can have final temperatures below the Doppler limit.

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I. INTRODUCTION

In the past two decades a great number of experimental and theoretical papers have been devoted to the study of laser cooling. It is well established [1] that with the usual cooling by scattering forces the minimum achievable temperature for two-level atoms is given by the "Doppler limit" $T_D = \hbar\Gamma/(2k_B)$, where Γ is the spontaneous decay rate of the excited atomic level. Over the past few years new mechanisms such as polarization gradient cooling [2] and dark state cooling [3] have been investigated, which have shown that temperatures much lower than T_D can be obtained. In all these mechanisms, the atom must have a multilevel structure. However, it has recently been proposed that very low temperatures can be reached with two-level atoms by using pulsed fields [4] or, alternatively, by squeezing the vacuum fluctuations of the electromagnetic reservoir damping the optical transition, [5, 6]. In particular Graham, Walls, and Zhang [6] have shown that in a traveling wave and a detuned standing-wave laser field, a two-level atom in a squeezed vacuum cannot be cooled to temperatures lower than in normal vacuum, while for a two-level atom in a resonant standing-wave temperatures somewhat lower than the Doppler limit are obtained. Here we investigate laser cooling of trapped ions in the presence of a squeezed vacuum. We will show that for a trap located at the node of a resonant standing wave and in a squeezed vacuum, ions will be cooled to the trap ground state, i.e., cooling to the lowest possible energy is achieved.

In Ref. [7] we have shown that the cooling rates and final temperatures of trapped ions can be expressed in terms of internal atomic correlation functions when the vibrational amplitude of the ion in the trap is much less than the typical wavelengths of the cooling laser (Lamb-Dicke limit). For two-level ions, denoting the classical absorption–gain coefficient used in three-wave mixing by $A(\nu)$, we find that cooling occurs when $A(\nu) > A(-\nu)$. This implies that the absorption spectrum must be asymmetric with respect to the laser frequency [1, 8]. In an ordinary vacuum reservoir, this asymmetry is achieved by detuning the laser frequency from the ion (internal) transition frequency. For traveling-wave laser fields this leads to a minimum temperature of T_D. For a standing-wave laser field the final temperature of the ion can be half the Doppler limit when the ion is located at the node of the standing wave, since at this position the ion remains in its ground level, and therefore there is no diffusion due to spontaneous emission [7]. However, this asymmetry in the spectrum can also be found when the two quadratures of the atomic dipole decay at different rates, as occurs when a two-level atom is damped by a squeezed reservoir, even on resonance [9, 10]. In this case, as the squeezing becomes stronger, the asymmetry increases, which would lead, in principle, to a lower temperature. However, as noticed by Graham, Walls, and Zhang [6], the quantum fluctuations experienced by the ion in the squeezed vacuum are a source of heating, and also increase with the squeezing.

The purpose of this paper is to analyze the cooling experienced by a trapped ion in a squeezed vacuum. We consider a two-level ion situated at the node of a standing-wave laser field. In our calculations we use the master equation derived earlier, which is valid in the Lamb-Dicke limit. Our results show final temperatures lower than the Doppler limit. We also show that the diffusion produced by the quantum fluctuations in the squeezed reservoir can be counteracted by increasing the intensity of the laser field, since it accelerates the cooling process without modifying the interaction between the ion and the squeezed reservoir.

II. MODEL

We consider an ion trapped in a harmonic potential. The ion is assumed to have two internal states that are resonantly excited by a laser standing wave and damped by an electromagnetic reservoir in a broadband squeezed vacuum state. The master equation for the density operator of both the internal and external (motional) degrees of freedom is, in a rotating frame at the laser frequency ω_L ($\hbar = 1$),

$$\frac{d}{dt}\rho = -i[H_{\rm HP} + V_{\rm dip}, \rho] + L_{\rm sq}\rho. \quad (1)$$

Here,

$$H_{\rm HP} = \frac{\hat{P}^2}{2m} + \tfrac{1}{2}m\nu^2\hat{R}^2 \quad (2)$$

is the Hamiltonian for the center-of-mass motion of an m mass ion in a harmonic potential of oscillator frequency ν, and

$$V_{\rm dip} = \frac{\Omega}{2}\sigma_x \sin(\kappa_L \hat{R}) \tag{3}$$

describes the dipole interaction between the ion and the standing wave when the trapping potential is centered at the node of the standing wave. $\hat{R}$ and $\hat{P}$ are the position and momentum operators of the ion, the sigmas are usual Pauli operators for a two-level system, Ω is the Rabi frequency of the ion-laser interaction, and $\kappa_L = \omega_L/c$. The processes induced by the squeezed reservoir are described by the term

$$\begin{aligned} L_{\rm sq}\rho = & \gamma(N+1)(2\sigma^- \rho_{+,-}\sigma^+ - \sigma^+\sigma^-\rho - \rho\sigma^+\sigma^-) \\ & +\gamma N(2\sigma^+\rho_{-,+}\sigma^- - \sigma^-\sigma^+\rho - \rho\sigma^-\sigma^+) \\ & +\gamma M e^{2i\psi} 2\sigma^+\rho_{-,-}\sigma^+ + \gamma M e^{-2i\psi} 2\sigma^-\rho_{+,+}\sigma^-, \end{aligned} \tag{4}$$

where $\Gamma = 2\gamma$ is the spontaneous decay rate into the (unsqueezed) vacuum, and

$$\rho_{\alpha,\beta} = \tfrac{1}{2}\int_{-1}^{1} dx\, \Theta(x) e^{\alpha i\kappa \hat{R} x} \rho e^{\beta i \kappa \hat{R} x} \quad (\alpha,\beta = +,-), \tag{5}$$

with $\Theta(x)$ being the angular distribution of spontaneous emission, which, for the dipole transitions considered here, is $\Theta(x) = \frac{3}{4}(1+x^2)$ [1]. In Eq. (4), ψ gives the relative phase between the squeezed vacuum and the laser field, and N and M are parameters characterizing the squeezing, fulfilling $M^2 \le N(N+1)$ [11].

In the Lamb-Dicke limit, where the motion of the ion in the trap is restricted to a spatial region much smaller than the dimension of the laser wavelength, the internal variables can be adiabatically eliminated, yielding a master equation for the reduced density operator of only the motional degrees of freedom [12]. We will also assume here that, in this limit, the parameters N, M, and ψ can be considered independent of position. In this case, following Ref. [7] we find that, if we write the mean energy of the ion in the harmonic trap as

$$E = \nu(\langle n\rangle + \tfrac{1}{2}), \tag{6}$$

the mean value $\langle n\rangle$ of the quantum oscillator number obeys the following equation:

$$\frac{d}{dt}\langle n\rangle = -(A_- - A_+)\langle n\rangle + A_+. \tag{7}$$

The transition rates $A_\pm$, which must be calculated for a two-level system at the position $R = 0$, are given by

$$A_\pm = \eta^2 \frac{\Omega^2}{2} S(\mp\nu) + 2D, \tag{8}$$

where

$$S(\nu) = {\rm Re}\int_0^\infty d\tau\, e^{i\nu\tau}\langle\sigma_x(\tau)\sigma_x(0)\rangle_{\rm SS}, \tag{9}$$

is proportional to the spectrum of fluctuations of the dipole force in the steady state, and

$$\begin{aligned} D = & \tfrac{2}{5}\eta^2\gamma\,{\rm Tr}[\ (N+1)\sigma^-\rho\sigma^+ + N\sigma^+\rho\sigma^- \\ & \qquad + M e^{i\psi}\sigma^+\rho\sigma^+ + M e^{-i\psi}\sigma^-\rho\sigma^-] \\ = & \tfrac{2}{5}\eta^2[\gamma(N+1)\langle\sigma_e\rangle_{\rm SS} + \gamma N\langle\sigma_g\rangle_{\rm SS}] \end{aligned} \tag{10}$$

gives the diffusion produced by the random recoil accompanying each transition induced by the squeezed vacuum. Here, $\eta = \kappa_L/\sqrt{2m\nu}$ is the small parameter in the Lamb-Dicke limit, and $\langle\sigma_e\rangle_{\rm SS}$ and $\langle\sigma_g\rangle_{\rm SS}$ are the stationary populations of the excited and ground states, respectively.

In view of Eqs. (7) and (8) we may write the cooling rate as

$$W = A_- - A_+ = \eta^2\frac{\Omega^2}{2}[S(\nu) - S(-\nu)], \tag{11}$$

and the corresponding final energy

$$E_{\rm SS} = \nu\left(\frac{A_+}{A_- - A_+} + \frac{1}{2}\right). \tag{12}$$

Cooling occurs when $S(\nu) > S(-\nu)$ (for an interpretation of this condition in terms of the absorption spectrum of three-wave mixing, see Ref. [7]). This condition is usually achieved by detuning the laser frequency from the internal transition frequency. In the usual vacuum this leads to a minimum temperature of half the Doppler limit; however, in a squeezed vacuum $S(\nu)$ can be asymmetric, even on resonance, when the laser field and the squeezed vacuum fulfill some phase condition. In Sec. III we show that this can lead to very low temperatures.

III. RESULTS AND DISCUSSION

In order to calculate the transition rates $A_\pm$ we need the Bloch equations for a two-level system resting at the node of a standing wave. These have the form [9]

$$\begin{aligned} \frac{d}{dt}\langle\sigma_x\rangle = & -\gamma[2N+1-2M\cos(2\psi)]\langle\sigma_x\rangle \\ & -\gamma 2M\sin(2\psi)\langle\sigma_y\rangle, \end{aligned} \tag{13a}$$

$$\begin{aligned} \frac{d}{dt}\langle\sigma_y\rangle = & -\gamma 2M\sin(2\psi)\langle\sigma_x\rangle \\ & -\gamma[2N+1+2M\cos(2\psi)]\langle\sigma_y\rangle, \end{aligned} \tag{13b}$$

$$\frac{d}{dt}\langle\sigma_z\rangle = -2\gamma(2N+1)\langle\sigma_z\rangle - 2\gamma. \tag{13c}$$

Note that the Rabi frequency Ω does not appear in these equations, since at the node of the standing wave the laser intensity is zero. The diffusion constant given in (10) can be calculated by solving the Bloch equations in the steady state, resulting in

$$D = \tfrac{4}{5}\eta^2\gamma\frac{N(N+1)}{2N+1}. \tag{14}$$

To determine $S(\nu)$ let us express Eqs. (13a) and (13b) in a reference frame that coincides with the principal directions of the squeezing ellipse in the phase space (see Fig. 1)

$$\frac{d}{dt}\langle\sigma_u\rangle = -\gamma_u\langle\sigma_u\rangle, \tag{15a}$$

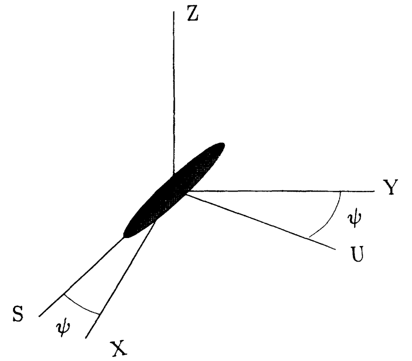

FIG. 1. Bloch space representation for a two-level system damped by a squeezed reservoir at the node of a laser standing wave in the direction X. The SUZ reference frame is obtained from the XYZ one by a rotation of angle ψ around Z. The squeezed fluctuations of the reservoir are represented by an elliptical distribution with the principal directions having a length of $\gamma(2N+1+2M)$ and $\gamma(2n+1-2M)$ along the S and U directions, respectively.

$$\frac{d}{dt}\langle\sigma_s\rangle = -\gamma_s\langle\sigma_s\rangle, \tag{15b}$$

where

$$\sigma_u = \sin(\psi)\sigma_x + \cos(\psi)\sigma_y, \tag{16a}$$
$$\sigma_s = \cos(\psi)\sigma_x - \sin(\psi)\sigma_y \tag{16b}$$

are the components of the Bloch vector in phase and out-of-phase with the maximally squeezed quadrature of the electromagnetic reservoir, respectively, and

$$\gamma_u = \gamma(2N+1+2M), \tag{17a}$$
$$\gamma_s = \gamma(2N+1-2M). \tag{17b}$$

Using relations (16), we have for the two-time correlation function

$$\langle\sigma_x(\tau)\sigma_x(0)\rangle_{\rm SS} = \sin(\psi)\langle\sigma_u(\tau)\sigma_x(0)\rangle_{\rm SS} + \cos(\psi)\langle\sigma_s(\tau)\sigma_x(0)\rangle_{\rm SS}, \tag{18}$$

which, with the help of the quantum regression theorem [13] and Eqs. (15) becomes

$$\langle\sigma_x(\tau)\sigma_x(0)\rangle = \sin(\psi)\langle\sigma_u\sigma_x\rangle_{\rm SS}e^{-\gamma_u\tau} + \cos(\psi)\langle\sigma_s\sigma_x\rangle_{\rm SS}e^{-\gamma_s\tau}. \tag{19}$$

Fourier transforming and taking the real parts, we obtain

$$S(\nu) = \cos^2(\psi)\frac{\gamma_s}{\gamma_s^2+\nu^2} + \sin^2(\psi)\frac{\gamma_u}{\gamma_u^2+\nu^2} + \tfrac{1}{2}\nu\langle\sigma_z\rangle_{ss}\sin(2\psi)\left(\frac{1}{\gamma_u^2+\nu^2} - \frac{1}{\gamma_s^2+\nu^2}\right). \tag{20}$$

Note that according to Eq. (20), $S(\nu)$ is asymmetric with respect to ν when $\sin(2\psi)\neq 0$. In Fig. 2 we have plotted $S(\nu)$ for $N=1$ and $\psi = 0$, $\pi/6$, $\pi/4$, $\pi/3$. In this figure, for $\psi \neq 0$ we have that $S(\nu) > S(-\nu)$, and therefore cooling will occur, even though there is resonance.

Substituting (20) in (11), we have for the cooling rate

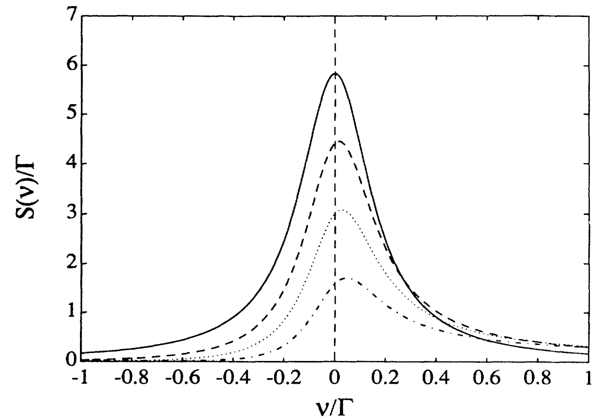

FIG. 2. $S(\nu)$ for $N=1$, $M=\sqrt{2}$ and ψ: 0 (1); $\pi/6$ (2); $\pi/4$ (3); $\pi/3$ (4).

$$W = \eta^2\frac{\Omega^2}{2}\nu\sin(2\psi)\frac{1}{2N+1}\left(\frac{1}{\gamma_s^2+\nu^2} - \frac{1}{\gamma_u^2+\nu^2}\right). \tag{21}$$

Then, cooling takes place when $W>0$, i.e., for $\sin(2\psi)>0$, while for $\sin(2\psi)\leq 0$ there is heating. On the other hand, W is proportional to Ω^2, so that the cooling rate does not saturate with the laser intensity. Note that this *exact* in the Lamb-Dicke limit (i.e., in second order in η).

The mean value $\langle n\rangle$ in the final state can be written as

$$\langle n\rangle_{\rm SS} = \frac{S(\nu)}{S(\nu)-S(-\nu)} + \frac{2D}{W}. \tag{22}$$

The first term does not depend on the Rabi frequency, while the second one decreases with Ω as $1/\Omega^2$, and thus, for intense laser fields, it can be neglected. Physically, for strong laser fields, the processes induced by the dipole force are faster than the diffusion due to spontaneous emission in the squeezed vacuum. Using (14) and (21), we can also write the condition $W \gg 2D$ as

$$\Omega^2 \gg \frac{16}{5}\frac{\gamma N(N+1)}{\nu\sin(2\psi)\left(\frac{1}{\gamma_s^2+\nu^2} - \frac{1}{\gamma_u^2+\nu^2}\right)}. \tag{23}$$

In the following, we will concentrate on the case in which the inequality (23) is fulfilled, since it is then that lowest temperatures are found. In this case, the final energy becomes

$$E_{\rm SS} = (N+\tfrac{1}{2}) \times\frac{\gamma_s(\gamma_u^2+\nu^2)\cot(\psi)+\gamma_u(\gamma_s^2+\nu^2)\tan(\psi)}{\gamma_u^2-\gamma_s^2}. \tag{24}$$

Figure 3 shows the dependence of $E_{\rm SS}$ as a function of N and ψ for a $\nu = 0.1\Gamma$. As for a free atom [6], there is a range of values of ψ where the final energy is lower than that imposed by the Doppler limit.

The minimum value of the energy is found for

$$\tan(\psi) = \sqrt{\frac{\gamma_s}{\gamma_u}\frac{\gamma_u^2+\nu^2}{\gamma_s^2+\nu^2}}, \tag{25}$$

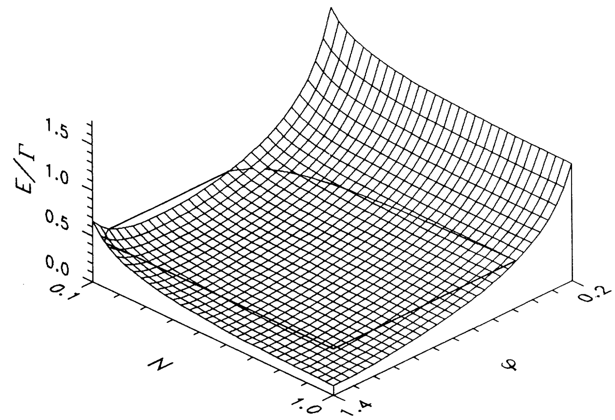

FIG. 3. Final energy E_{SS} as a function of the squeezing parameter N and the phase ψ for $\nu = 0.1\Gamma$ and an ideal squeezed vacuum $[M^2 = N(N+1)]$. The plane $E_{SS} = \Gamma/2 = k_B T_D$ is also drawn in the figure.

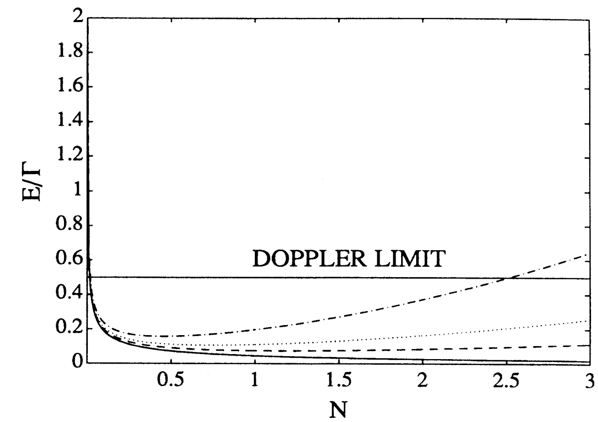

FIG. 4. Miminum final energy $E_{\min}$ as a function of the squeezing parameter N for $\nu = 0.01\Gamma$ and $M/\sqrt{N(N+1)}$: 1 (solid line), 0.975 (dashed line), 0.95 (dotted line), and 0.9 (dashed-dotted line).

giving

$$E_{\min} = (2N+1)\frac{\sqrt{\gamma_u\gamma_s(\gamma_u^2+\nu^2)(\gamma_s^2+\nu^2)}}{\gamma_u^2-\gamma_s^2}. \tag{26}$$

In Fig. 4 we have plotted $E_{\min}$ as a function of N for $\nu = 0.01\Gamma$ and $M/\sqrt{N(N+1)} = 1$, 0.975, 0.95, and 0.9. Note that for a given N, as the ratio $M/\sqrt{N(N+1)}$ decreases, the minimum energy increases, this effect being more pronounced for large N. Then, the lowest energy is found for an ideal squeezed vacuum $M^2 = N(N+1)$. In this case, Eq. (26) can be written as

$$E_{\min} = \tfrac{1}{2}\sqrt{\nu^2 + \frac{(\gamma^2+\nu^2)^2}{16\gamma^2 N(N+1)}}. \tag{27}$$

Hence, as N increases, the minimum energy decreases, tending to $\nu/2$ for a strong squeezed vacuum ($N \to \infty$). In this limit we have that $\langle n\rangle_{SS} \to 0$, even when $\Gamma > \nu$, contrary to what occurs in the normal vacuum [1]. Then, we conclude that for a fixed Γ, temperatures below the Doppler limit can be achieved when $\nu < \Gamma$.

IV. CONCLUSIONS

We have studied the cooling rate and the final energy of a trapped ion when it is embedded in a squeezed-vacuum reservoir. We have shown that for very intense laser fields, the final temperature can be lower than the Doppler limit when the ion is situated at the node of a standing wave, even on resonance. This is a direct consequence of the asymmetry appearing in the atomic spectrum when some phase condition between the squeezed reservoir and the laser field is fulfilled. We have also shown that lowest energies are found for an ideal squeezed state. In this case, for a strong squeezing, the minimum achievable energy tends to the trap ground state $\hbar\nu/2$. This behavior is less pronounced if the ion does not interact exclusively with squeezed modes of the electromagnetic field, which represents a significant practical problem. However, this problem could be circumvented by using optical cavities, which open new possibilities of having atoms interacting with one-dimensional beams of squeezed light [14, 15].

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