Target dressing effects in laser-assisted x-ray photoionization

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X-ray photoionization of hydrogen in the presence of a low-frequency laser field is studied using an approach which partially takes into account the radiative stimulated corrections to both bound and continuum states. Special attention is paid to the cases in which one low-frequency photon is exchanged (absorbed or emitted) between the atomic system and the laser field. It is demonstrated that atomic "dressing" effects are important close to the ionization threshold.

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I. INTRODUCTION

Two-color ionization refers to processes in the course of which an atom is ionized as the result of simultaneous interactions with two different radiation fields. Interesting physical effects, distinct from the ones observed in standard (single-mode) multiphoton experiments, are expected to be observed when a high- and a low-frequency field are present. In this context a high frequency, which will be denoted ω_X, is such that $\omega_X > |E_i|$, where E_i is the binding energy of the considered initial atomic state (atomic units will be used throughout this paper, unless otherwise mentioned). This means that ionization is possible through the absorption of one photon from the high-frequency field. On the other hand, the low-frequency ω_L is such that $\omega_L \ll |E_i|$, so that one-photon ionization is not possible. Here the indices X and L refer to situations in which one considers x-ray photoionization in the presence of a laser field. Such situations are quite different from the ones which have been considered up to now in actual experiments, in which not so different frequencies were used [1,2]. However, the recent observations of higher harmonic generation in dense gases [3,4] open the possibility to create correlated fields with very different frequencies ranging between the infrared and soft-x-ray domains. We shall address here the possible modifications of the x-ray photoionization cross section induced by the presence of the laser, modifications which strongly depend on the characteristics of the latter.

The theory of such processes [5–11] has already been considered by several authors who have resorted to various approximation schemes, whose common features may be schematized as follows. The interaction between the atom and the weak high-frequency field is treated perturbatively to first order. As a result of this interaction, the atomic electron is brought, via the absorption of the one photon, into a positive energy state belonging to the continuous spectrum. Although the above-mentioned references take into account the influence of the laser field on the positive-energy electron, they differ in the approximate treatments used. Also common to these treatments are the facts that one neglects the effects of the intense laser on the atomic states and of the weak high-frequency field on the ejected electron. Although the latter assumption can be easily justified because of the assumed weakness of the high-frequency source, it is not clear to which extent the dressing effects of the atomic system by the laser field can be neglected. To discuss this latter point we will introduce an improved approximation, directly derived from a previous work on laser-assisted $(e,2e)$ collisions [12], which allows us, although only partially, to consistently take into account such dressing effects.

Note that, recently, different nonperturbative approaches have been presented, based either on the Floquet theory [13,14] or on the variational approximation [15]. As they do not discuss the same issues as the ones considered here we have not made specific comparisons.

The main idea behind our approach is to consistently account for the dressing of the atomic states, belonging to either the discrete or continuous spectrum, via the use of first-order corrected wave functions [16–19]. A step further, specific to positive-energy states, is to apply this procedure to Coulomb-Volkov waves [20–22], in order to partially account for the influence of the whole atomic spectrum into the overall dressing of a Coulomb wave in the presence of the field.

One of the main outcomes of our study is to demonstrate that such atomic dressing effects contribute significantly to the laser-assisted differential cross section. In this respect, the situation is similar to the one found in the case of laser-assisted electron-atom collisions, in which the inclusion of the dressing of the atomic target

can completely modify the differential cross sections [23–26]. We will demonstrate the importance of these effects by considering the ionization of a hydrogen atomic system initially in its ground state. Although somewhat simplified, the model possesses the distinctive advantage of allowing "exact" calculations of the needed transition amplitudes, which helps to delineate the limits of validity of the theory.

The organization of the paper is as follows: in the next section we shall present the theoretical background. We shall also introduce and discuss the various approximation levels which can be used to compute the relevant matrix elements; at the same time we shall outline the main features of our approach. In Sec. III we shall establish the expressions of the matrix elements entering the differential cross section which are used in the present calculation. The results will be presented and discussed in Sec. IV.

II. THEORY

A convenient way to compute transition probabilities for two-color ionization of an atom in the presence of a weak (x-ray) high-frequency field and a strong (laser) low-frequency field is to consider the corresponding S-matrix element for one-photon x-ray ionization, between atomic states "dressed" by the laser field:

$$S=-i\int_{-\infty}^{+\infty}dt\left\langle\phi_{\mathbf{k}}(t)\left|\frac{1}{c}\mathbf{A}_X(t)\cdot\mathbf{p}\right|\phi_i(t)\right\rangle , \tag{1}$$

where $\mathbf{A}_X(t)$ is the vector potential associated to the high-frequency field. The kets $|\phi(t)\rangle$, which represent the dressed atomic states, are solutions of the following time-dependent Schrödinger equation:

$$i\frac{\partial|\phi(t)\rangle}{\partial t}=\left[\frac{p^2}{2}-\frac{1}{r}+\frac{1}{c}\mathbf{A}_L(t)\cdot\mathbf{p}\right]|\phi(t)\rangle , \tag{2}$$

where $\mathbf{A}_L(t)$ is the vector potential associated to the laser field. More precisely, $|\phi_i(t)\rangle$ is a time-dependent wave function representing the atomic ground state in the presence of the (supposedly adiabatically turned on) laser field. Similarly $|\phi_{\mathbf{k}}(t)\rangle$ represents a positive energy electron with asymptotic momentum $\mathbf{k}$, submitted to both the Coulomb field of the nucleus and the low-frequency laser. We have chosen the interaction Hamiltonian between the fields and charged particles as

$$H_{\text{int}}=\frac{1}{c}\mathbf{A}\cdot\mathbf{p} , \tag{3}$$

where $\mathbf{A}$ is the classical vector potential of either field. We have further,

$$\mathbf{A}(t)=\mathbf{A}_0\cos\omega t,\quad \mathbf{A}_0=\frac{c\mathbf{F}_0}{\omega} ,$$

where $\mathbf{F}_0$ is the (observable) electric field strength amplitude. Note that in the expression of the interaction Hamiltonian, Eq. (2), we have omitted the field-dependent term $A_L^2/2c^2$, which, within the dipole approximation used here, does not act on atomic variables and only contributes to an unobservable shift of the energy scale. As in our study, we consider the frequency ω_L much smaller than ω_X, and we restrict the discussion to the exchange of one laser photon, the question of the relative phase of the two fields [1,14] is irrelevant.

A. Ground-state dressing

As long as the laser field intensity remains moderate, the dressing of the ground state can be safely described to first order in the perturbation

$$|\phi_{1s}^{(1)}(t)\rangle=e^{-iE_{1s}t}\left\{|\phi_{1s}\rangle+\frac{\omega_L}{2}[G_c(E_{1s}-\omega_L)\boldsymbol{\alpha}_0\cdot\mathbf{p}|\phi_{1s}\rangle e^{i\omega_L t}+G_c(E_{1s}+\omega_L)\boldsymbol{\alpha}_0\cdot\mathbf{p}|\phi_{1s}\rangle e^{-i\omega_L t}]\right\} , \tag{4}$$

where $|\phi_{1s}\rangle$ represents the stationary ground-state wave function and G_c is the Coulomb resolvent operator:

$$G_c(\Omega)=(\Omega-H_c)^{-1}=\sum_n\frac{|\phi_n\rangle\langle\phi_n|}{\Omega-E_n} . \tag{5}$$

Here $H_c=p^2/2-1/r$ is the Coulomb Hamiltonian with eingenkets $|\phi_n\rangle$ such that

$$H_c|\phi_n\rangle=E_n|\phi_n\rangle , \tag{6}$$

and, in Eq. (5), the sum over the index n runs over the discrete and continuous spectra. For the sake of future convenience, we have also introduced the parameter $\boldsymbol{\alpha}_0$,

$$\boldsymbol{\alpha}_0=\frac{1}{c}\frac{\mathbf{A}_{0L}}{\omega_L}=\frac{\mathbf{F}_{0L}}{\omega_L^2} . \tag{7}$$

It is important, at this stage of our presentation, to make more precise our statement regarding the laser intensity which should be kept at a *moderate* level, in order to ensure the validity of the above expression, Eq. (4). For a ground-state atom, a good criterion is to compare to the atomic unit of intensity $I_0=(c/8\pi)|\mathcal{E}_0|^2$ ($I_0\approx3.5\times10^{16}$ W/cm^2), where $\mathcal{E}_0$ is the atomic unit of field strength intensity, $\mathcal{E}_0=e/a_0^2$ ($\approx5.14\times10^9$ V/cm). It has been observed that if the laser intensity becomes a sizable fraction of this value, multiphoton ionization becomes dominant in most atomic systems [27]. More precisely, at laser intensities $I_L\approx10^{13}$ W/cm^2, and quite independently of the frequency, most atomic systems with ionization energies $|E_i|\approx10$ eV are ionized, either through a tunnel process at lower frequencies (e.g., CO_2 laser, $\lambda=10.6$ μm) [28(a)], or through multiphoton absorption at higher frequencies [e.g., Nd-YAG (YAG denotes yttrium aluminum garnet) laser, $\lambda=1.06$ μm] [27]. Note, however,

that tunneling can become important even at higher frequencies, but at higher intensities [28(b)]. In both cases lowest-order perturbative approaches have been shown to be invalid, which provides us with an upper limit for the laser intensity I_L which, accordingly, must be such that $I_L \leq 10^{-3}$ a.u. or $F_0 \leq 3\times 10^{-2}$ a.u., in order to ensure the validity of Eq. (4). It is worth mentioning that in most previous treatments of two-color ionization the "dressing" has been neglected and plain ground-state wave functions have been used. We wish to stress that such a simplified approach, which, as explained above, is certainly not correct at higher intensities, can be questionable even at moderate intensities. We shall discuss this point later.

B. Continuum-state dressing

The situation is somewhat different when considering the wave function of the positive-energy electron. The intensity of the laser is no longer the unique criterion which allows one to choose an adequate approximation for the wave function. Indeed, under the influence of the field a classical free electron, initially with a momentum $\mathbf{k}$, acquires a time-dependent momentum:

$$\mathbf{k}\rightarrow\mathbf{k}-\frac{1}{c}\mathbf{A}(t) , \tag{8}$$

which gives rise to a quiver motion with amplitude $\boldsymbol{\alpha}_0$, defined in Eq. (7).

It appears that the dynamics of a laser-assisted process resulting in the ionization of the target will strongly depend on the magnitude of $\boldsymbol{\alpha}_0$, which in turn not only depends on the field strength F_0 but also on the frequency as ω^{-2}. One expects, in particular, that at a given intensity, the smaller the frequency, the larger the influence of the laser on the motion of the ejected electron. As it is difficult to assess quantitatively the validity of such heuristic arguments, it can be useful to discuss the various levels of approximations which can be contemplated. We present here, in order of increasing sophistication, several approximations used for the description of the dressed positive-energy electron. Our work, based on an approach discussed below, will be compared to these simpler treatments.

(i) The crudest approximation uses nonrelativistic Volkov waves [29]. The exact solution of the Schrödinger equation for an electron with asymptotic momentum $\mathbf{k}$ in the presence of a single-mode, linearly polarized laser field is

$$\chi_V(t)=e^{-i\mathbf{k}\cdot\boldsymbol{\alpha}(t)}\chi_0(t) , \tag{9}$$

where

$$\boldsymbol{\alpha}(t)=\boldsymbol{\alpha}_0\sin\omega t ,$$

and

$$\chi_0(t)=(2\pi)^{-3/2}\exp[i(\mathbf{k}\cdot\mathbf{r}-k^2t/2)]$$

represents a plane wave.

Jain and Tzoar [6], Fonseca and Nunes [8], and later on, Kalman [10] have used $\chi_V(t)$ as an approximation to $\phi_{\mathbf{k}}(t)$ in Eq. (1). The neglect of the influence of the Coulomb field of the nucleus on the final state of the electron cannot be a satisfactory approximation, as in the absence of the laser field, the photoionization cross section is not well approximated by such a zero-order approximation, which becomes appropriate only at high electron energies, i.e., far from the x-ray absorption edge [30]. Exact analytic results for the photoionization amplitude of the hydrogenic states have been known for a long time [31]. Also it is known that the use of the plane wave for K-shell photoeffect in the context of the length gauge leads to an incorrect result when compared with the lowest-order approximation in $1/k$ of the exact formula [32,33].

(ii) A way to partially account for the influence of the Coulomb field is to use an ansatz initially proposed by Jain and Tzoar [20], which consists of multiplying a Coulomb wave function by a Volkov-like term, which leads to the so-called Coulomb-Volkov wave

$$\tilde{\phi}_{\mathbf{k}}(t)=e^{-i\mathbf{k}\cdot\boldsymbol{\alpha}(t)}\phi^{(-)}_{c,\mathbf{k}}(t) , \tag{10}$$

where

$$\phi^{(-)}_{c,\mathbf{k}}(t)=(2\pi)^{-3/2}e^{-ik^2t/2}e^{i\mathbf{k}\cdot\mathbf{r}}e^{\pi/2k}\Gamma(1+i/k) \times {}_1F_1(-i/k,1,-i(kr+\mathbf{k}\cdot\mathbf{r})) \tag{11}$$

is a pure incoming Coulomb wave. This ansatz has been widely used in the multiphoton literature [21,34] as well as in two-color ionization calculations [9,11]. Indeed, via a Fourier expansion of the time-dependent Volkov factor and after a δ-function time integration, the S-matrix element (1) becomes

$$S=\sum_{n=-\infty}^{\infty} S^{(n)}\delta(E_{\mathbf{k}}-E_{1s}-\omega_X+n\omega_L) . \tag{12}$$

In Eq. (12) $S^{(n)}$ is the S-matrix element for the exchange of n laser photons (in addition to the absorption of one x-ray photon) with the expression

$$S^{(n)}=-\pi i J_n(\mathbf{k}_n\cdot\boldsymbol{\alpha}_0)\left\langle\phi^{(-)}_{c,\mathbf{k}_n}\left|\frac{1}{c}\mathbf{A}_{0X}\cdot\mathbf{p}\right|\phi_{1s}\right\rangle , \tag{13}$$

where J_n is a Bessel function and the magnitude of the final momentum $\mathbf{k}_n$ satisfies the energy conservation relation:

$$\frac{k_n^2}{2}=E_{1s}+\omega_X-n\omega_L . \tag{14}$$

In the above equation $n<0$ corresponds to the absorption of n photons and $n>0$ to stimulated emission. As mentioned before, the bound-free atomic matrix element entering Eq. (13) is known in closed form [31] and the corresponding cross section can be calculated without further approximation. One notes that, within this framework, besides a change of the final asymptotic momentum, the effect of the laser field is entirely contained in the Bessel function which modulates the field-free cross section in close analogy with Kroll and Watson's result obtained for laser-assisted electron scattering [35]. A slightly more general form of this an-

satz has been suggested [7] which consists of replacing the momentum $\mathbf{k}$ in the Coulomb wave by the instantaneous time-dependent momentum, Eq. (8). The difficulty with such an extension is that it impedes the ability to perform the time integration of the S matrix in closed form and imposes the use of numerical integration (see the discussion in Ref. [7]).

The main limitation of these approaches is that the possible coupling of the ejected photoelectron with the atomic spectrum, via the laser field, is completely neglected. Now, one expects that such a coupling can become important in the case of a slow photoelectron, typically with an energy of a few eV in hydrogen, in the presence of ~ 1 eV photons (Nd:YAG laser). We present next a way to account, at least partially, for the possible influence of the atomic spectrum on the photoelectron state.

(iii) Banerji and Mittleman [22] have suggested improving the Coulomb-Volkov ansatz by replacing the Coulomb function in Eq. (10) by a modified form, including first-order radiative corrections. The calculation proceeds as follows: one assumes that the positive-energy solutions of the time-dependent Schrödinger equation (2) are of the general form

$$\phi_{\mathbf{k}}(t)=e^{-i\mathbf{k}\cdot\alpha(t)}V_{\mathbf{k}}(t)\ , \tag{15}$$

which is substituted in Eq. (2) and solved perturbatively with respect to the field for the unknown function $V_{\mathbf{k}}(t)$. The result of this analysis is the following form [12]:

$$|\phi_{\mathbf{k}}(t)\rangle=e^{-i\mathbf{k}\cdot\alpha(t)}\left\{|\phi_{c,\mathbf{k}}^{(-)}(t)\rangle-\frac{\omega_L}{2}\left[G_c\left[\frac{k^2}{2}-\omega_L\right]\boldsymbol{\alpha}_0\cdot(\mathbf{k}-\mathbf{p})|\phi_{c,\mathbf{k}}^{(-)}(t)\rangle e^{i\omega_L t}\right.\right.$$
$$\left.\left.+G_c\left[\frac{k^2}{2}+\omega_L\right]\boldsymbol{\alpha}_0\cdot(\mathbf{k}-\mathbf{p})|\phi_{c,\mathbf{k}}^{(-)}(t)\rangle e^{-i\omega_L t}\right]\right\}\ . \tag{16}$$

This is the approach we shall adopt in the present calculation. Note that Banerji and Mittleman have in fact used a simplified version of this general expression, only valid in the low-frequency limit $\omega_L\to 0$. Note also that, if one omits the second term in the curly brackets, one recovers the Coulomb-Volkov ansatz. The terms containing the Coulomb Green's functions include the contributions of the discrete as well as continuous atomic spectrum to the dressing of the ejected electron. Although higher-order corrections [36] can be formally written, the corresponding terms become of increasing complexity and difficult to handle in actual calculations.

A common feature of the above-mentioned wave functions, which *are not exact solutions* of the time-dependent Schrödinger equation, is that the dressing of the ejected electron is taken into account in a mixed way. More precisely the electron-field interaction is treated to all orders in the Volkov exponential factor Eqs. (9), (10), and (16), and at best to first order in the Coulomb wave function Eq. (16). Let us mention that for moderate laser intensities, and if one is interested in the exchange of only one laser photon, a correct, or at least consistent, treatment of the problem requires the use of a Coulomb wave corrected to first order in the field as

$$|\phi_{c,\mathbf{k}}^{(1)}(t)\rangle=|\phi_{c,\mathbf{k}}^{(-)}(t)\rangle+\frac{\omega_L}{2}\left[G_c\left[\frac{k^2}{2}-\omega_L\right]\boldsymbol{\alpha}_0\cdot\mathbf{p}|\phi_{c,\mathbf{k}}^{(-)}(t)\rangle e^{i\omega_L t}+G_c\left[\frac{k^2}{2}+\omega_L\right]\boldsymbol{\alpha}_0\cdot\mathbf{p}|\phi_{c,\mathbf{k}}^{(-)}(t)\rangle e^{-i\omega_L t}\right]\ , \tag{17}$$

which corresponds to Eq. (4) for the case of positive-energy states.

III. CALCULATIONS

According to the description of our method given in Secs. I and II, we use the expressions (4) and (16) respectively, for the ground-state and the continuum-state wave functions. The S-matrix element (1) becomes

$$S=-\frac{i}{2}\int_{-\infty}^{+\infty}dt\,e^{i[\mathbf{k}\cdot\boldsymbol{\alpha}_0\sin\omega_L t+(k^2/2-E_{1s}-\omega_X)t]}$$
$$\times\left\langle\phi_{c,\mathbf{k}}^{(-)}\right|\left\{1+\frac{\omega_L}{2}\boldsymbol{\alpha}_0\cdot(\mathbf{p}-\mathbf{k})\left[G_c\left[\frac{k^2}{2}-\omega_L\right]e^{-i\omega_L t}+G_c\left[\frac{k^2}{2}+\omega_L\right]e^{i\omega_L t}\right]\right\}\frac{1}{c}\mathbf{A}_{0X}\cdot\mathbf{p}$$
$$\times\left\{1+\frac{\omega_L}{2}\boldsymbol{\alpha}_0\cdot\mathbf{p}[G_c(E_{1s}-\omega_L)e^{i\omega_L t}+G_c(E_{1s}+\omega_L)e^{-i\omega_L t}]\right\}\left|\phi_{1s}\right\rangle\ .$$

After Fourier expanding the sinusoidal exponential and performing the time integration, one gets the same structure of the S-matrix element as in (12), but now the matrix element $S^{(n)}$ for the photoionization associated to the exchange of n laser photons is

$$S^{(n)}=-\pi i(f_{\mathrm{I}}+f_{\mathrm{II}}+f_{\mathrm{III}})\ , \tag{18}$$

with

$$f_{\mathrm{I}}=J_n(\lambda)\left\langle\phi^{(-)}_{c,\mathbf{k}_n}\left|\frac{1}{c}\,\mathbf{A}_{0X}\cdot\mathbf{p}\right|\phi_{1s}\right\rangle \tag{19a}$$

$$f_{\mathrm{II}}=J_{n+1}(\lambda)\left\{\frac{\omega_L}{2}\left[\left\langle\phi^{(-)}_{c,\mathbf{k}_n}\left|\boldsymbol{\alpha}_0\cdot\mathbf{p}G_c\left[\frac{k_n^2}{2}-\omega_L\right]\frac{1}{c}\,\mathbf{A}_{0X}\cdot\mathbf{p}\right|\phi_{1s}\right\rangle + \left\langle\phi^{(-)}_{c,\mathbf{k}_n}\left|\frac{1}{c}\,\mathbf{A}_{0X}\cdot\mathbf{p}G_c(E_{1s}+\omega_L)\boldsymbol{\alpha}_0\cdot\mathbf{p}\right|\phi_{1s}\right\rangle\right]+\frac{\lambda}{2}\left\langle\phi^{(-)}_{c,\mathbf{k}_n}\left|\frac{1}{c}\,\mathbf{A}_{0X}\cdot\mathbf{p}\right|\phi_{1s}\right\rangle\right\}, \tag{19b}$$

$$f_{\mathrm{III}}=J_{n-1}(\lambda)\left\{\frac{\omega_L}{2}\left[\left\langle\phi^{(-)}_{c,\mathbf{k}_n}\left|\boldsymbol{\alpha}_0\cdot\mathbf{p}G_c\left[\frac{k_n^2}{2}+\omega_L\right]\frac{1}{c}\,\mathbf{A}_{0X}\cdot\mathbf{p}\right|\phi_{1s}\right\rangle + \left\langle\phi_{c,\mathbf{k}_n}\left|\frac{1}{c}\,\mathbf{A}_{0X}\cdot\mathbf{p}G_c(E_{1s}-\omega_L)\boldsymbol{\alpha}_0\cdot\mathbf{p}\right|\phi_{1s}\right\rangle\right]-\frac{\lambda}{2}\left\langle\phi^{(-)}_{c,\mathbf{k}_n}\left|\frac{1}{c}\,\mathbf{A}_{0X}\cdot\mathbf{p}\right|\phi_{1s}\right\rangle\right\}, \tag{19c}$$

where $\lambda=\boldsymbol{\alpha}_0\cdot\mathbf{k}_n$ and the magnitude of $\mathbf{k}_n$ is given by Eq. (14). In order to derive the above expressions for f_{II} and f_{III} we have used the fact that

$$\left\langle\phi^{(-)}_{c,\mathbf{k}_n}\left|\boldsymbol{\alpha}_0\cdot\mathbf{k}_nG_c\left[\frac{k_n^2}{2}\pm\omega_L\right]\frac{1}{c}\,\mathbf{A}_{0X}\cdot\mathbf{p}\right|\phi_{1s}\right\rangle = \pm\frac{\boldsymbol{\alpha}_0\cdot\mathbf{k}_n}{\omega_L}\left\langle\phi^{(-)}_{c,\mathbf{k}_n}\left|\frac{1}{c}\,\mathbf{A}_{0X}\cdot\mathbf{p}\right|\phi_{1s}\right\rangle .$$

Before going further, it is interesting to discuss the structure of these general expressions and to compare them with the ones obtained within more restricted approximations. First, the reduced form, Eq. (13), obtained by using simpler Coulomb-Volkov functions, is recovered on retaining only the contribution f_{I} as given in Eq. (19a). Discarding the terms (19b) and (19c) then amounts to completely neglecting target dressing effects. More precisely, these terms containing the Coulomb Green's function account for the influence of the hydrogenic spectrum or, in other words, of the dynamic polarizability of the atom either in its ground state or in its (positive-energy) final state. Note that we have included counter-rotating wave contributions in these terms which account for the "dressing" of the atomic states. As we shall show below, the reduced form (19a) becomes a fair approximation only if the outgoing electron has a relatively high kinetic energy.

The differential cross section corresponding to the exchange of n laser photons, normalized to the flux of incident x-ray photons, is obtained from $S^{(n)}$ in Eq. (18), as

$$\frac{d\sigma^{(n)}}{d\Omega}=\frac{1}{\pi}\frac{\omega_X}{I_X}|S^{(n)}|^2 . \tag{20}$$

One can recover also the lowest-order perturbative limit, in which only one laser photon is exchanged, by specializing to the cases $|n|=1$ and retaining only the first-order term in the laser field strength (i.e., in α_0) in the Bessel function expansions and (or) in the matrix elements. In this limit, one obtains the following expression for the S-matrix element in the case of the absorption of one laser photon ($n=-1$) in addition to that of the x-ray photon:

$$S^{(-1)}=-i\pi(f_A^{(-1)}+f_B^{(-1)}) . \tag{21}$$

$f_A^{(-1)}$ and $f_B^{(-1)}$ are explicitly

$$f_A^{(-1)}=4\pi\alpha\frac{(I_XI_L)^{1/2}}{\omega_X\omega_L}\times\left\langle\phi^{(-)}_{c,\mathbf{k}_{-1}}\left|\frac{1}{c}\boldsymbol{\epsilon}_X\cdot\mathbf{p}G_c(E_{1s}+\omega_L)\boldsymbol{\epsilon}_L\cdot\mathbf{p}\right|\phi_{1s}\right\rangle , \tag{22a}$$

$$f_B^{(-1)}=4\pi\alpha\frac{(I_XI_L)^{1/2}}{\omega_X\omega_L}\times\left\langle\phi^{(-)}_{c,\mathbf{k}_{-1}}\left|\frac{1}{c}\boldsymbol{\epsilon}_L\cdot\mathbf{p}G_c\left[\frac{k_1^2}{2}-\omega_L\right]\boldsymbol{\epsilon}_X\cdot\mathbf{p}\right|\phi_{1s}\right\rangle , \tag{22b}$$

with k_{-1} given by (14) when $n=-1$. Here $\alpha\sim\frac{1}{137}$ is the fine structure constant, I_L and I_X are the laser and x-ray intensities, and $\boldsymbol{\epsilon}_L$ and $\boldsymbol{\epsilon}_X$ are the corresponding field polarization vectors. We have introduced these latter parameters for the sake of future discussion of their influence on the cross section. A similar analysis, not reproduced here, can be done for the case of stimulated emission ($n=1$).

One remarks that the second-order perturbative matrix elements, Eqs. (22), are formally identical to those entering the general expression, Eqs. (19b) and (19c). Since the pioneering works of Zernik [37], Klarsfeld [38], and Gavrila [39], several techniques have been used for analytical and numerical evaluation of such two-photon matrix elements in bound-free transitions [40]. The difference with respect to these previous works is that they considered the absorption of two identical photons, i.e., the "one-color" case. Note also that the process in which the laser photon is emitted is, in fact, a stimulated Compton scattering on a bound electron, namely, the stimulated version of the process already discussed by Gavrila [39].

Nevertheless, the regime of frequencies we have to consider is different from that treated before.

For the computation of the second-order amplitudes we have chosen two distinct methods, allowing us to check our numerical results. In the first method, we started form analytic expressions [39] of the two-photon matrix element between the ground state and a continuum state, implying numerical evaluations of several Appell hypergeometric functions of two variables F_1. These functions have been evaluated using either their standard integral representation, or the appropriate expansion as series of Gauss functions [41]. The second method used is the Sturmian implementation of the so-called Dalgarno method, described in Taïeb *et al.* [42]. It is worth emphasizing that two different gauges have been used in our calculations, namely, the velocity gauge in the first case and the length gauge in the second one. The results obtained through these two methods coincide, in agreement with the general theory of gauge invariance for multiphoton transition amplitudes [43].

IV. RESULTS AND DISCUSSION

We have chosen to mainly discuss here the importance of the dressing of the target states on the angular distribution of the ejected photoelectron, the differential cross section being indeed most sensitive to such effects. Similar situations have been met in atomic bremsstrahlung calculations [44,45] and also in laser assisted (e-2e) collisions [12], where the results depend in particular on the dynamical polarizability of the atomic system, not only in its initial state but also in its final state belonging to the continuum.

Within this framework, we have first considered in detail the perturbative regime, corresponding to the case of low or moderate laser intensities, and to physical processes in which one photon from the laser field is exchanged with the atomic system. Two possibilities can then arise, according to whether the laser photon is absorbed (two-photon absorption) or emitted via stimulated emission (stimulated Compton scattering). It appears in fact that, within the perturbative regime, if one considers the exchange of only one laser photon, a second-order calculation is fully consistent, so long as one can safely discard higher-order contributions which, in our case, would be fourth order at least.

In a second part we shall consider nonperturbative corrections to these effects and will discuss the limitations of simplified approaches which do not take into account target dressing effects. We shall briefly discuss also the possible range of validity of our model.

Although the target dressing effects are important and can strongly modify the angular dependence of the differential cross sections, in both the perturbative and nonperturbative regimes, it should be kept in mind, however, that the laser polarization also plays an important role. We will also illustrate this point by considering different orientations of the laser polarization with respect to the one of the "x-ray" photon, first in the perturbative regime and then in the nonperturbative one.

A. Perturbative results

Within the perturbative regime, the S-matrix element for the exchange of one laser photon, in addition to the absorption of the "x-ray" photon, is given in Eqs. (21) and (22). We have computed the corresponding cross sections for various frequencies of the "x-ray" photon, keeping fixed the laser frequency at $\hbar\omega_L = 1.17$ eV ($\omega_L = 0.043$ a.u.), which is the one of the Nd:YAG laser. We have also kept the orientation of the laser polarization ϵ_L fixed, defining the Oz axis in the following analysis. At the same time we have considered x rays of definite polarization, denoted ϵ_X. The Ox axis has been chosen along the propagation direction of both fields; θ and φ represent the polar angles of the ejected electron. For parallel polarizations ($\epsilon_L \| \epsilon_X$) the angular distributions are of the general form [46]

$$\frac{d\sigma}{d\Omega} \approx I_L(\alpha + \beta\cos^2\theta + \gamma\cos^4\theta) \ . \tag{23a}$$

Note that in the case of orthogonal polarizations

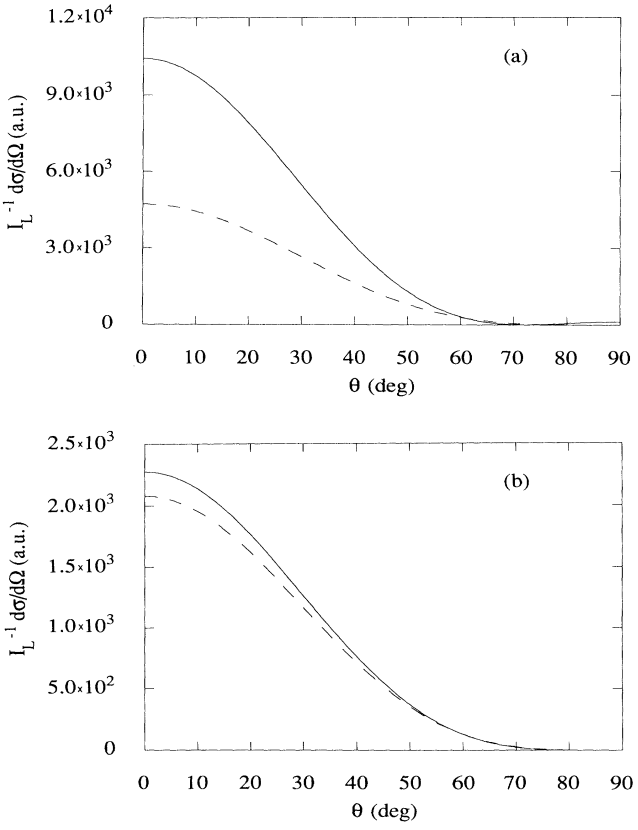

FIG. 1. The differential cross section for the photoionization of atomic hydrogen, as defined in Eq. (20) and normalized with respect to the intensity of the laser field (see text), in the perturbative regime, as a function of the polar angle θ. The case of two-photon absorption ($n = -1$) is considered for $\hbar\omega_L = 1.17$ eV with $\hbar\omega_X = 16$ eV (a) and $\hbar\omega_X = 50$ eV (b). The polarization vectors are parallel: $\epsilon_L \| \epsilon_X$. The full line represents the results of the present calculation, Eqs. (21) and (22); the broken line corresponds to the simplified form in Eq. (24).

($\epsilon_L \perp \epsilon_X$), the differential cross section also has a dependence on φ, so that one obtains

$$\frac{d\sigma}{d\Omega} \approx I_L \delta \sin^2\theta \cos^2\theta \sin^2\varphi \ . \tag{23b}$$

Keeping these parameters fixed allowed us to explore the effects of the field on the angular distribution of photoelectrons with different kinetic energies and to assess the validity of some approximations.

In Fig. 1 are displayed the differential cross sections for two-photon absorption as a function of the angle between the photoelectron asymptotic momentum and the polarization vectors of the "x-ray" and laser fields which are both directed along the Oz axis, ($\epsilon_L \| \epsilon_X$). Since, to lowest order in the laser field, the cross section is proportional to the laser intensity I_L, we have reported the ratio $(d\sigma/d\Omega)/I_L$. In these figures we compare the results as obtained from our exact computation of the second-order matrix elements, Eqs. (21) and (22), with those obtained in the low-intensity limit if one neglects the dressing of the ground state and if the continuum final state is described via a Coulomb-Volkov wave. The corresponding S-matrix element would then reduce to the amplitude f_I as given in Eq. (19a). In the low-intensity limit this expression simplifies further, as one can retain only the lowest-order term in the Bessel function expansion:

$$\tilde{S}^{(n)} = -i\pi \frac{\mathbf{k}_n \cdot \boldsymbol{\alpha}_0}{2} \left\langle \phi^{(-)}_{c,\mathbf{k}_n} \left| \frac{1}{c} \mathbf{A}_{0X} \cdot \mathbf{p} \right| \phi_{1s} \right\rangle \ . \tag{24}$$

In fact, this expression coincides with the "soft-photon" limiting form ($\omega_L \to 0$) of the second-order amplitude (22b), which describes the absorption of the "x-ray" photon *followed* by the absorption of the low-frequency one. In this case, the two-photon amplitude factorizes as a result of the Low theorem [47] which is reminiscent of the infrared divergence of quantum electrodynamics (see also Refs. [48,49] for the case of two-photon continuum-continuum transitions).

We have first considered the case of a high-frequency field with a photon energy $\hbar\omega_X = 16$ eV, so that the photoelectron would be ejected with a relatively low kinetic energy ($E_{\mathbf{k}} = 2.4$ eV) in the absence of the laser field. It appears then that the limiting form (24) is very inaccurate and underestimates the cross section although the laser frequency considered here is quite low and belongs to the IR range. This shows in fact that the soft-photon approximation cannot be used without precaution, in particular for slow electrons, i.e., just above the ionization threshold. More generally, this result indicates that predictions based on the use of Coulomb-Volkov waves are inaccurate close to the threshold.

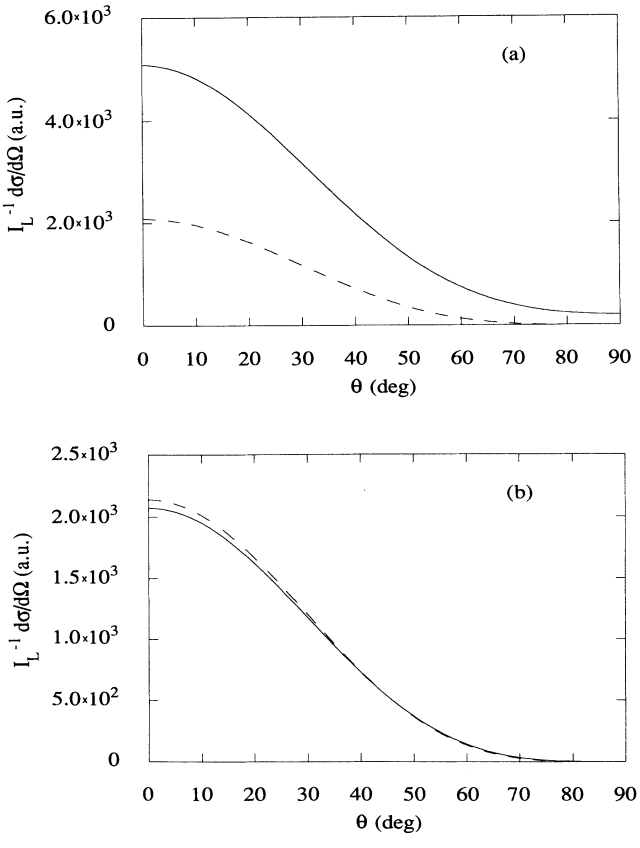

FIG. 2. Same as Fig. 1, but for the case of one-photon stimulated emission (stimulated Compton effect, $n = 1$). (a) $\hbar\omega_X = 16$ eV; (b) $\hbar\omega_X = 50$ eV.

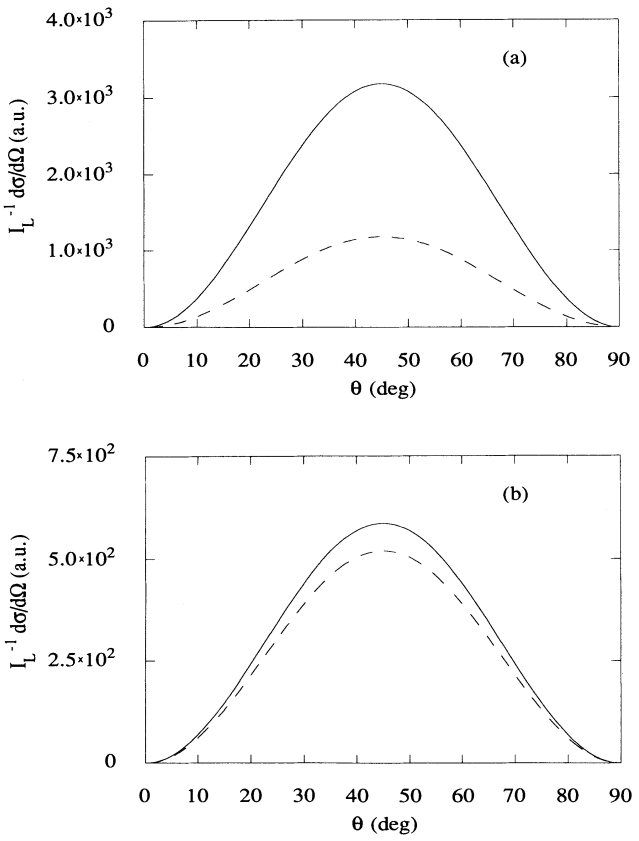

FIG. 3. Same as Fig. 1, but the polarization vectors are orthogonal, $\epsilon_L \perp \epsilon_X$, and the azimuthal angles $\varphi = \pi/2$. (a) $\hbar\omega_X = 16$ eV; (b) $\hbar\omega_X = 50$ eV.

If one keeps the laser frequency fixed and increases the "x-ray" photon frequency, the soft-photon approximation becomes more reliable. This is shown in Fig. 1(b), where angular distributions are displayed for $\hbar\omega_X = 50$ eV, which implies that the ejected electron would have an energy of 36.4 eV in the absence of the laser. One observes indeed that the cross sections, as obtained from the "exact" Eqs.(21), (22), and approximate Eq. (24) are much closer to each other.

This behavior is also observed for the "stimulated Compton effect" in which one laser photon is emitted, see Fig. 2. The overall shape of the angular distribution is similar, since the angular momentum algebra is identical to the one in the preceding case. Again, one observes that the soft-photon approximation, Fig. 2(b), is much better in the high-energy range. In addition, one verifies by comparing Figs. 1(b) and 2(b) that absorption and stimulated emission of one laser photon have almost the same probabilities since the cross sections become comparable at high photoelectron energies. This feature, which is in fact a consequence of the Low theorem [47], contrasts with the low-energy domain in which the magnitudes of the "exact" cross sections differ significantly for emission and absorption; compare Figs. 1(a) and 2(a).

Another general observation which can be drawn from our results is the overall decrease of the cross sections for increasing energies of the "x-ray" photon. This is illustrated by comparing Figs. 1(a) [2(a)] and 1(b) [2(b)]. This clearly shows that the presence of the low-frequency laser does not modify substantially the general trend of photoelectric cross sections which decrease with increasing x-ray photon energies.

Still within the perturbative regime, we show next that the choice of the respective orientations of the polarization directions of the fields plays an important role in the angular distribution of the ejected photoelectron. In Fig. 3(a) are shown the differential cross sections for two-photon absorption in the low-energy regime for conditions similar to those in Fig. 1(a), except that the laser polarization has been chosen as $\epsilon_L \perp \epsilon_X$. The angular distri-

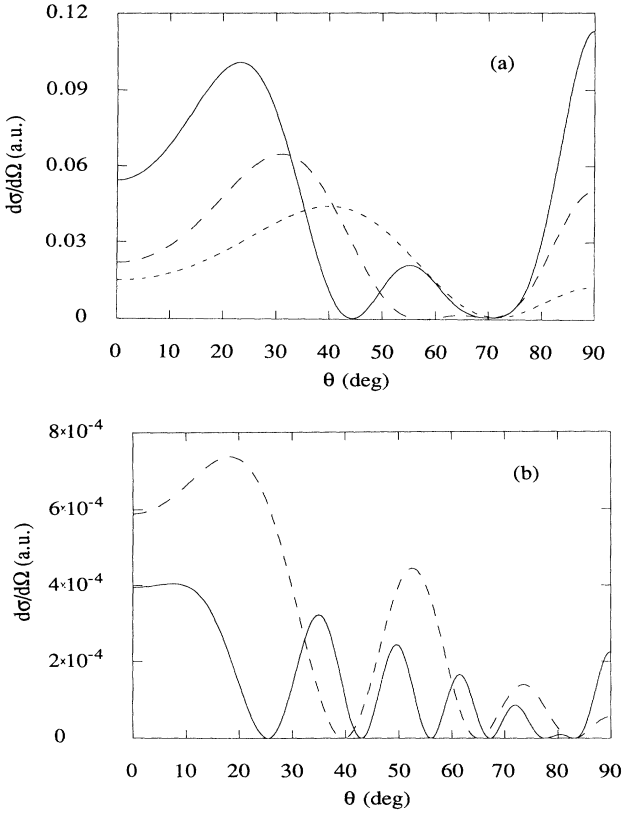

FIG. 4. (a) The differential cross section for the photoionization of atomic hydrogen, Eq. (20), as computed from the present formalism, Eqs. (18) and (19), as a function of the polar angle θ, in the case of the absorption of one laser photon ($n=-1$), for $\hbar\omega_L = 1.17$ eV and $\hbar\omega_X = 16$ eV. The polarization vectors are parallel: $\epsilon_L \| \epsilon_X$. The full line corresponds to a laser field strength $F_{0L} = 3\times10^{-2}$ a.u., the broken line to $F_{0L} = 2\times10^{-2}$ a.u., and the dotted line to $F_{0L} = 10^{-2}$ a.u. (b) Same as (a), but $\hbar\omega_X = 50$ eV; the full line corresponds to a value of the field strength $F_{0L} = 2\times10^{-2}$ a.u., the broken line to $F_{0L} = 10^{-2}$ a.u.

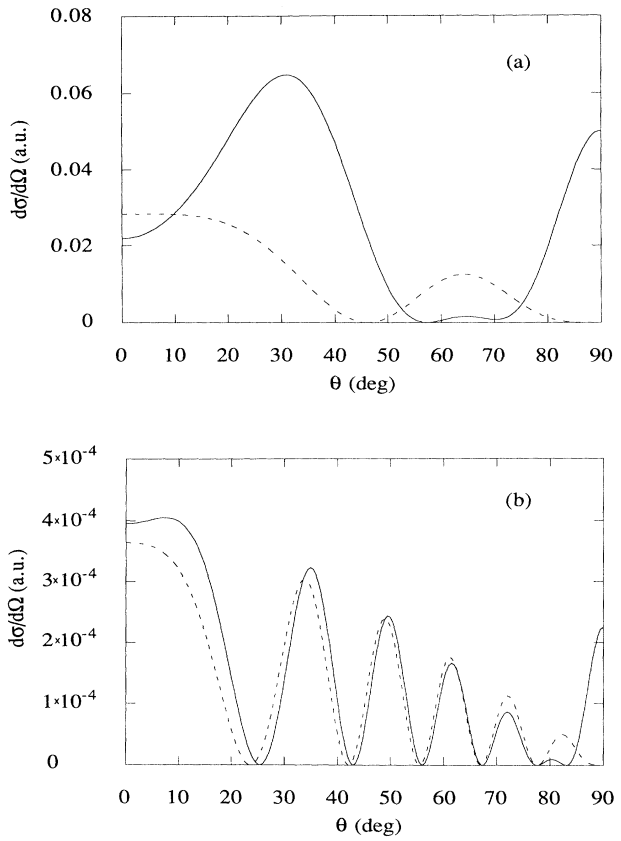

FIG. 5. (a) The differential cross section for the photoionization of atomic hydrogen, defined in Eq. (20), as a function of the polar angle θ, in the case of one laser photon absorption ($n=-1$), for energies and polarizations as in Fig. 4(a). The value of the laser field strength is $F_{0L} = 2\times10^{-2}$ a.u. The full line corresponds to the S-matrix element computed from Eqs. (18) and (19), the broken line to that deduced from the simplified form, Eq. (13). (b) Same as (a), but $\hbar\omega_X = 50$ eV.

butions now have a bell-shaped form with maximum at $\theta=\pi/4$, being zero for $\theta=0$ and $\theta=\pi/2$, instead of the monotonic variations of the preceding cases. The origin of this behavior can be traced back to the angular momentum algebra. Besides these notable differences, the above conclusions, regarding the validity of the soft-photon approximation, still hold, as observed when comparing Figs. 3(a) and 3(b). Note that we have not displayed the results obtained for the stimulated Compton process, as the angular distributions, as well as the conclusions regarding the applicability of the Low theorem, are much similar.

We now turn to the discussion of nonperturbative effects which become important at higher laser intensities.

B. Nonperturbative results

If one increases the laser intensity, the above approach is no longer valid and one has to account for nonperturbative effects which are known to become important at laser field strengths of about 10^{-2} a.u. ($I_L \approx 10^{12}$ W/cm^2) in hydrogen. These high-intensity effects, which are included in the general expressions (18) and (19), significantly affect the angular distributions of the photoelectrons, as compared to those obtained in the perturbative regime. This is illustrated first in Fig. 4(a) in which the differential cross section, as obtained from the full expressions (18) and (19), is shown for three laser field strengths: $F_0=10^{-2}$, 2×10^{-2}, and 3×10^{-2} a.u. The position of the maximum of the distribution is now shifted from the origin and is strongly intensity dependent. Secondary maxima appear, as the intensity increases and the magnitude of the cross section becomes significant at $\theta=\pi/2$ in contrast to the perturbative case. The oscillatory character of the variations of the cross section is even more marked when the "x-ray" photon frequency is larger, i.e., when the photoelectron energy increases. This is shown in Fig. 4(b) which displays the angular distribution in the same conditions as in Fig. 1(b). The cross section is now computed from the nonperturbative expressions Eqs. (18) and (19) for laser field strengths $F_0=10^{-2}$ and 2×10^{2} a.u. We note that the fact that the differential cross section is not small at $\theta=\pi/2$ is a signature of the occurrence of target dressing effects.

A generalized version of the Low theorem in the soft-photon limit, valid in the intense field regime, is known to apply to laser-assisted potential scattering [35], and also to the case of e-atom excitation [50]. The question arises of its applicability in the context of two-color ionization, in which case it leads to the simplified expression, Eq. (19a). Our computations of the complete expressions of the S-matrix elements (18) and (19), when compared to those deduced from this simplified form, clearly show the

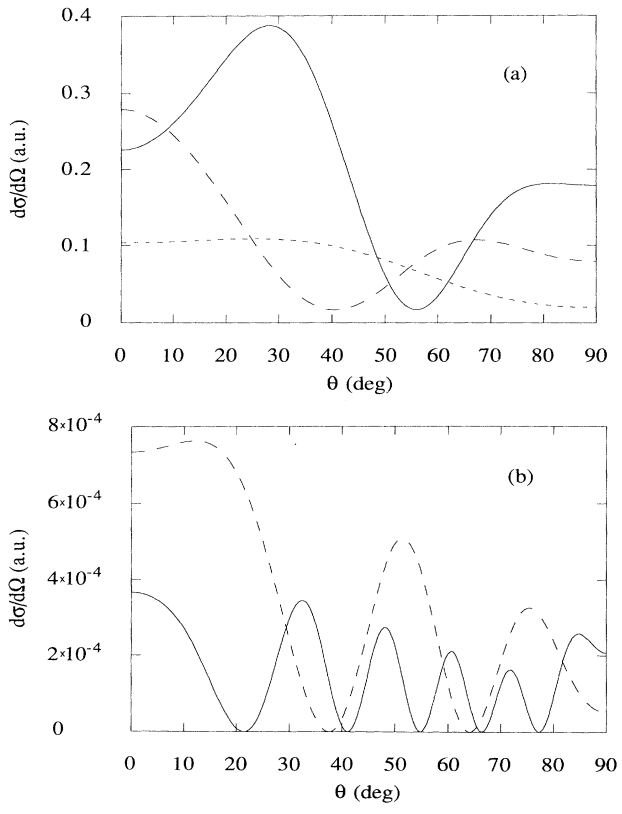

FIG. 6. Same as Fig. 4, but in the case of one-photon stimulated emission (stimulated Compton effect, $n=1$). (a) $\hbar\omega_X=16$ eV; (b) $\hbar\omega_X=50$ eV.

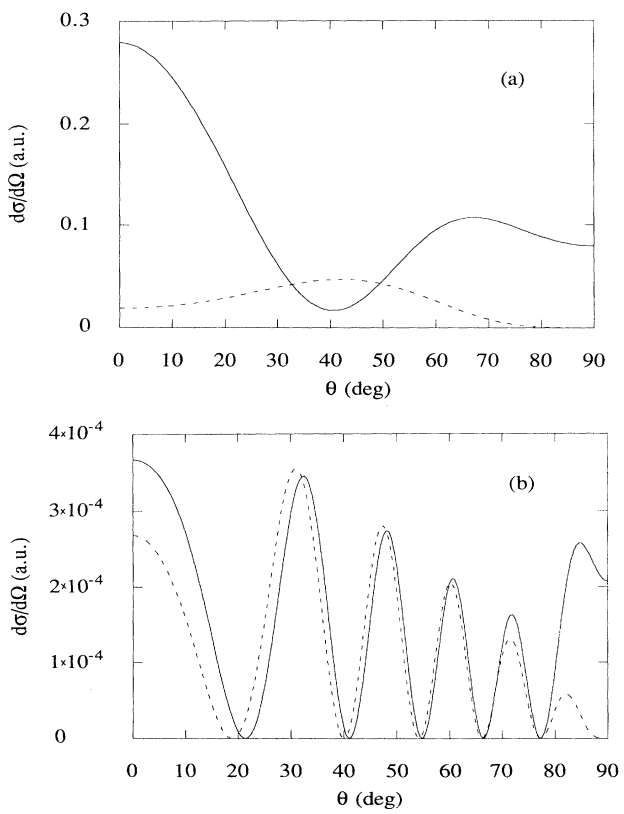

FIG. 7. Same as Fig. 5, but in the case of one-photon stimulated emission (stimulated Compton effect, $n=1$). (a) $\hbar\omega_X=16$ eV; (b) $\hbar\omega_X=50$ eV.

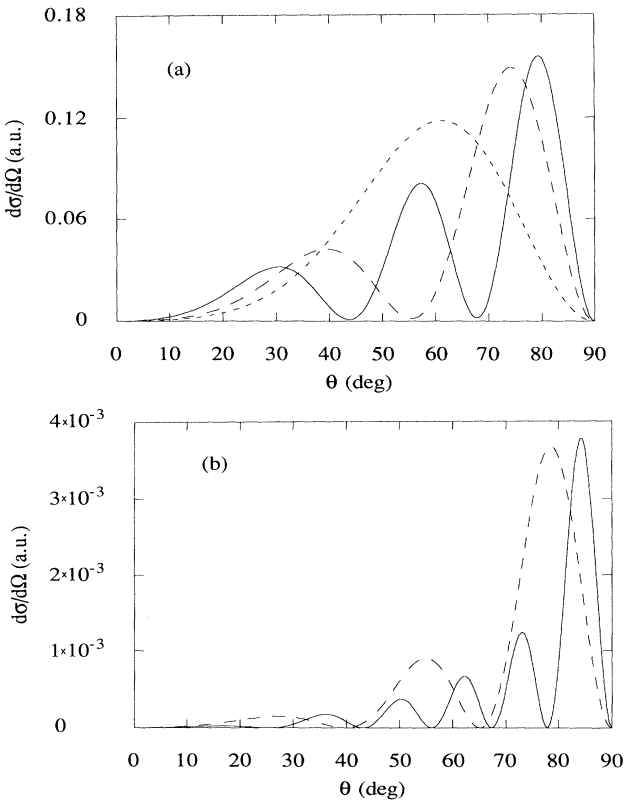

FIG. 8. Same as Fig. 4, but for orthogonal polarization vectors $\epsilon_L \perp \epsilon_X$, and $\varphi = \pi/2$. (a) $\hbar\omega_X = 16$ eV; (b) $\hbar\omega_X = 50$ eV.

inadequacy of the latter if the photoelectrons are relatively slow, see Fig. 5(a). Note that, in this figure, the conditions are the same as in Fig. 1(a), except for the laser field strength which is $F_0 = 2 \times 10^{-2}$ a.u. It is, in fact, only at higher photoelectron kinetic energies that the simplified form Eq. (19a) regains some validity. This is illustrated in Fig. 5(b), with the same conditions as those for Fig. 1(b), except for the laser field strength which is the same as in Fig. 5(a). Here, one observes that the oscillations of the angular distributions are quite well reproduced by the approximation, with again a notable exception at $\theta = \pi/2$, where the approximate calculation is zero while ours is not.

The same conclusions hold also for one-photon stimulated emission as shown in Figs. 6 and 7. Again, the simplified treatment, based on the high-intensity version of the Low theorem, is only approximately valid at relatively high photoelectron energies.

The role of the choice of the respective polarizations of the fields is illustrated in Fig. 8, where the differential cross sections are shown at various energies and laser field strengths for $\epsilon_L \perp \epsilon_X$ in the case of two-photon absorption ($n = -1$). The main influence of nonperturbative effects, as compared to the perturbative regime of Fig. 3, is to introduce oscillations and to shift the position of the maximum of the distribution. This shift, as well as the oscillatory character of the angular distribution, is clearly a signature of nonperturbative effects.

The main conclusion which may be drawn from this set of results is that target dressing effects and, more precisely, the role of the atomic spectrum, can be very important when discussing two-color ionization in the presence of a strong laser field. This is particularly so when the photoelectron energy is relatively small, i.e., close to the ionization threshold. In this respect, our results demonstrate that simplified treatments based on the sole use of pure Coulomb-Volkov waves (and *a fortiori* of simpler Volkov waves), can be very inaccurate, depending on the photoelectron energy and laser frequency. Our results indicate also that the signature of atom dressing effects in the higher laser intensity range is the nonvanishing of the differential cross section at some selected angles, a phenomenon which should be easily observable. Eventually, we have also shown that the polarization orientation of the laser field plays an important role on the angular distribution of the ejected photoelectron. Note also that, in the case of circularly polarized laser field, additional features of the differential cross section can be predicted [51].

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