Role of atomic coherence in lasing without inversion

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A noninversion laser system which is closed lifetime broadened and free of the restriction on atomic decay rates is proposed. For the realization of lasing without inversion, the Rabi frequency of the applied field must exceed a lower threshold. Moreover, the coherence term in dressed states must enhance the gain. These results are obtained beyond the specific approximations.

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Some time ago, it was believed that population inversion was a necessary condition for laser action. Recent advances [1–15] in this field have shown that some laser systems may operate in the absence of population inversion. The essential point for this possibility is to modify the emissive and absorptive profiles, with the help of a quantum-interference effect.

In these laser systems [1–15], if atomic coherence, a kind of quantum interference, is produced in lower levels, it leads to an absorption cancellation. If atomic coherence exists in upper levels, Agarwal [16] studied the role of atomic coherence in gain. In particular, for the Λ type of closed lifetime-broadened system proposed recently by Imamoğlu, Field, and Harris [15], Agarwal found that in the strong-coherence–driving-field case the gain originates from the contribution of the coherence term in dressed states, i.e., the coherence-term-induced enhancement.

In this paper, we propose a noninversion laser system which is, as in the model of Imamoğlu, Field, and Harris [15], closed lifetime broadened. Atomic coherence is produced by an electromagnetic (em) field in two lower levels. This noninversion laser system has three separate features: (i) A lower threshold exists for the applied em field and an analytical expression is obtained for this threshold. (ii) The appearance of lasing without inversion requires only the proper arrangements of the external conditions; no requirement is placed on atomic decay rates. (iii) The coherence term in dressed states is proved positive in gain beyond the specific approximations (e.g., the strong em field approximation).

The model system considered is shown in Fig. 1, where the probe field and em field are assumed to be on resonance with corresponding atomic transitions. The equation of motion for the atomic density matrix in the rotating frame may be written in the form [16]

$$\begin{aligned}\dot{\rho}=&-r_1(A_{11}\rho+\rho A_{11}-2\rho_{11}A_{22})-r_2(A_{22}\rho+\rho A_{22}-2\rho_{22}A_{33})\\&-\lambda_1(A_{22}\rho+\rho A_{22}-2\rho_{22}A_{11})-\lambda_2(A_{33}\rho+\rho A_{33}-2\rho_{33}A_{22})+ig_c[A_{23}+A_{32},\rho]+ig_p[A_{12}+A_{21},\rho]\ ,\end{aligned} \tag{1}$$

where $A_{ij}=|i\rangle\langle j|$. r_1 (r_2) is the spontaneous decay rate and λ_1 (λ_2) is the incoherently pumping rate, respectively. $2g_c$ and $2g_p$ are the Rabi frequencies of the em field used to create atomic coherence and a weak probe field, respectively, and may be assumed to be real positive for simplicity.

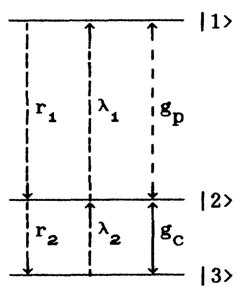

FIG. 1. The energy-level diagram.

In order to find the conditions under which lasing occurs in the absence of population inversion, we first need to identify the condition of noninversion, i.e., the population in the single upper level is less than that in each of two lower levels. This may be easily performed by writing Eq. (1) in the form of matrix elements of density operator and then solving it in steady state. The populations in the three levels are found to be

$$\begin{aligned}\rho_{11}&=\frac{\lambda_1[g_c^2+\lambda_2(r_2+\lambda_1+\lambda_2)]}{W}\ ,\\ \rho_{22}&=\frac{r_1[g_c^2+\lambda_2(r_2+\lambda_1+\lambda_2)]}{W}\ ,\\ \rho_{33}&=\frac{r_1[g_c^2+r_2(r_2+\lambda_1+\lambda_2)]}{W}\ ,\end{aligned} \tag{2}$$

where $W=(\lambda_1+2r_1)g_c^2+(r_2+\lambda_1+\lambda_2)(r_1r_2+r_1\lambda_2+\lambda_1\lambda_2)$. Thus the populations are not inverted when

$$\lambda_1 < \min\left[r_1, \left[1-\frac{(\lambda_2-r_2)(r_2+\lambda_1+\lambda_2)}{g_c^2+\lambda_2(r_2+\lambda_1+\lambda_2)} \right] r_1 \right] . \quad (3)$$

In the following, we study the gain condition of the weak probe field. For the purpose of studying the role of atomic coherence in gain, we work in the dressed states which are defined by

$$|0\rangle=|1\rangle, \quad |\pm\rangle=\frac{1}{\sqrt{2}}(|2\rangle\pm|3\rangle|) . \quad (4)$$

By transferring Eq. (1) into dressed states, we obtain

$$\begin{aligned}
\dot{\rho}_{\pm\pm} &= -\frac{\lambda_1}{2}(\rho_{++}+\rho_{--}) \mp \frac{r_2+\lambda_1+\lambda_2}{2}(\rho_{++}-\rho_{--}) \\
&\quad + r_1\rho_{00} - \frac{\lambda_1}{2}(\rho_{+-}+\rho_{-+}) , \\
\dot{\rho}_{\pm\mp} &= \left[\pm ig_c - r_2 - \frac{\lambda_1}{2} - \lambda_2 \right](\rho_{+-}+\rho_{-+}) \\
&\quad + \left[ig_c \mp \frac{r_2+\lambda_1+\lambda_2}{2} \right](\rho_{+-}-\rho_{-+}) \\
&\quad + r_1\rho_{00} - \left[r_2 + \frac{\lambda_1}{2} - \lambda_2 \right](\rho_{++}+\rho_{--}) , \\
\dot{\rho}_{0\pm} &= -\frac{r_1+r_2+\lambda_1 \pm ig_c}{2}(\rho_{0+}+\rho_{0-}) \\
&\quad \mp \frac{r_1+\lambda_2 \pm ig_c}{2}(\rho_{0+}-\rho_{0-}) \\
&\quad - \frac{ig_p}{\sqrt{2}}(\rho_{00}-\rho_{\pm\pm}) + \frac{ig_p}{\sqrt{2}}\rho_{\mp\pm} ,
\end{aligned} \quad (5)$$

where the contributions of the weak probe field to atomic population $\dot{\rho}_{\pm\pm}$ and coherence $\dot{\rho}_{\pm\mp}$ have been omitted.

If the loss rate due to the leakage of the cavity is small, the equation of motion for the probe field is

$$\dot{E} = 2\pi i \omega_p N d_{21} \rho_{12} , \quad (6)$$

where ω_p is the probe field frequency, N is atomic density, and d_{21} is the matrix element of atomic dipole transition. The weak probe field will be amplified when

$$G = -\mathrm{Im}\rho_{12} = -\mathrm{Im}\frac{\rho_{0+}+\rho_{0-}}{\sqrt{2}} > 0 . \quad (7)$$

Solving Eqs. (5) in steady state and then substituting $\rho_{0+}+\rho_{0-}$ into Eq. (7), we get

$$G = \frac{g_p}{2}\frac{r_1+\lambda_2}{g_c^2+(r_1+r_2+\lambda_1)(r_1+\lambda_2)}(T_1+T_2) , \quad (8)$$

where T_1 ($=2\rho_{00}-\rho_{++}-\rho_{--}$) comes from atomic populations and T_2 from the coherence terms (ρ_{+-} and ρ_{-+}), respectively, and are given by

$$\begin{aligned}
T_1 &= \frac{2(\lambda_1-r_1)g_c^2+(r_2+\lambda_1+\lambda_2)[-r_1(r_2+\lambda_2)+2\lambda_1\lambda_2]}{W} , \\
T_2 &= \frac{[2g_c^2-(r_2+\lambda_1+\lambda_2)(r_1+\lambda_2)]r_1(\lambda_2-r_2)}{W(r_1+\lambda_2)} .
\end{aligned} \quad (9)$$

The gain condition is then $T_1+T_2>0$.

From Eqs. (3) and (9), we obtain the necessary and sufficient conditions for lasing without inversion as follows:

$$\begin{aligned}
&\left[1-\frac{(\lambda_2-r_2)(r_2+\lambda_1+\lambda_2)}{g_c^2+\lambda_2(r_2+\lambda_1+\lambda_2)} \right] r_1 > \lambda_1 \\
&\qquad > \left[1-\frac{g_c^2(\lambda_2-r_2)}{(r_1+\lambda_2)[g_c^2+\lambda_2(r_2+\lambda_1+\lambda_2)]} \right] r_1 , \\
&\lambda_2 > r_2 , \\
&g_c^2 > (r_1+\lambda_2)(r_2+\lambda_1+\lambda_2) ,
\end{aligned} \quad (10)$$

where the two inequalities on λ_2 and g_c are for the compatibility of noninversion and gain conditions. (Here we note that direct optical pumping from level $|3\rangle$ to level $|2\rangle$ can not achieve $\lambda_2>r_2$ since it is bidirectional. This condition may be realized in practice, for example, by using a incoherent light to couple level $|3\rangle$ with an auxiliary upper level from which atoms quickly decay to the $2\rangle$.) If the first inequality on λ_1 is fulfilled, atomic populations are not inverted. If the second is fulfilled, the probe field may have a gain. All conditions in Eqs. (10) can be satisfied simultaneously; this point becomes more obvious if the applied EM field is strong. In this case ($g_c \gg r_1, r_2, \lambda_1, \lambda_2$), Eqs. (11) reduces to the simple form

$$\begin{aligned}
&r_1 > \lambda_1 > \left[1+\frac{r_2-\lambda_2}{r_1+\lambda_2} \right] r_1 , \\
&\lambda_2 > r_2 .
\end{aligned} \quad (11)$$

We then make some comments on this noninversion laser system. We find that a lower threshold exists for the applied EM field. Previously, Narducci *et al.* [13] and Fearn *et al.* [14] demonstrated the existence of a similar lower threshold in their energy-level configurations from numerical calculations. Here we are able to give an analytical expression for this threshold. As an estimate, we set $\lambda_1 \sim 0$ and $\lambda_2 \sim r_2$ and thus obtain

$$g_c^2 > 2r_2(r_1+r_2) . \quad (12)$$

Another important feature is the fact that the proper choices of external conditions are sufficient for the system to lase without inversion, without the requirement for a certain condition on atomic decay rates, which is essential for the Λ type of closed lifetime-broadened system [15,16]. This advantage provides a greater possibility in practical realizations of lasing without inversion.

The third feature is the enhancement of gain by the coherence term. The two inequalities on λ_2 and g_c^2 in Eqs. (10) ensure a positive contribution of the coherence term (T_2) to gain. In fact, we may write the gain condition as

$$2\rho_{00}-\rho_{++}-\rho_{--}+T_2>0 . \quad (13)$$

In the absence of population inversion ($\rho_{11}<\rho_{22}$ and $\rho_{11}<\rho_{33}$, which leads to $2\rho_{00}-\rho_{++}-\rho_{--}<0$ since $\rho_{11}=\rho_{00}$ and $\rho_{22}+\rho_{33}=\rho_{++}+\rho_{--}$), the gain condition requires

$$T_2 > 0 \ , \tag{14}$$

which tells us that the coherence term must enhance the gain. Apparently, if the coherence term (T_2) is ignored, $G>0$ can not be achieved for noninverted population. Therefore, it is the coherence term in dressed states that leads to the feasibility of lasing action in the absence of population inversion.

We note that previously Narducci *et al.* [17] and Whitley and Stroud [18] have analyzed the gain in the present atomic configuration except that there is no incoherent pumping. Our considerations show that with incoherent pumping included, a gain may exist in the absence of population inversion.

In summary, we have suggested a laser system which may operate in steady state in the absence of population inversion. The realization of lasing without inversion requires that the Rabi frequency of applied em field exceeds a lower threshold. The proper choices of external conditions (λ_1, λ_2, and g_c) may lead to the appearance of lasing without inversion, without a special requirement for atomic decay rates. Furthermore, it is found that the coherence term in dressed states must enhance the gain; this result is true beyond the specific approximations, e.g., the strong em field approximation.

[1] S. E. Harris, Phys. Rev. Lett. **62**, 1033 (1989); A. Imamoğlu, Phys. Rev. A **40**, 2835 (1989).

[2] M. O. Scully, S. Y. Zhu, and A. Gavrielides, Phys. Rev. Lett. **62**, 2813 (1989); E. E. Fill, M. O. Scully, and S. Y. Zhu, Opt. Commun. **77**, 36 (1990).

[3] B. R. Mollow, Phys. Rev. A **5**, 1522 (1972).

[4] V. C. Arlhipkin and Yu. I. Heller, Phys. Lett. **98**A, 12 (1983).

[5] A. Lyras, X. Tang, P. Lambropoulos, and J. Zhang, Phys. Rev. A **40**, 4131 (1989).

[6] S. E. Harris and J. J. Macklin, Phys. Rev. A **40**, 4135 (1989); A. Imamoğlu and S. E. Harris, Opt. Lett. **14**, 1344 (1989).

[7] N. Lu, Opt. Commun. **73**, 419 (1989).

[8] G. S. Agarwal, S. Ravi, and J. Cooper, Phys. Rev. A **41**, 4721 (1990); **41**, 4727 (1990).

[9] O. A. Kocharovskaya and P. Mandel, Phys. Rev. A **42**, 523 (1990); Opt. Commun. **84**, 179 (1991).

[10] S. Basile and P. Lambropoulou, Opt. Commun. **78**, 163 (1990).

[11] G. S. Agarwal, Phys. Rev. A **42**, 686 (1990); Opt. Commun. **80**, 37 (1990).

[12] Zhen-Fei Luo and Zhi-Zhan Xu, Phys. Lett. A **169**, 389 (1992); Phys. Rev. A **45**, 8292 (1992).

[13] L. M. Narducci *et al.*, Opt. Commun. **81**, 379 (1991).

[14] H. Fearn *et al.*, Opt. Commun. **87**, 323 (1992).

[15] A. Imamoğlu, J. E. Field, and S. E. Harris, Phys. Rev. Lett. **66**, 1154 (1991).

[16] G. S. Agarwal, Phys. Rev. A **44**, R28 (1991).

[17] L. M. Narducci, M. O. Scully, G.-L. Oppo. P. Ru, and J. R. Tredicce, Phys. Rev. A **42**, 1630 (1990).

[18] R. M. Whitley and C. R. Stroud, Jr., Phys. Rev. A **14**, 1498 (1976).