Laser-assisted nuclear γ excitation by the inverse electronic-bridge process

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A method to generate low-energy and high-multipolarity nuclear transitions through a laser-assisted, resonant, inverse electronic-bridge process is presented. The cross section of the suggested nuclear-excitation process is calculated in a simple model. The yield is compared with the yields of ordinary γ-ray absorption and Coulomb excitation. The excitation of the ^{235m}U isomeric state of energy 73.5 eV by an $E3$ transition is traced numerically. The excitation of an electron from the O_4 ($5d_{3/2}$) electronic shell of binding energy 105 eV can be followed by a P_3 ($6p_{3/2}$)$\rightarrow O_4$ transition, which can be tuned by the laser to resonance with the nuclear transition. In case of resonance, the electronic excitation energy is transferred to the nucleus with a high efficiency.

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I. INTRODUCTION

The effect of optical laser fields on atomic nuclei has been widely investigated but no observable increment in the nuclear β [1] and γ [2] decay rates caused by intense optical lasers was predicted. It seems to be a good rule of thumb that a measurable effect of optical lasers on decay rates can only be expected in those nuclear processes that have a characteristic energy that is comparable to the laser photon energy [3]. It was found, e.g., that laser-assisted internal conversion may be a candidate for nuclear processes which can be enhanced by optical lasers [4]. Similarly, it was quite recently shown that the electronic-bridge process, which was observed a few years ago [5], can also be influenced by intense laser beams if certain resonance conditions are met [6]. Our work was partly motivated by other recent papers [7] in which the possibility of excitation of nuclei by interaction with atomic electrons that are forced to oscillate by intense optical lasers was theoretically studied.

Now it is known that the electronic cloud is easy to deform by an external electromagnetic, e.g., x-ray field. The electronic cloud that loses its spherical symmetry will generally produce an electromagnetic field of higher multipolarity at the nucleus. On the other hand, studies concerning the internal-conversion process indicate that the multipole interaction of the electronic cloud and the nucleus can play an essential role in nuclear deexcitation. Furthermore, earlier considerations showed that the nuclear deexcitation process can be significantly enhanced if one of the electronic transitions is resonant to a nuclear transition, which can be achieved by applying a laser of appropriate photon energy and intensity [6].

In view of the above considerations we propose to study the possibility of the process of nuclear isomeric excitation with the help of the electronic cloud by simultaneously applying an x ray and a laser beam. The underlying idea is that the x-ray radiation can directly excite an inner electron in a dipole transition, producing a vacant state in the corresponding shell of energy E_0. If another electron of energy $E_1 > E_0$ and of angular momentum appropriately different from that of the vacant state fills the vacancy, then during the transition a strong electromagnetic field of multipole character appears at the nucleus. If the energy of the electronic transition $E_1 - E_0$ is close to the energy of the nuclear transition $E_b - E_a$ [see Fig. 1(a)] then the resonant excitation of the nucleus can be effected by the addition (or subtraction) of the energy of a number of laser photons (the addition of one laser photon

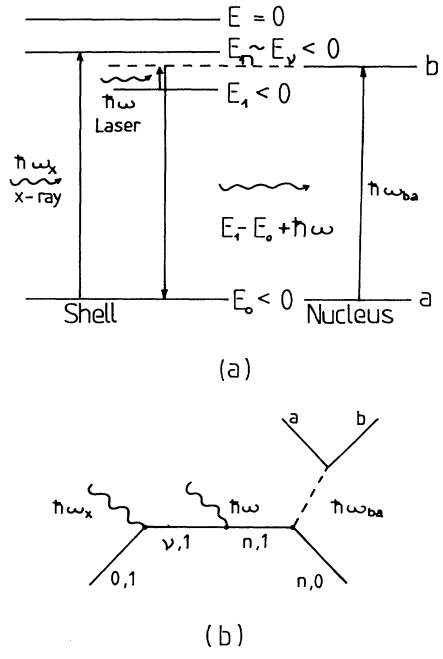

FIG. 1. Level scheme (a) and diagram (b) of the resonant laser-assisted inverse electronic-bridge process. E_0, E_1, E_ν, and E_n are electronic energies, $\hbar\omega_x$, $\hbar\omega$, and $\hbar\omega_{ba}$ are the x-ray, laser-photon, and γ-ray transition energies. The wavy lines represent x-ray and laser photons, the solid lines stand for electron (at the bottom) or nuclear (at the top) states, and the dashed line indicates the Coulomb interaction between the electron cloud and the nucleus.

is discussed here in detail). The absorption of the energy by the shell in the first step is more effective than the direct absorption by the nucleus since the electronic transition, unlike the multipole nuclear transition, is a dipole one. At the same time the width of the excited electronic state is much larger as compared to the nuclear isomeric width. In the second step of the process the input energy is converted into the energy of a strong electromagnetic field of higher multipolarity. Finally, the application of the laser field makes it possible to tune to resonance, fulfilling the requirement of energy conservation in this step also. So we study the effect of the laser field on the so-called inverse electronic-bridge mechanism in the case of two resonances. The schematic diagram of the process is given in Fig. 1(b).

Our paper is structured as follows. In Sec. II a simple model is presented, in which the nuclear γ excitation by the laser-assisted inverse electronic-bridge process (LAIEBP) can be treated. Here an approximate solution of bound electronic wave functions in the presence of a laser field is also given. The cross section of the LAIEBP is calculated in Sec. III. Section IV contains the relative yields of the LAIEBP compared with the yields of nuclear excitation by customary γ-ray absorption and by Coulomb excitation. The numerical results and conclusion can be found in Sec. V.

II. DESCRIPTION OF THE MODEL

The model is defined with the following Hamiltonian that consists of five terms:

$$H=H_0+H_1+H_2+H_3+H_N \ . \tag{1}$$

Terms H_0 to H_3 are

$$H_0=-\frac{\hbar^2}{2\kappa}\nabla^2-\frac{Ze^2}{r} \ , \tag{2}$$

$$H_1=e\mathbf{r}\cdot\mathbf{E}_L(t) \ , \tag{3}$$

$$H_2=\frac{Ze^2}{r}-\sum_{p=1}^{Z}\frac{e^2}{|\mathbf{r}-\mathbf{x}_p|} \ , \tag{4}$$

and

$$H_3=e\mathbf{r}\cdot\mathbf{E}_x(t) \ , \tag{5}$$

where the following notation is used: κ is the rest mass of the electron; e is the absolute value of the elementary charge; Z is the proton number of the nucleus; $\mathbf{x}_p$, $p=1,\ldots,Z$, denotes the proton coordinates; and r stands for the electron coordinate. The last term H_N in Eq. (1) is the Hamiltonian of the nucleus.

The laser radiation is treated as a classical field. Its electric-field strength is

$$\mathbf{E}_L(t)=E_0\hat{\mathbf{z}}\cos(\omega t) \ , \tag{6}$$

where E_0 and ω are the amplitude and the angular frequency of the laser radiation, which is polarized linearly parallel to direction z. Finally,

$$\mathbf{E}_x(t)=i\sum_{\omega_x,\epsilon}\left[\frac{2\pi\hbar\omega_x}{V}\right]^{1/2}\epsilon(ae^{-i\omega_x t}-a^{\dagger}e^{i\omega_x t}) \tag{7}$$

is the operator of the quantized x-ray field, where $\hbar\omega_x$ is the x-ray photon energy, ϵ determines the state of linear polarization, a and $a^{\dagger}$ are the photon annihilation and creation operators V is the volume of normalization.

So in the model H_1 describes the laser-electron interaction, H_2 is responsible for the electron-nucleus interaction, and H_3 gives the interaction of the electron with the quantized electromagnetic field of x rays.

The calculations are made in Göppert-Mayer gauge and in dipole approximation. The behavior of the many-electron system of the atom is approximated by one-electron direct-product states, but the effect of the other electrons is taken into account, applying effective nuclear charges $Z_{\text{eff}}(n)$ in the one-electron eigensolutions of H_0. We use a dressed-state perturbation calculation, taking H_2 and H_3 as perturbations, and calculate their effect on the laser plus Coulomb solutions. Considering, however, that there are no exact solutions of the Coulomb plus laser problem available, we use an approximate solution to describe the motion of the electrons under the joint influence of the Coulomb and laser fields.

If the electron in question is in an outer shell it seems to be a reasonable simplification that it feels the Coulomb field of a modified pointlike charge, and its motion is mainly determined by the Hamiltonian H_0+H_1. Thus the one-electron-state wave function Ψ can be approximately written for a given subshell of principal quantum number n as [8]

$$\Psi(nn_1n_2m,\varepsilon_n)=\Phi_{nn_1n_2m}\sum_{N=-\infty}^{\infty}J_N(\lambda_{n_1n_2})\times e^{-i(E_n+N\hbar\omega)t/\hbar} \ , \tag{8}$$

where $\Phi_{nn_1n_2m}$ is a hydrogen-type solution in parabolic coordinates and J_N denotes a Bessel function of the first kind. N is the number of absorbed or emitted laser photons and $E_n=\hbar\varepsilon_n$ is the energy of the state. The variable of J_N is

$$\lambda_{n_1n_2}=\frac{\frac{3}{2}n(n_1-n_2)E_0ea_B}{Z_{\text{eff}}\hbar\omega} \ , \tag{9}$$

where $n=n_1+n_2+|m|+1$, with m denoting the magnetic quantum number and n_1 and n_2 the parabolic quantum numbers [9], while a_B is the Bohr radius.

III. CROSS SECTION OF LASER-ASSISTED γ EXCITATION BY THE INVERSE ELECTRONIC-BRIDGE PROCESS

First we make some general considerations that help to simplify the evaluation of the S matrix related to the graph in Fig. 1(b). The mechanism to be discussed here can be traced up to all orders of the laser-electron interaction using dressed electronic states [Fig. 2(a)]. However, if we pick from all the possible processes the doubly resonant one depicted in Fig. 1(b) then significant

simplifications can be made in the calculation. Recalling that in the case of intermediately intense laser fields the dressed electronic Green function can be given in terms of the orders of laser-electron interaction [Fig. 2(b)], the graph in Fig. 1(b) can be approximately substituted by the one given in Fig. 2(c).

Initially the two-electron system is assumed to be in the state

$$|i(t)\rangle = U(n_1 j_1 l_1 m_1, \varepsilon_1)\varphi(n_0 l_0 m_0', \varepsilon_0)|a(t)\rangle |N_x \omega_x\rangle \ , \tag{10}$$

where $U(n_1 j_1 l_1 m_1, \varepsilon_1)$ is a one-electron state of the form

$$U(n_1 j_1 l_1 m_1, \varepsilon_1) = u_1(\mathbf{r})\exp(-i\varepsilon_1 t) \ , \tag{11}$$

where $u_1(\mathbf{r})$ is a hydrogen-type Pauli solution of quantum numbers n_1, j_1, l_1, m_1 and energy of $E_1 = \hbar\varepsilon_1$. The other one-electron state is

$$\varphi(n_0 l_0 m_0', \varepsilon_0) = \varphi_0(\mathbf{r}) e^{-i\varepsilon_0 t} \tag{12}$$

with $\varphi_0(\mathbf{r})$ being a nonrelativistic, spinless electron state of quantum numbers n_0, l_0, m_0' and energy $E_0 = \hbar\varepsilon_0$. It is assumed that due to shielding the effect of the laser on the innermost electron can be neglected. Thus the space-dependent part φ_0 of φ will have the form of a hydrogen-type eigenstate of a spinless electron.

$$|a(t)\rangle = |a\rangle e^{-i\varepsilon_a t} \tag{13}$$

is the initial (ground) nuclear state with $|a\rangle$ denoting its space-dependent part and $|N_x \omega_x\rangle$ is a photon-number state of angular frequency ω_x.

If the laser field does not induce transitions between electronic states of different principal quantum numbers then the S-matrix element of the process can be approximately written

$$S_{fi} = (i\hbar)^{-2} \int_{t_0}^{t} dt_2 \int_{t_0}^{t_2} dt_1 \sum_{\nu_1' \nu_2' \rho m_1'} \langle f(t_2)|H_2|k(t_2)\rangle \langle k(t_1)|H_3(t_1)|i(t_1)\rangle \ , \tag{14}$$

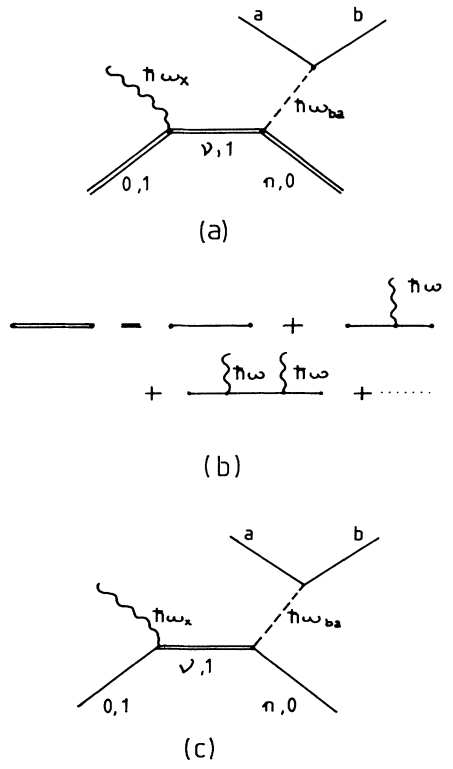

FIG. 2. Diagram necessary for the calculation. (a) Diagram describing the interaction with the laser. Double lines in the graph represent laser-dressed two-electronic states, which means (b) the usual expansion in terms of the increasing order of laser-electron interaction. (c) shows the graph actually calculated.

where the final state

$$|f(t)\rangle = U(n_0 j_0 l_0 m_0, \varepsilon_0)\Phi(n\nu_1\nu_2 m, \varepsilon_n) \times |b(t)\rangle |(N_x - 1)\omega_x\rangle \ , \tag{15}$$

with U of the form of Eq. (11) but with quantum numbers n_0, j_0, l_0, m_0 and of energy $E_0 = \hbar\varepsilon_0$. The second function

$$\Phi(n\nu_1\nu_2 m, \varepsilon_n) = \Phi_{n\nu_1\nu_2 m}\exp(-i\varepsilon_n t) \tag{16}$$

is a customary hydrogen-type solution in parabolic coordinates with quantum numbers n, ν_1, ν_2, m and energy $E_n = \hbar\varepsilon_n$. The intermediate state is

$$|k(t)\rangle = U(n_1 j_1 l_1 m_1', \varepsilon_1)\Psi(\nu\nu_1'\nu_2'\rho, \varepsilon_\nu) \times |a(t)\rangle |(N_x - 1)\omega_x\rangle \ , \tag{17}$$

with quantum numbers n_1, j_{1j}, l_1, m_1' of states given by Eq. (11) and with $\nu, \nu_1', \nu_2', \rho$ of solutions given by Eq. (8). Furthermore,

$$|b(t)\rangle = |b\rangle e^{-i\varepsilon_b t} \tag{18}$$

is the final (excited) nuclear state with $|b\rangle$ denoting its space-dependent part and $|(N_x - 1)\omega_x\rangle$ is a photon-number state of angular frequency ω_x. The initial and final nuclear states have energies $E_a = \hbar\varepsilon_a$ and $E_b = \hbar\varepsilon_b$, respectively. The space-dependent parts of the electronic states U will be denoted by $u_0(\mathbf{r})$ and $u_1(\mathbf{r})$ and their energies are $E_0 = \hbar\varepsilon_0$ and $E_1 = \hbar\varepsilon_1$. u_0 and u_1 describe inner states; therefore it is supposed that the laser has no effect on them so that they can be given by Eq. (11). Thus u_0 and u_1 are hydrogenic-type Pauli solutions in the usual spherical coordinates.

The choice of the initial, intermediate, and final two-electron states given by Eqs. (10), (15), and (17) can be explained as follows. We search for resonance-type processes so that the cross term of the S-matrix element can be neglected in Eq. (14). Therefore the x-ray absorption takes place in the first step, which is followed by the nuclear excitation. Thus the x-ray absorption selects the one-electron states φ of energy $\hbar\varepsilon_0$ and Ψ of energy $\hbar\varepsilon_\nu$ in the initial and in the intermediate states. State U of energy $\hbar\varepsilon_1$, which appears in the initial and intermediate states as well, is selected through the angular-momentum selection rule by the definite multipole order of the nuclear transition generated in the second step by the electronic transition $\hbar\varepsilon_1 \rightarrow \hbar\varepsilon_0$ through the Coulomb interaction. Naturally, due to the presence of the laser field the intermediate electronic state is dressed according to the solution given by Eq. (8), which makes possible the absorption of a laser photon by the intermediate electron before the nuclear excitation and the tuning of the second step to resonance also.

As in this simple model, only H_2 is used to describe the electron-nucleus interaction [10]; the γ-photon exchange between the nucleus and the electron cloud does not appear explicitly in the formalism.

Carrying out the time integrations in Eq. (14) the following result is obtained:

$$S_{fi}=(i\hbar)^{-2}\sum_{N,K} H_{2,01,ba}\,H_{3,n0}2\pi J_N(\lambda_{\nu_1\nu_2})J_K(\lambda_{\nu_1\nu_2}) \times (N_x)^{1/2}\frac{\delta(\omega_{ba}+[N-K]\omega+\omega_{n1}-\omega_x)}{\omega_x-\omega_{\nu 0}-N\omega+i\gamma_\nu/2}\ , \tag{19}$$

where $\omega_{ba}=\varepsilon_b-\varepsilon_a$, $\omega_{n1}=\varepsilon_n-\varepsilon_1$, $\omega_{\nu 0}=\varepsilon_\nu-\varepsilon_0$. In order to handle the resonance process in deducing Eq. (19), a complex energy $E_\nu=\hbar\varepsilon_\nu-i\hbar\gamma_\nu/2$ was introduced for the intermediate state, which is a customary *ad hoc* method used, e.g., in the calculation of nuclear resonance fluorescence cross sections [11]. Here γ_ν is the width of the excited electronic shell if a vacancy in the shell of energy E_0 is created. In the second step, H_2 makes change only in the U part of the electronic wave function, which leads to $n=\nu$, $\nu_1=\nu_1'$, $\nu_2=\nu_2'$, and $m=\rho$.

The following notation can also be introduced:

$$H_{3,n0}=\sum_{l=0}^{n-1}\langle lm\,|\mu_1\mu_2\rangle H_{3,nlm0}\ , \tag{20}$$

where $\langle lm\,|\mu_1\mu_2\rangle$ is a Clebsch-Gordan coefficient

$$\left\langle lm\left|\frac{n-1}{2}\mu_1\,\frac{n-1}{2}\mu_2\right\rangle\right.$$

with $\mu_1=(m+\nu_1-\nu_2)/2$ and $\mu_2=(m-\nu_1+\nu_2)/2$ [12]. Here

$$H_{3,nlm0}=\int e\Phi_{nlm}\,i(2\pi\hbar\omega_x/V)^{1/2}\boldsymbol{\epsilon}\cdot\mathbf{r}\varphi_0(\mathbf{r})d^3r\ , \tag{21}$$

where Φ_{nlm} is a hydrogen-type wave function in the usual spherical coordinates.

For the first factor in Eq. (19) we have [13]

$$H_{2,01,ba}=\frac{4\pi}{2L+1}\sum_{M=-L}^{L}Q_{LM}(a,b)I_{LM,01}\ , \tag{22}$$

where $Q_{LM}(a,b)$ is the matrix element of the multipole moment of order L,M between nuclear states $|a\rangle$ and $|b\rangle$ which, applying the Wigner-Eckart theorem [14], can be expressed in terms of its reduced matrix element $\langle b\|Q_L\|a\rangle$ as

$$Q_{LM}(a,b)=(-1)^{j_b-m_b}\langle b\|Q_L\|a\rangle\begin{pmatrix} j_b & L & j_a\\ -m_b & M & m_a\end{pmatrix}. \tag{23}$$

j_b,m_b,j_a,m_a denote the total angular momentum and its third-component quantum numbers of the final and initial nuclear states. L is determined by the nuclear transition. The second quantity in Eq. (22) is

$$I_{LM,01}=e\int u_0^\dagger(\mathbf{r})Y_{LM}u_1(\mathbf{r})r^{1-L}dr\,d\Omega\ . \tag{24}$$

Here Y_{LM} is a spherical harmonic and Ω denotes the solid angle. Separating the radial and the angular integrals in Eq. (24), the radial part yields the dimensionless quantity

$$\hat{I}^{n_1l_1}_{L,n_0l_0}=a_B^{L+1}\int R_{n_1l_1}(Z_{\rm eff}(n_1),r) \times R_{n_0l_0}(Z_{\rm eff}(n_0),r)r^{1-L}dr\ , \tag{25}$$

where $R_{n_1l_1},R_{n_0l_0}$ are the radial parts of $u_1(\mathbf{r})$ and $u_2(\mathbf{r})$.

The angular dependence of the functions $u(\mathbf{r})$ is described by the spinors

$$\mathbf{Y}^{(\pm)}_{jlm}=(2l+1)^{-1/2}\begin{pmatrix}(l\pm m+\frac{1}{2})^{1/2}Y_{l,m-\frac{1}{2}}\\ (l\mp m+\frac{1}{2})^{1/2}Y_{l,m+\frac{1}{2}}\end{pmatrix}, \tag{26}$$

where the upper and lower signs refer to states for which $j=l+\frac{1}{2}$ or $j=l-\frac{1}{2}$.

The angular integral in Eq. (24) will become

$$F^{LM}_{j_0l_0m_0\,j_1l_1m_1}=\left[\frac{4\pi}{2L+1}\right]^{1/2} \times\int\mathbf{Y}^{\dagger(\pm)}_{j_0l_0m_0}Y_{LM}\mathbf{Y}^{(\pm)}_{j_1l_1m_1}d\Omega\ , \tag{27}$$

which gives

$$F^{LM}_{j_0l_0m_0\,j_1l_1m_1}=\left[\{(l_1\pm m_1+\tfrac{1}{2})(l_0\pm m_0+\tfrac{1}{2})\}^{1/2}\begin{pmatrix} l_1 & l_0 & L\\ m_1-\frac{1}{2} & -m_0+\frac{1}{2} & M\end{pmatrix}(-1)^{m_0-1/2} +\{(l_1\mp m_1+\tfrac{1}{2})(l_0\mp m_0+\tfrac{1}{2})\}^{1/2}\begin{pmatrix} l_1 & l_0 & L\\ m_1+\frac{1}{2} & -m_0-\frac{1}{2} & M\end{pmatrix}(-1)^{m_0+1/2}\right]\begin{pmatrix} l_1 & l_0 & L\\ 0 & 0 & 0\end{pmatrix}. \tag{28}$$

In order to evaluate Eq. (21) we use the identity $\epsilon\cdot\mathbf{r}=(4\pi/3)r\sum_s Y^*_{1s}(\mathbf{r})Y_{1s}(\epsilon)$. The radial part of the integral leads to the formula

$$\hat{r}_{nl}^{n_0l_0}=a_B^{-1}\int R_{n_0l_0}(Z_{\rm eff}(n_0),r)R_{nl}(Z_{\rm eff}(n),r)r^3dr \tag{29}$$

and the angular integral results in a formula that contains, among others, combinations of $3j$ symbols.

To find the cross section $\sigma_{\rm LEB}$ of the LAIEBP the following manipulations must be done. The $|S_{fi}|^2$ must be calculated; furthermore, the average over the initial states and the sum over the final states have to be taken. The condition of orthogonality of the $3j$ symbols and the spherical harmonics [14] has to be utilized and the identity for the δ-functions

$$\delta(\omega_{ba}+\omega_{n1}+[N-K]\omega-\omega_x)\delta(\omega_{ba}+\omega_{n1}+[N'-K']\omega-\omega_x)=\delta(0)\delta_{K-N,K'-N'}\delta(\omega_{ba}+\omega_{n1}+[N-K]\omega-\omega_x)$$

has to be used. Since the electronic cloud finally remains in an excited state the energy ε_n will have a distribution function

$$\rho_{\varepsilon_n}=\frac{\gamma_n/(2\pi)}{(\varepsilon_n-\varepsilon_\nu)^2+\gamma_n^2/4}\ , \tag{30}$$

where γ_n ($<\gamma_\nu$) is the width of the deexcitation of the electronic cloud if a vacant state in the shell of energy $\hbar\varepsilon_1$ is present. Finally, dividing the transition probability per unit time of the LAIEBP by the x-ray flux $\Phi_x=cN_x/V$ we obtain for the leading term of the cross section in the channel N,K set by the resonances

$$\sigma_{\rm LEB}=\frac{16\pi^2|\langle b\|Q_L\|a\rangle|^2\alpha^2c\omega_x g_0g_1a_B^{-2L}\gamma_n(\hat{I}^{n_1l_1}_{L,n_0l_0})^2}{3(2L+1)^2(2j_a+1)(2j_1+1)\hbar(\Delta'^2+\gamma_\nu^2/4)(\Delta^2+\gamma_n^2/4)}F_{NK}\ . \tag{31}$$

Here α is the fine-structure constant; g_0,g_1 denote the number of the electrons in the shells of quantum numbers n_0,j_0,l_0 and n_1,j_1,l_1; and the quantity F_{NK} is introduced so that

$$F_{NK}=F_LS_{nNK}\ , \tag{32}$$

where

$$F_L=\sum_{m_1,m_0,M}(F^{LM}_{j_0l_0m_0j_1l_1m_1})^2 \tag{33}$$

and

$$S_{nNK}=\sum_{\substack{l=0\\ \mu_1,\mu_2}}^{n-1}|\langle lm|\mu_1\mu_2\rangle|^2|\hat{r}_{nl}^{n_0l_0}|^2\begin{bmatrix}l&1&l_0\\0&0&0\end{bmatrix}^2$$

$$\times J_N^2(\lambda\hat{\nu}_1\hat{\nu}_2)J_K^2(\lambda\hat{\nu}_1\hat{\nu}_2)\delta_{l,l_0\pm1}\ . \tag{34}$$

Since ω_x is fixed in the first step, it is the detuning

$$\Delta'=\omega_x-N\omega-\omega_{\nu0} \tag{35}$$

in the denominator of Eq. (31) that sets N ($N=0$ here) to meet the condition of resonance. The other factor in the denominator contains

$$\Delta=\varepsilon_n-\varepsilon_\nu=\omega_x-\omega_{ba}-(N-K)\omega-\omega_{\nu1}\ , \tag{36}$$

which restricts K ($K=1$ here) in the case of the second resonance.

Although the quantum numbers of the intermediate and the final one-electron states Ψ and Φ are the same, in consequence of their finite widths their energies $\hbar\varepsilon_\nu$ and $\hbar\varepsilon_n$ can be different, which is particularly advantageous in the process investigated. The order of magnitude of the product of the x-ray source flux spectral density $\Phi_\omega=d\Phi_x/d\omega_x$ and the width γ_ν of the intermediate electronic state is the quantity which determines the effectivity of the energy input. The nuclear width Γ of an isomeric state is much smaller than the electronic widths γ_ν and γ_n. All this means that without the electronic process a much smaller fraction of the x-ray flux would be effective in the direct nuclear excitation. It is the remaining uncertainty in the energy of the final, excited electronic state that makes possible the efficient transmission of the energy of the electronic excitation of the first step to nuclear excitation.

IV. RELATIVE YIELDS

In order to characterize the yield of nuclear excitation by LAIEBP relative to the yield of normal nuclear excitation by γ-ray absorption we introduce the quantity η_γ, which is defined as $\eta_\gamma=W_{fi}^{\rm LEB}/W_{fi}^{\rm abs}$, where $W_{fi}^{\rm LEB}$ and $W_{fi}^{\rm abs}$ are the transition probabilities per unit time of the LAIEBP and the spontaneous γ-ray absorption. η_γ can be expressed with the cross sections as

$$\eta_\gamma=\frac{\int\sigma_{\rm LEB}\Phi_\omega d\omega_x}{\int\sigma_{\rm abs}\Phi_\omega d\omega_x}\ . \tag{37}$$

Here $\Phi_\omega=d\Phi_x/d\omega_x$ and the cross section $\sigma_{\rm abs}$ of nuclear γ absorption [10] is

$$\sigma_{\rm abs}=(\pi/2)k_{ba}^{2L-1}\frac{|\langle b\|Q_L\|a\rangle|^2}{2j_a+1}\frac{8\pi(L+1)}{L[(2L+1)!!]^2}$$

$$\times\hbar^{-1}\frac{\Gamma}{(\omega_{ba}-\omega_x)^2+\Gamma^2/4}\ , \tag{38}$$

where Γ is the width of the nuclear state $|b\rangle$ and $k_{ba}=\omega_{ba}/c$. From this, introducing $\omega_{x0}=\varepsilon_\nu-\varepsilon_0+N\omega$ one has

$$\eta_\gamma=\eta_{\gamma 0}F_{NK} , \tag{39}$$

where F_{NK} contains the laser intensity dependence (see Fig. 3) as given by Eqs. (32)–(34) and

$$\eta_{\gamma 0}=\frac{32\pi L[(2L-1)!!]^2}{3(L+1)(2j_1+1)}(\hat{I}^{n_1 l_1}_{L,n_0 l_0})^2\alpha^2 g_0 g_1 \times\frac{\omega_{x0}\omega_{ba}}{(\gamma_n+\gamma_\nu)\gamma_\nu(a_B k_{ba})^{2L}} . \tag{40}$$

The order of magnitude of $\eta_{\gamma 0}$ is basically determined by the last fraction in Eq. (40). Since $\omega_{x0}\sim\omega_{ba}$ and $\gamma_\nu>\gamma_n$, it can be estimated as $(a_B k_{ba})^{-2L}(\omega_{ba}/\gamma_\nu)^2$. The first factor in this estimation arises because of the reduction of multipolarity in the energy input step, i.e., the direct γ excitation has a multipolarity L in contrast with the dipole excitation of the electronic shell. This can cause an increase if $a_B k_{ba}<1$, i.e., for nuclear transitions which have nuclear transition energy $E_b-E_a<3.7$ keV. The reason for the second factor is the possibility of the energy difference $\varepsilon_n-\varepsilon_\nu$, i.e., the fact that the final electronic state can have an energy uncertainty extends drastically the effectively usable part of the x-ray flux.

The other process to the yield of which it is worth comparing the one investigated by us is the Coulomb excitation. In this case the relative yield η_C is defined as

$$\eta_C=\frac{\int\sigma_{\mathrm{LEB}}\Phi_\omega d\omega_x}{\sigma_{EL}\Phi_{Z_1}} , \tag{41}$$

where Φ_{Z_1} is the flux of projectiles of charge $Z_1 e$ and

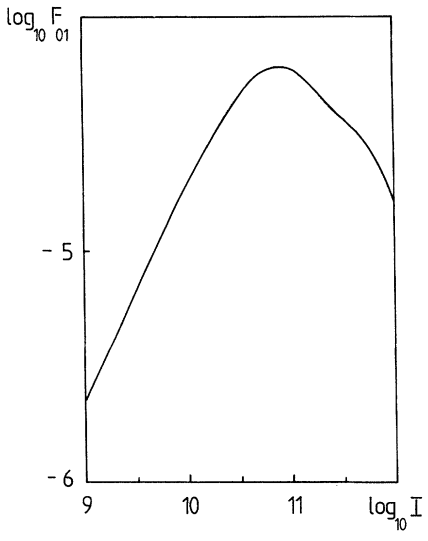

FIG. 3. The laser-intensity- (I) dependent factor (F_{01}) [given by Eq. (32)] of the γ-excitation cross section of the resonant, laser-assisted inverse electronic-bridge process. I is measured in W/cm^2. The index 01 refers to the number of laser photons absorbed in the first and second steps.

σ_{EL} is the Coulomb-excitation cross section of an EL transition [15]

$$\sigma_{EL}=\left[\frac{Z_1 e}{\hbar v_i}\right]^2 A^{-2L+2}\frac{|\langle b\|Q_L\|a\rangle|^2}{(2j_a+1)}f_{EL}(u_i,u_f) , \tag{42}$$

with the notation

$$A=\frac{Z_1 Z_2 e^2}{M_0 v_1 v_f},\quad u_i=\frac{Z_1 Z_2 e^2}{\hbar v_i},\quad u_f=\frac{Z_1 Z_2 e^2}{\hbar v_f} . \tag{43}$$

In these formulas $Z_2 e$ is the charge of the target nucleus, v_i and v_f are the initial and final velocities of the projectile of rest mass M_0. The quantity $f_{EL}(u_i,u_f)$ has the following form in Born approximation

$$f_{EL}(u_i,u_f)=\frac{16\pi^2}{(L-1)[(2L+1)!!]^2}\times\{(u_i+u_f)^{2(L-1)}-(u_f-u_i)^{2(L-1)}\} . \tag{44}$$

If we suppose that the velocity of the projectile after collision is much smaller than before it, then we obtain for the Coulomb-excitation cross section

$$\alpha_{EL}=\frac{32\pi^2}{(L-1)[(2L+1)!!]^2}\left[\frac{Z_1 e}{\hbar c}\right]^2\times(c/v_i)^2 q_i^{2L-2}\frac{|\langle b\|Q_L\|a\rangle|^2}{(2j_a+1)} , \tag{45}$$

which results in a relative yield

$$\eta_C=\frac{4\pi(L-1)[(2L-1)!!]^2}{3(2j_1+1)Z_1^2 a_B^2}(\hat{I}^{n_1 l_1}_{L,n_0 l_0})^2\times\alpha g_0 g_1 F_{NK}\frac{\omega_{x0}v_i^2(\Phi_\omega/\Phi_{Z_1})}{(\gamma_n+\gamma_\nu)\gamma_\nu(a_B q_1)^{2L-2}} . \tag{46}$$

Here $q_i=M_0 v_i/\hbar$ is the initial wave number of the projectile.

V. NUMERICAL RESULTS AND CONCLUSION

For numerical calculations the ^{235m}U isomer, which has a γ-photon energy 73.5 eV and multipolarity $L=3$, seems to be an appropriate sample material that may be accessible for experimental investigations. Its electrons have 105 and 32.3 eV binding energies in the O_4 ($5d_{3/2}$) and P_3 ($6p_{3/2}$) shells [16,17]. An intermediate state of binding energy 2.14 eV can be excited from the O_4 shell by the absorption of a soft-x-ray photon of energy 103 eV if no additional laser photon is absorbed ($N=0$ case). The effective charges and the principal and angular momentum quantum numbers of the electronic states used are $Z_{\mathrm{eff}}=13.89$, $n_0=5$, $l_0=2$ ($j_0=\frac{3}{2}$); $Z_{\mathrm{eff}}=3.173$, $n=8$, $l=3$ or 1; and $Z_{\mathrm{eff}}=9.245$, $n_1=6$, $l_1=1$ ($j_1=\frac{3}{2}$). The energy misfit is 1 eV. It means that the energy of the $6p_{3/2}\rightarrow 5d_{3/2}$ transition plus about 1 eV equals the energy 73.5 eV of the nuclear transition ($\hbar\omega\sim 1$ eV, $K=1$).

The other quantities we need are $\hat{r}^{83}_{52}=0.411$ $\hat{r}^{81}_{52}=-0.199$, and $\hat{I}^{61}_{3,52}=5.715$. The widths $\hbar\gamma_n$ and

$\hbar\gamma_v$ are estimated to be not larger than 0.1 eV [18]. For such values of natural width the power broadening can be neglected in the intensity range below 10^{13} W/cm^2 [19].

Using the above data in Eq. (40) one obtains $\eta_{\gamma 0} \gtrsim 5\times 10^{16}$. Since at laser intensity $I=10^{11}$ W/cm^2 the quantity $F_{01}\sim 6\times 10^{-5}$ (see Fig. 3) the relative yield η_γ can reach, in this case, a value as high as 3×10^{12}.

The order of magnitude of the ratio η_γ can be checked, comparing its value with that of the total internal-conversion coefficient α_T [20]. α_T is also a ratio which probably has the same order of magnitude since in the internal-conversion process the nucleus deexcites in consequence of the electron-nucleus interaction similarly to the process discussed here. In Ref. [21] $\alpha_T=2\times 10^{19}$ was obtained for the $E3$ transition of the ^{235m}U of energy 73.5 eV, which shows that our values ($\eta_{\gamma 0}=5\times 10^{16}$ and $\eta_\gamma\sim 3\times 10^{12}$) do not seem to be unacceptably large.

From the results obtained in our simplified model one can draw the following conclusion. For nuclear excitation, the combined application of an x-ray source and an intense laser beam can be more effective than the direct γ-excitation in cases when the following conditions hold. The multipolarity of the transition must be higher than dipole and its energy must be less than 3.7 keV. The electron cloud must have two levels with energy difference close to the nuclear excitation energy, which makes possible the tuning of the electron transition to resonance by applying an intense laser of appropriate photon energy and intensity. In cases where these requirements can be met, the laser-assisted inverse electronic-bridge mechanism seems to be a workable mechanism of nuclear excitation.

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$$\frac{1}{|\mathbf{r}-\mathbf{x}_p|}=\sum_{L=0}^{\infty}\sum_{M=-L}^{L}\frac{4\pi}{2L+1}\sum_{p=1}^{Z}\frac{x_p^L}{r^{L+1}}Y_{LM}(\vartheta,\varphi)Y_{LM}^*(\vartheta_p,\varphi_p)$$

can be used.

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$$\alpha_n=\frac{2\omega}{\hbar}\sum_j\frac{|\langle\varphi_n|-e\mathbf{r}\cdot\mathbf{e}_0|\varphi_j\rangle|^2}{\omega_{nj}^2-\omega^2},$$

which is measured, similarly to the atomic polarizability in units of 10^{-24} cm^3 [G. Ferrante, C. Leone, and F. Trombetta, J. Phys. B **15**, L475 (1982)]. Though an experimentally observed value of α_n is not yet available, its order of magnitude can be estimated to have a value of 10^{-24} cm^3.

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