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Effect of a partial Siberian snake on an "rf-induced" depolarizing resonance

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A recent experiment studied the effect of an rf solenoid magnet and a partial Siberian snake on a 120-MeV polarized proton beam. We measured the frequencies of the "rf-induced" depolarizing resonance for different values of the snake strength; this frequency measurement determined the spin tune ν_{sp}, which is the number of spin precessions in one turn around the ring. A 4% snake increased the frequency of the rf-induced depolarizing resonance by the predicted 11 kHz and thus shifted the spin tune by the predicted $\Delta\nu_{sp} = 0.006\,88 \pm 0.000\,04$; as expected, the 4% snake also tilted the stable spin direction by more than 38° from the vertical.

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Accelerating polarized proton beams to tens of GeV at the zero-gradient synchrotron [1] and the alternating-gradient synchrotron [2] required correction dipoles and fast pulsed quadrupoles to separately overcome each depolarizing resonance. This time-consuming and difficult individual resonance correction technique would be impractical at a TeV accelerator with thousands of depolarizing resonances [3]. The Siberian snake technique [4] rotates the spin of each proton by 180° on each turn around an accelerator ring; this simple and elegant technique should correct all depolarizing resonances with essentially no tuning.

Our earlier experiments at the Indiana University Cyclotron Facility (IUCF) Cooler Ring showed that a Siberian snake could overcome both imperfection and intrinsic depolarizing resonances with essentially no loss of polarization [5, 6]. An imperfection resonance occurs when the protons' spin precession frequency is synchronized with the frequency of passing through the ring's imperfection horizontal fields. An intrinsic depolarizing resonance is caused by the vertical betatron oscillations that periodically move the protons into the horizontal fields of the ring's focusing quadrupoles. These resonances can depolarize a spin-polarized beam whenever the kinetic energy T satisfies the condition

$$G\gamma = n + m\nu_y, \tag{1}$$

where n and m are integers, ν_y is the vertical betatron tune, $\gamma = 1 + T/(938.272\text{ MeV})$, and $G = 1.792\,847$ is the anomalous magnetic moment of the proton. The imperfection resonances occur when $m = 0$, while the first-order intrinsic resonances occur when $m = \pm 1$.

A high-power radio-frequency (rf) solenoid magnet was built and installed in the IUCF Cooler Ring; we used the solenoid's oscillating longitudinal magnetic field tc create an "rf-induced" depolarizing resonance at a frequency f_r, which is related to the spin tune ν_{sp} by

$$f_r = f_c(k \pm \nu_{sp}), \tag{2}$$

where k is an integer and f_c is the protons' circulation frequency in the ring. The spin tune ν_{sp} is equal to $G\gamma$ when no snake is present. A weak type-3 Siberian snake, which rotates the spin about the vertical axis, inadvertently exists in the IUCF Cooler Ring; this type-3 snake shifted the spin tune by about $\delta_3 \simeq 0.0036$ in an earlier experiment [7].

A partial Siberian snake should be able to overcome the relatively weak imperfection depolarizing resonances in accelerators of about 10 GeV such as the Fermilab Booster [8]. With a partial Siberian snake of strength s, the spin tune should obey the equation [9]

$$\cos(\pi\nu_{sp}) = \cos(\pi[G\gamma + \delta_3])\cos\left(\frac{\pi s}{2}\right), \tag{3}$$

where $s = 1$ corresponds to a 100% snake, which rotates the spin by 180° around the longitudinal direction. We measured the rf resonant frequency and thus the spin

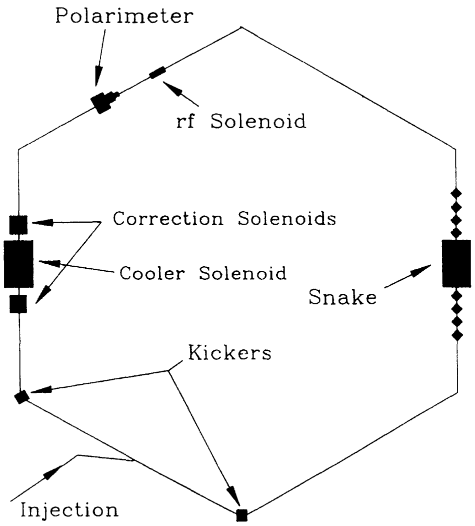

FIG. 1. Location of the Siberian snake and the related hardware in the IUCF Cooler Ring.

tune for partial snake strengths between 0% and 4%.

The polarimeter and the operation of the IUCF Cooler Ring with polarized protons were discussed earlier [5–7]; we will only describe the new partial Siberian snake solenoid and the new rf solenoid used in this experiment. Figure 1 shows these solenoid magnets placed in the IUCF Cooler Ring along with the other relevant hardware.

Our usual superconducting solenoid [5] was much stronger than necessary for the partial Siberian snake; therefore, we instead used a 1200-turn warm solenoid. With this weaker solenoid we did not need the eight quadrupoles that normally make our strong solenoid snake optically transparent. The snake strength s for a solenoid magnet of NI ampere turns is given by

$$s = \frac{\mu_0(1+G)}{10.479p} NI, \tag{4}$$

where p is the proton momentum in GeV/c and $\mu_0 = 4\pi \times 10^{-7}$ T m/A. The current of the snake power supply was stable to better than 0.01%; this stabilized each resonance frequency to better than $\pm$ 2 Hz.

The new rf solenoid magnet was a 21-turn copper coil wrapped around a ceramic vacuum chamber; the coil was part of a resonant circuit with an adjustable high-voltage capacitor and a 10-kW rf power supply. The frequency range was 0.8 to 2.8 MHz and the optimal Q was about 400. The solenoid inductance was about 18 μH and the maximum voltage across the solenoid was 25 kV peak to peak. The maximum oscillating longitudinal magnetic field integral was $\int Bdl = 0.0018$ T m corresponding to a resonance strength of about $\epsilon = 5 \times 10^{-4}$. A feedback circuit, which monitored the rf magnetic field, used the adjustable capacitor to automatically tune the phase of the resonant circuit; a related feedback system stabilized the rf magnetic field to better than 1%.

Each proton's spin precessed around some stable spin direction, which was vertical with no snake and horizontal with a 100% snake. With a partial snake the stable spin direction at injection made an angle α with the vertical injected polarization direction; therefore, the measured vertical polarization was proportional to $\cos^2\alpha$. Near $G\gamma = 2$, the angle α approached 90°.

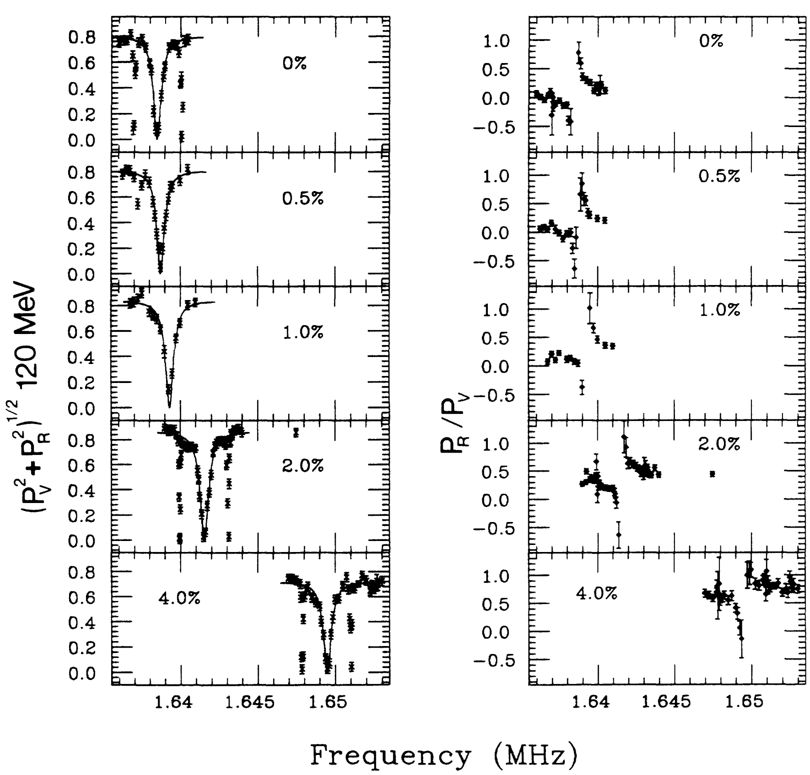

FIG. 2. The measured total transverse polarization (left) and the measured polarization ratio P_r/P_v (right) are plotted against the frequency of the rf solenoid magnet for each indicated partial snake strength at 120 MeV. The curves are the fits to Eq. (5), which determine the resonance frequency at each snake strength. The rf solenoid voltage was 22 kV.

Choosing $T = 120.02 \pm 0.03$ MeV, which corresponds to $G\gamma = 2.0222$, was a compromise between maximizing the spin tune shift by staying near $G\gamma = 2$ and minimizing the depolarization at injection due to the snake-induced angle α.

We first adjusted the cooling correction solenoids to compensate for the large imperfection field caused by the main cooling solenoid, which confines the electrons that provide the cooling; with vertically polarized injected protons and with the snake off, we varied the current of the correction solenoids to maximize the measured vertical polarization. Because 120 MeV is fairly far from the $G\gamma = 2$ imperfection resonance, the vertical polarization curve was rather flat. We set the correction solenoids near the peak of this curve to minimize the strength of the $G\gamma = 2$ resonance.

We next determined the frequency of the rf-induced depolarizing resonance by measuring the vertical and radial polarization components P_v and P_r versus the rf frequency for snake strengths of about 0%, 0.5%, 1%, 2%, and 4%. The data are shown in Fig. 2, where the total transverse polarization $P_t = \sqrt{P_v^2 + P_r^2}$ and the ratio P_r/P_v are plotted against the rf frequency. The frequency of the rf-induced resonance f_r is taken to be the measured center of the main dip in each P_t curve. The exact f_r value was obtained by minimizing the χ^2 value in each plotted fit to the resonance equation

$$\frac{P_i}{P_0} = \frac{(f_i - f_r)^2}{(f_i - f_r)^2 + \Gamma^2}, \tag{5}$$

where P_i is the measured P_t value at each measured frequency f_i, while P_0 and Γ are fits to the initial polarization and the resonance width, respectively. These measured resonance frequencies and widths at each snake strength and their total errors are listed in Table I along with each corresponding $\Delta\nu_{sp}$, which is the shift in the spin tune from its value at $s = 0$. Notice that f_r and $\Delta\nu_{sp}$ both increase as the snake strength increases; the width appears to be independent of s.

The measured IUCF Cooler Ring circulation frequency f_c was 1 597 952 Hz at 120.02 MeV and $f_r^0 = 1\,638\,490 \pm 30$ Hz was the measured resonant frequency with $s = 0$. Using Eqs. (2) and (3) these two frequencies indicate that the contribution of the type-3 snake was now $\delta_3 \simeq 0.0032$; the small change from the earlier value [7] of 0.0036 indicates that the cooling section magnets were now probably

TABLE I. The measured frequency f_r and width Γ of the "rf-induced" depolarizing resonance and the corresponding spin tune shift $\Delta\nu_{sp}$ are listed for each partial snake strength s.

s	f_r (kHz)	Γ (kHz)	$\Delta\nu_{sp}$ (units of 10^{-4})
0	1638.49 ± 0.03	0.61 ± 0.06	
0.5%	1638.75 ± 0.03	0.59 ± 0.07	1.6 ± 0.4
1.0%	1639.32 ± 0.04	0.62 ± 0.09	5.2 ± 0.5
2.0%	1641.55 ± 0.03	0.65 ± 0.06	19.1 ± 0.4
4.0%	1649.49 ± 0.03	0.62 ± 0.06	68.8 ± 0.4

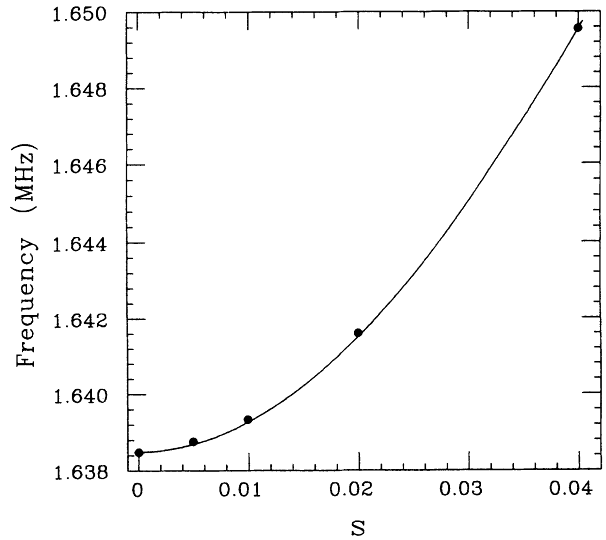

FIG. 3. The measured frequency of the "rf-induced" depolarizing resonance f_r is plotted against the snake strength s. The curve is the resonance frequency calculated from Eq. (6).

tuned to slightly different values. One can also use Eqs. (2) and (3) to calculate the resonance frequency f_r with a partial snake of strength s:

$$\cos(\pi f_r/f_c) = \cos(\pi f_r^0/f_c)\cos(\pi s/2). \tag{6}$$

The measured resonance frequency is plotted against the snake strength in Fig. 3 along with Eq. (6), which is in good agreement with the data. The growth in f_r is approximately quadratic in s; note that Eq. (6) can be

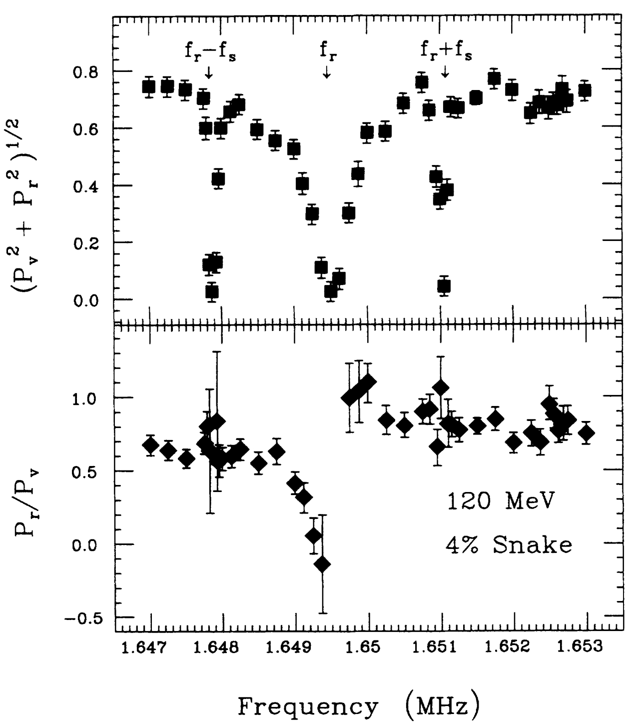

FIG. 4. The total transverse polarization $P_t = \sqrt{P_v^2 + P_r^2}$ and the ratio P_r/P_v at 120 MeV are plotted against the rf solenoid frequency for a 4% snake with $V_{rf} = 22$ kV

expanded in quadratic form for small s.

The acceleration cavity in the IUCF Cooler Ring caused synchrotron oscillations of frequency f_s, which induced synchrotron depolarizing resonances [6]; these should appear as sideband dips in the polarization curves at frequencies near

$$f_\pm = f_r \pm f_s. \quad (7)$$

These synchrotron resonances can be seen in Fig. 2 in the 0%, 2%, and 4% snake data, where we varied the rf solenoid frequency in fine steps. An expanded plot of the measured polarization with a 4% snake is shown in Fig. 4. The synchrotron resonances can now be seen very clearly; the measured frequencies of these dips agree with the indicated values given by Eq. (7). The synchrotron frequency was measured by a low bandwidth wall gap monitor to be $f_s = 1.62 \pm 0.04$ kHz.

At 120 MeV the radial polarization was substantial for the 4% snake; therefore, the stable spin direction was significantly tilted away from the vertical. Only the polarization component along the stable spin direction survived injection; the ratio of the surviving radial to vertical polarization components measured at the position of our polarimeter is approximately given by [5]

$$P_r/P_v = \tan\left(\frac{\pi s}{2}\right) \frac{\sin\left(\frac{\pi G\gamma}{3}\right)}{\sin(\pi G\gamma)}. \quad (8)$$

This gives $P_r/P_v = 0.77$ for a 4% snake at 120 MeV; this calculation is consistent with the average measured ratio shown in Fig. 4. However, notice the low-frequency to high-frequency asymmetry in the P_r/P_v ratio. This asymmetry may be related to an asymmetry found in the radial polarization curves when the cooling correction solenoid was varied [5–7]. Note that the average ratio $P_r/P_v = 0.77$ for a 4% snake corresponds to the stable spin direction being tilted by an angle of about 38° from the vertical in the transverse plane at the position of the polarimeter; the longitudinal component was unmeasurable.

We will next use this rf solenoid to study the properties of overlapping depolarizing resonances by varying the rf frequency to move the rf-induced resonance until it overlaps with the $G\gamma = 2$ imperfection resonance. Overlapping depolarizing resonances may be a significant problem at TeV proton accelerators such as SSC (Superconducting Super Collider), LHC (Large Hadron Collider), UNK (Serpukhov's Collider), HERA (Hadron Electron Ring Accelerator), and the Tevatron (at Fermilab) [3].

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[1] T. Khoe *et al.*, Part. Accel. **6**, 213 (1975).

[2] F. Z. Khiari *et al.*, Phys. Rev. D **39**, 45 (1989).

[3] *Polarized Beams at SSC (June 10–15, 1985, in Ann Arbor, Michigan)*, Proceedings of the Workshops on Polarized Beams at the SSC and Polarized Antiproton Sources, edited by A. D. Krisch, A. M. T. Lin, and O. Chamberlain, AIP Conf. Proc. No. 145 (AIP, New York, 1986).

[4] Ya. S. Derbenev and A. M. Kondratenko, Zh. Eksp. Teor. Fiz. **62**, 430 (1972) [Sov. Phys. JETP **35**, 230 (1972)]; Part. Accel. **8**, 115 (1978).

[5] A. D. Krisch *et al.*, Phys. Rev. Lett. **63**, 1137 (1989).

[6] J. E. Goodwin *et al.*, Phys. Rev. Lett. **64**, 2779 (1990).

[7] M. G. Minty *et al.*, Phys. Rev. D **44**, 1361 (1991).

[8] SPIN Collaboration, Acceleration of Polarized Protons to 120 and 150 GeV in the Fermilab Main Injector, University of Michigan Report, 1992 (unpublished).

[9] Equation (3) can be derived using equations given in M. G. Minty, Ph.D. thesis, Indiana University (1991); and T. Roser, *High-Energy Spin Physics: Eighth International Symposium*, edited by Kenneth J. Heller, AIP Conf. Proc. No. 187 (AIP, New York, 1989), p. 1442.