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Transient bimodality in turbulence-1–turbulence-2 transition in electrohydrodynamic convection in nematic liquid crystals

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In electrohydrodynamic convection, the turbulence-1–turbulence-2 (T1–T2) transition is observed when the applied voltage is very high. The transient property at the T1–T2 transition is studied through the evolution of the probability distribution of the area of T2. At a voltage close to the T1–T2 threshold the distribution transiently shows double peaks, i.e., a transient bimodality.

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The turbulence-turbulence transition is not a very common phenomenon and a few examples are found such as convective states in helium (He) gas [1], in superfluid He [2] and in the electrohydrodynamic convection (EHC) in nematic liquid crystals [3,4]. Turbulence-1-turbulence-2 (T1–T2) transition in superfluid ^{3}He is induced by a topological instability, due to the formation of a characteristic density of vortex filaments by an increase in the counterflow between the superfluid and the normal fluid of liquid He. Unfortunately direct visual observation is not possible in liquid He. In EHC the corresponding phenomenon is a transition between two turbulent convection states, the so-called dynamic scattering modes I and II (DSM1 and DSM2) transition [4]. Both differ by a different density of disclinations in the director field, whereas the director field describes the macroscopic orientational order in nematic liquid crystals [3]. The change of the disclination density is accompanied by a change in the light transmittance through the convective cell. These disclinations are directly visible under a microscope in contrast to the T1–T2 transition in He.

The DSM1-DSM2 (hereafter T1–T2) transition is a local one, via nucleation of the T2 phase and we found no hysteresis [4].

Additionally, several uncommon facts were observed: The relaxation time becomes very long at the T1–T2 threshold V_2 [5]. The growth velocity v of T2 nulei does not show the dependence proportional to $v \propto \epsilon_2^{1/2}$ typical for a second-order phase transition (supercritical bifurcation) or a ϵ_2-dependence characteristic for a first-order phase transition (subcritical one) [6]. However, we found a nonclassical dependence, $\epsilon_2^{0.7}$ [4]. Here $\epsilon_2=(V-V_2)/V_2$, and V is the applied voltage and V_2 is the threshold for the onset of T2. A similar dependence $v \propto \epsilon_2^{0.75}$ has recently been obtained from the amplitude equation containing higher-order derivatives [7].

In this article we report results observed on the transient behavior of the T1–T2 transition in EHC where we discover the transient bimodality. Such phenomena have been observed in electrical [8] and optical systems [9] experimentally and in combustion systems theoretically [10]. In systems with spatial degrees of freedom such as convective systems, however, there have been no reports of this, to our knowledge.

The sample used in the present study was the room-temperature nematic liquid crystal MBBA [4,5]. The thickness of enclosed cells ranged from 25 to 100 μm of which surfaces were coated by transparent electrodes. In_2O_3. The four samples were prepared whose threshold voltage V_2 for the onset of T2 turbulence and whose critical frequency f_c for the dielectric regime were as follows: (A) $V_2=37.8$ V (at $f=50$ Hz) and $f_c=100$ Hz, (B) $V_2=27.5$ V and $f_c=250$ Hz, (C) $V_2=77.7$ V and $f_c=70$ Hz, and (D) $V_2=27$ V and $f_c=500$ Hz. As all samples show qualitatively similar results, we mainly describe the results for the sample (A). The experimental setup was the conventional one [4,11]. The visual observation was performed with a microscope whose magnified image was simultaneously recorded on a magnetic tape through a video camera in order to analyze the size and the growth velocity of the T2 nucleus by a digital image analyzer. The temporal evolution of the convective state was also obtained by detecting the evolution of the light transmittance under a microscope. For the statistical measurement a steplike field from below threshold to above threshold was applied repeatedly to the sample 1024 times in order to obtain the distribution and the variance of its growth response [11]. The details on these experimental procedures have already been reported in Refs. [4,5,11].

Figure 1 shows the hysteresis in the light transmittance near T1–T2 (DSM1-DSM2) transition point V_2, which is related to the density of disclinations induced by the turbulent flow [4]. Here I is a transmitted light intensity through a sample for the convection state and I_{max} for one without convection. The rate of change (ramp rate r) of applied voltage is 25 mV/s. The T1–T2 transition can be observed near 37.8 V for 50 Hz and 60 V for 150 Hz. The width of the hysteresis at 150 Hz is wider than that at 50 Hz obviously. As already reported, however, the hysteresis width depends on the ramp rate r, decreases with decreasing r, and finally disappears [4].

In Fig. 2, as an example, the temporal development of the probability distribution of the transmitted light intensity (proportional to the area of T2) is shown after the steplike voltage is repeatedly applied from below V_2 to above. For small deviations $V-V_2$ it shows transiently two peaks in the distribution in Fig. 2(a), that is a bimodality, while only a single peak is observed during the evolution in Fig. 2(b) for a larger deviation of $V-V_2$. As shown in Fig. 2(a), starting with a distribution centered

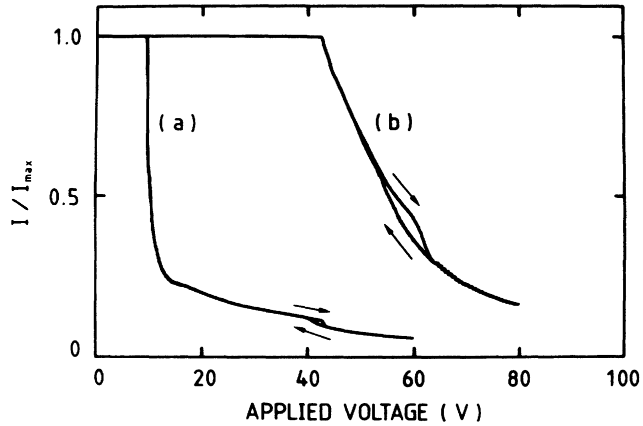

FIG. 1. Dynamic hysteresis in light transmittance near the T1–T2 transition point V_2. I denotes transmitted light intensity (arbitrary unit); I_{max}, transmitted light intensity without convection (arbitrary unit); ramp rate, $r=25$ mV/s. (a) $f=50$ Hz, (b) 150 Hz. $\Gamma=800$, $d=25$ μm.

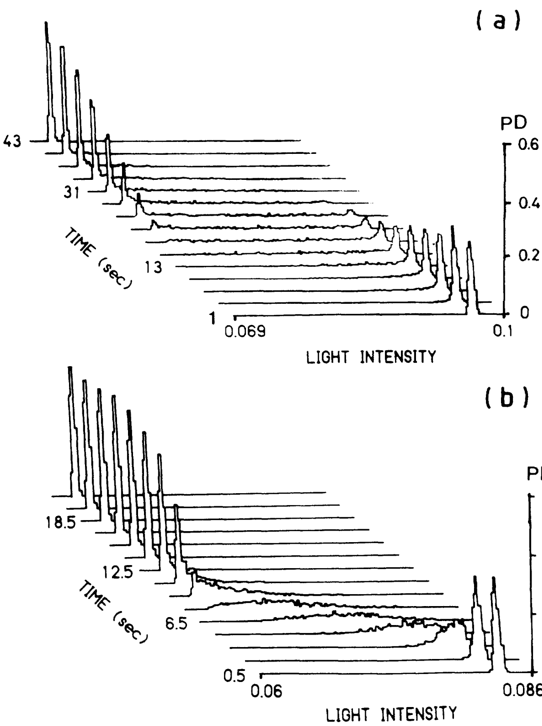

FIG. 2. Stochastic distribution in transient processes (observation area, 1.5 mm in diameter). (a) $\epsilon_2=3$, (b) $\epsilon_2=4$. The transmitted light intensity is dimensionless (I/I_{max}). PD is a probability density (dimensionless).

on a high transmission state (T1), the distribution gradually decreases its height, develops a long tail, and subsequently a transient bimodality, before collapsing again to a unimodal distribution centered on the stable attractor that is a low transmission state (T2). This clearly indicates the existence of two different time scales of transient dynamics. In Fig. 2(b), on the other hand, starting with a distribution centered on a high transmission state, its peak is gradually dispersed and moves into a low transmission state (i.e., the high density state of disclination [4]).

Figure 3 shows the temporal developments of the statistical variance σ_0 and the total statistical mean value P_0, and those of two local peaks P_F and P_S [11];

$$P_0(t)=\frac{1}{N}\sum_{i=1}^{N}P_i(t)\,, \tag{1}$$

$$\sigma_0^2(t)=\frac{1}{N-1}\sum_{i=1}^{N}[P_i(t)-P_0(t)]^2\,. \tag{2}$$

Here $P_i(t)=I_i(t)/I_{max}$ and I_i is the transmitted intensity of light with turbulence at the ith repetition of an applied stepfield through an EHD cell. P_F and P_S indicate the first peak observed at the beginning and the second appearing peak, respectively. To check the influence of local nucleation of T2, the dependence on the observation area was studied by varying from 6 mm to 200 μm in diameter.

Figure 3 shows P_0, P_F, P_S, and σ_0 for a wide observation area (1.5 mm in diameter; see Fig. 2). In Figs. 3 and 4 circles and squares show the highest peak (we call this hereafter the major peak) and the lower peak (the minor peak), respectively. At the beginning, only P_F can be observed. The bimodal distribution starts to appear around half the time to reach a steady state, for instance, $t\approx 20$ s in Fig. 3 where the system reaches a steady state at $t\approx 40$ s. In other words, the bimodality appears slightly before $t_{1/2}$ at which the order parameter (the area of T2) develops until half the value of a steady-state value and at which the variance also shows the maximum. Then the exchange of the amplitudes of those peaks, P_F and P_S, takes place at $t\approx t_{1/2}$ for a wide observation area. Nevertheless, the statistical average P_0 and the statistical variance σ_0 show a monotonical behavior. The specific behavior like a transient bimodality therefore cannot be observed when only the mean aspects are measured. The transient bimodality is more pronounced near the threshold V_2. The centered peaks P_F and P_S do not move from their initial locations and instead both heights exchange with time. Then P_F becomes the minor distribution peak finally replaced by P_S completely. For the widest observation (6 mm) the result is almost qualitatively identical with those in Fig. 3.

In Fig. 4, P_0, P_F, and P_S for a small observation area (200 μm) measurement are shown. The details of transient aspects are slightly different from those in Fig. 3. In Fig. 4(a) the transient bimodality for smaller ϵ_2 is shown and in Fig. 4(b) only a single peak exists in the whole transient period. The original peak P_F, however, appears continuously from the initial high transmission state with the highest amplitude until $t=38$ s. The second peak P_S appears to separate (bifurcate) from P_F after $t\approx 7$ s with the much smaller height. The height exchange takes place around $t\approx 38$ s. Thus for a small observation area both peaks P_F and P_S can coexist for a long period and the travels of the locations of these centered peaks can be observed during transition. The temporal development of σ_0 is a common one; that is, it increases at first and then decreases in the late stage. P_0 monotonically changes as might be expected from a simple evolution equation [11].

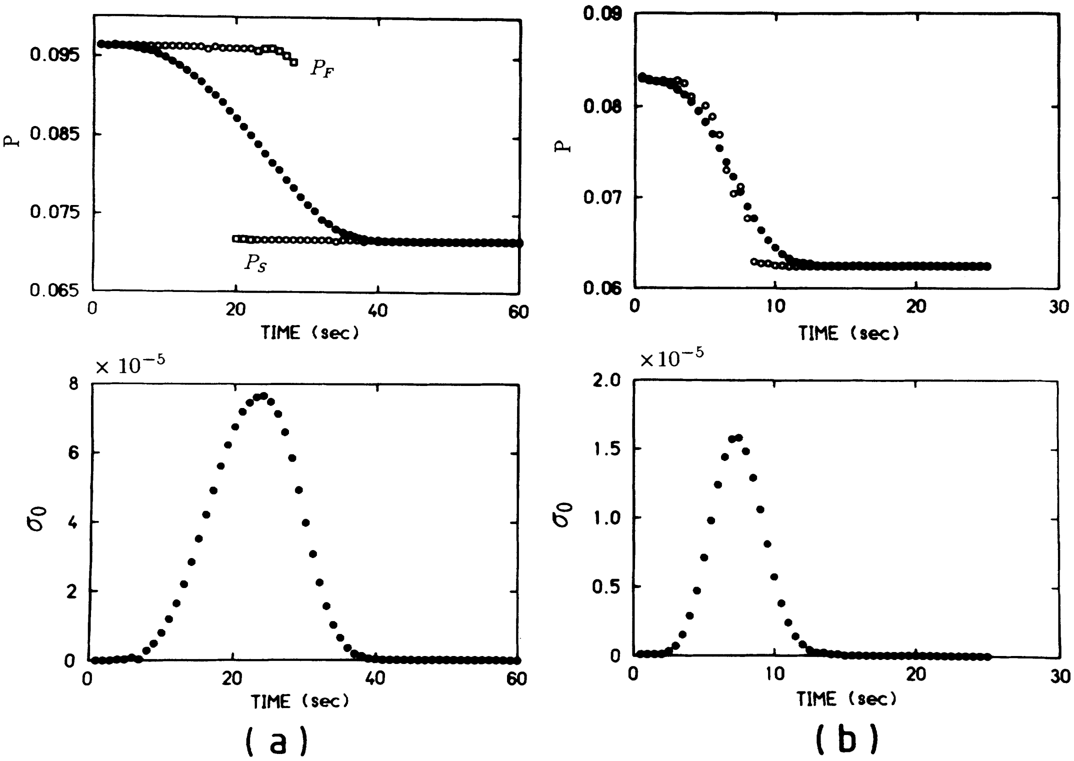

FIG. 3. Temporal developments of two peaks P_F and P_S (major peak, ○; minor peak, □) and statistical mean values (P_0, ●), and ones of variance σ_0 in a large observation area (1.5 mm in diameter) for (a) $\epsilon_2=3$ and (b) 4. Upper panels show the peaks; lower panels show the variance. All P and σ_0 are dimensionless.

We tried to fit these by the simple time-dependent Ginzburg-Landau equations with a noise term and linear and cubic terms of the order parameter [11]. However, it has not yet succeeded and is very difficult to obtain a perfect fitting. These characteristic features may suggest the existence of square terms of the order parameter and strong noise in addition to the importance of a very flat slope (marginal state) in its potential near T1–T2 transition. These may be explained similarly with optical and combustion systems [8,12]. When the initial δ-like distribution sits for a long time due to the flat slope of potential, it broadens and develops a tail in the direction of the potential well as a consequence of influence of noise (T1). Then the edge of the tail quickly transfers into the bottom of the potential and makes a second peak. That is, the symmetry breaking is induced due to the deterministic randomness already existing as a nonperiodic flow (T1). The property of "no hysteresis" can be observed commonly in supercritical (forward) bifurcation. On the other hand, properties such as the nucleation phenomenon, no real divergence of relaxation time, and the transient bimodality rather belong to properties in subcritical (backward) bifurcation, namely, the T1–T2 transition shows the mixed properties. Considering the observation of

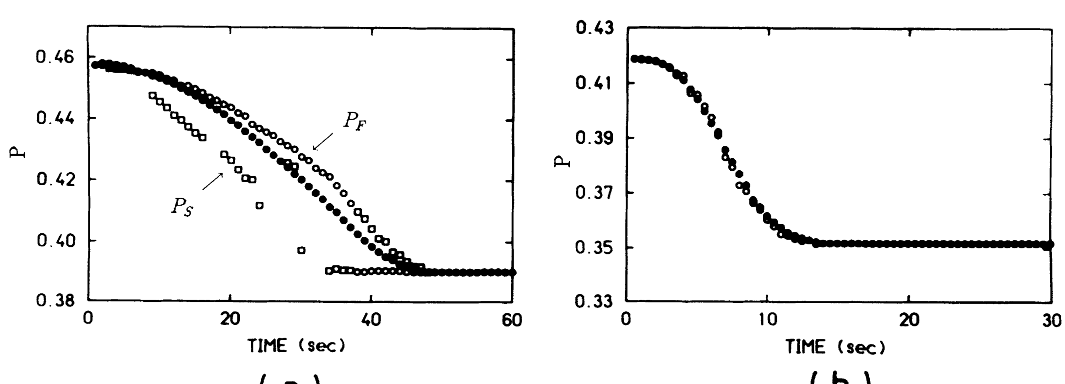

FIG. 4. Temporal developments of two peaks P_F and P_S (major peak, ○; minor peak, □) and statistical mean values (P_0,●) in small observation area (200 μm in diameter) for (a) $\epsilon_2=3$ and (b) 4. P_F, P_S, and P_0 are dimensionless.

transient bimodality, the bifurcation seems to be a subcritical-type bifurcation. These strange transition aspects therefore may be due to the influence of T1 (deterministic noise). The detailed analysis including further fitting is in progress and will be reported in the near future.

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