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Quantum trajectory simulations of the two-state behavior of an optical cavity containing one atom

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Under conditions of strong dipole coupling an optical cavity containing one atom behaves as a two-state system when excited near one of the "vacuum" Rabi resonances. A coherent driving field induces a dynamic Stark splitting of the vacuum Rabi resonance. We demonstrate this two-state behavior in computer experiments based on quantum trajectory simulations.

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Early experiments in cavity quantum electrodynamics focused on microwave transitions which are particularly suited to the need for large dipole coupling strengths. It was shown later that experiments at optical frequencies are feasible. Although it is more difficult to achieve dipole coupling strengths as large as spontaneous emission rates at optical frequencies, optical systems have their own special attraction. Detectors exist for optical photons, and this opens up the possibility for studying photon statistics.

There are many theoretical papers on the subject of photon statistics and quantum fluctuations for optical fields interacting with atoms inside cavities. Theories of the laser, optical bistability, and intracavity squeezing provide some of the examples. Most of this work is based on the diffusion model of quantum noise. The diffusion model assumes that changes by quantum—one photon emitted, one atom excited—are negligible on the scale of the number of particles needed to probe the intrinsic nonlinearity of the system. This number depends on the system size. For example, the photon number in a laser cavity scales as a power of the saturation photon number, $n_{\text{sat}}=\gamma^2/8g^2$, where g is the dipole coupling constant and γ is the spontaneous emission rate; n_{sat} increases with system size since g depends inversely on cavity volume; n_{sat} is typically very large and this justifies the standard diffusion model for laser noise.

A large saturation photon number means small dipole coupling. In cavity quantum electrodynamics it is just this weak-coupling condition that is relaxed. Thus, in cavity quantum electrodynamics the standard treatment of quantum fluctuations does not hold. In this paper we address this issue with two specific objectives in mind: First, to show just how much the quantum fluctuations can change in the strong-coupling limit. Second, to propose a new approach, the quantum trajectory approach, for describing these fluctuations.

We consider an optical cavity containing one atom. The atom has a dipole transition that matches one of the cavity resonances and the cavity mode is driven by a coherent laser field. With quantum fluctuations neglected this system is described by the semiclassical theory of optical bistability. In the diffusion picture we expect fluctuations to broaden the semiclassical states, and for weak dipole coupling this is exactly what happens [1]. But for strong dipole coupling it is not; semiclassical results are changed completely so that where an amplitude bimodality is expected a phase bimodality occurs [2]. The reason for the change is that the idea of diffusion around semiclassical states starts from a factorization—from a factorized cavity mode and atom. But for strong dipole coupling the factorization is not valid, not even approximately so. Strong dipole coupling forms two quantum systems into a new quantum system, just as two atoms form a molecule when they strongly interact. One signature of this change is the so-called "vacuum" Rabi splitting. But the mere splitting of a resonance can be described within semiclassical theory. Here we will provide a more convincing demonstration of the *quantum* character of the new system formed by a coupled cavity mode and atom. We tune the driving field to one of the vacuum Rabi resonances and observe the behavior of a single two-state system. In particular, we show that the light transmitted by the cavity has the photoelectron counting statistics of resonance fluorescence and a Mollow-triplet spectrum for strong driving field intensities. This behavior occurs because the energy levels of the coupled cavity mode and atom are unequally spaced, which, for sufficiently strong coupling allows a two-state approximation to be made.

We use the quantum trajectory approach to demonstrate two-state behavior in simulated measurements of the photoelectron waiting-time distribution, the photoelectron counting distribution, and the optical spectrum. The quantum trajectory approach is built around the theory of photoelectric detection and the master equation theory of photoemissive sources. Using the two theories one can relate the statistics of photoelectron emissions to a dynamical process involving photon emissions taking place at the source [3,4]. The source then follows a quantum stochastic evolution described in the mathematical language of the theory of continuous quantum measurement [5]. We have developed these ideas into a general theory that can be applied to any photoemissive source [6,7]. In this theory a stochastic wave function describes the state of the source conditioned on the realization of particular stochastic signals at idealized detectors that monitor its radiated fields. An ensemble average, or time average, taken with respect to this *conditioned wave function* reproduces the results of a standard master equation calculation. Quantum trajectories have also been introduced by others in slightly different ways [8–10].

We denote the conditioned wave function by $|\psi_c(t)\rangle$. It is convenient to also introduce an unnormalized wave function $|\bar{\psi}_c(t)\rangle$, with

$$|\psi_c(t)\rangle = |\bar{\psi}_c(t)\rangle / [\langle \bar{\psi}_c(t)|\bar{\psi}_c(t)\rangle]^{1/2}.$$

The conditioned wave function satisfies a coherent evolution between photon emissions governed by a Schrödinger equation with a non-Hermitian Hamiltonian, interrupted by instantaneous collapses at the times of the photon emissions: Between photon emissions we have the Schrödinger equation

$$|\dot{\bar{\psi}}_c\rangle = (1/i\hbar)\hat{H}|\bar{\psi}_c\rangle, \tag{1}$$

where for our current application, in a frame rotating at the frequency ω_L of the laser,

$$\hat{H} = \hbar\delta(\hat{\sigma}_z/2 + \hat{a}^\dagger\hat{a}) + i\hbar g(\hat{a}^\dagger\hat{\sigma}_- - \hat{a}\hat{\sigma}_+) + i\hbar\mathcal{E}(\hat{a}^\dagger - \hat{a}) - i\hbar(\gamma_I/2)\hat{\sigma}_+\hat{\sigma}_- - i\hbar\kappa\hat{a}^\dagger\hat{a}; \tag{2}$$

$\delta = \omega_0 - \omega_L$ is the detuning of the cavity mode and atom from the laser field, $\mathcal{E}^2$ is the photon flux from the laser into the cavity, g is the dipole coupling constant, γ_I is the cavity-inhibited spontaneous emission rate for the atom, and 2κ is the photon loss rate from the cavity. Photon emissions occur at random times at a rate determined by $|\psi_c(t)\rangle$. Emission out the sides of the cavity occurs at the rate

$$r_A(t) = \gamma_I\langle\psi_c(t)|\hat{\sigma}_+\hat{\sigma}_-|\psi_c(t)\rangle \tag{3}$$

and is accompanied by the wave-function collapse

$$|\bar{\psi}_c(t)\rangle \to \hat{\sigma}_-|\bar{\psi}_c(t)\rangle. \tag{4}$$

The emission of a photon through the cavity mirrors occurs at the rate

$$r_C(t) = 2\kappa\langle\psi_c(t)|\hat{a}^\dagger\hat{a}|\psi_c(t)\rangle, \tag{5}$$

and is accompanied by the collapse

$$|\bar{\psi}_c(t)\rangle \to \hat{a}|\bar{\psi}_c(t)\rangle. \tag{6}$$

Using a computer, sequences of collapses are readily simulated in a Monte Carlo fashion while simultaneously integrating Eq. (1). In this way we produce realizations of the quantum trajectories $|\psi_c(t)\rangle$. An example is shown in Fig. 1 where we plot the *conditioned* averages that appear on the right-hand sides of Eqs. (3) and (5).

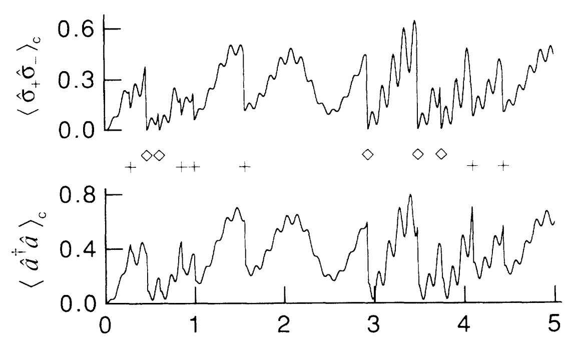

FIG. 1. Sample quantum trajectory for $\gamma_I/2\kappa = 1$, $g/\kappa = 25$, and $\mathcal{E}/\kappa = 5$. ◇ and + mark the times of photon emissions from the atom and cavity, respectively.

The quantum trajectories defined by Eqs. (1)–(6) come from a decomposition of the master equation based on direct photoelectric detection. We call such a decomposition an *unraveling* of the master equation. Other idealized measurements give different unravelings, and quite different quantum trajectories; for example, an unraveling based on homodyne detection produces a continuous evolution of the conditioned wave function [6,7].

The quantum trajectory shown in Fig. 1 was obtained with the laser tuned to the lower vacuum Rabi resonance, as illustrated in Fig. 2. It shows a fast coherent modulation at the detuning frequency $2g$ from the upper vacuum Rabi resonance. It also shows an oscillation at a much lower frequency (seen most clearly for $1.5 < \kappa t < 3.0$). This is caused by the dynamic Stark splitting indicated by the dashed lines in Fig. 2 [11]. The simplest model for this behavior is the two-state approximation,

$$\hat{H} = \hbar(\mathcal{E}/\sqrt{2})(\hat{l}_+ - \hat{l}_-) - i\hbar\tfrac{1}{2}(\kappa + \gamma_I/2)\hat{l}_+\hat{l}_-, \tag{7}$$

where $\hat{l}_+ = |1,l\rangle\langle g|$ and $\hat{l}_- = (\hat{l}_+)^\dagger$. Equations (3)–(6) are now read with $\hat{\sigma}_- \to (1/\sqrt{2})\hat{l}_-$ and $\hat{a} \to (-i/\sqrt{2})\hat{l}_-$. The Rabi frequency for this two-state system is $\sqrt{2}\mathcal{E}$, which agrees quite well with the modulation frequency observed in Fig. 1. Note that this frequency is independent of the dipole coupling constant g.

The two-state approximation is not suitable for a quantitative study. For example, the modulation at frequency $2g$ is absent in the two-state approximation. Also, in the two-state approximation every emission collapses the coupled system to its ground state $-\langle\hat{\sigma}_+\hat{\sigma}_-\rangle_c \to 0$, $\langle\hat{a}^\dagger\hat{a}\rangle_c \to 0$; this does not happen in Fig. 1 (although emissions from the atom do collapse the atom to its lower state $-\langle\hat{\sigma}_+\hat{\sigma}_-\rangle_c \to 0$). The results that follow were obtained using a basis truncated at the six-photon level.

We check first whether the coupled cavity mode and atom saturates like a two-state system. Figure 3 shows results for different dipole coupling strengths, where we expect saturated steady-state averages $\langle\hat{\sigma}_+\hat{\sigma}_-\rangle = \langle\hat{a}^\dagger\hat{a}\rangle = \frac{1}{4}$ for the $|g\rangle \to |1,l\rangle$ transition. The results were calculated by direct inversion of the standard density matrix equation, and in our simulations time averaging conditioned averages produced the same answers. We see that two-state saturation occurs over a substantial range of $\mathcal{E}/\kappa$ for large values of g. Of course, it eventually breaks down as

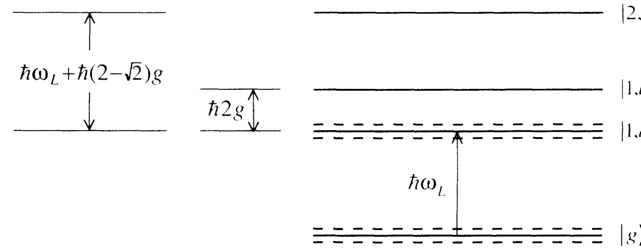

FIG. 2. Energy-level diagram for a cavity mode coupled to one atom. The dashed lines show the dynamic Stark splitting induced by a laser tuned to the lower vacuum Rabi resonance.

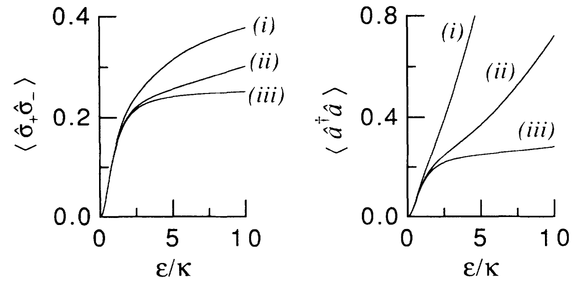

FIG. 3. Approximate two-state saturation for $\gamma_l/2\kappa=1$ and (i) $g/\kappa=10$, (ii) $g/\kappa=25$, and (iii) $g/\kappa=100$.

$\mathcal{E}/\kappa$ continues to increase as two-photon transitions to the state $|2,l\rangle$ become important. We use $g/\kappa=25$ and $\gamma_l/2\kappa=1$ in the following simulations.

Figures 4 and 5 show the results of simulated photoelectron counting experiments. $W_A(\tau)$ and $W_C(\tau)$ are the photoelectron waiting-time distributions for separate ideal detectors monitoring photon emissions from the atom and cavity, respectively. $W_{AC}(\tau)$ is the waiting-time distribution obtained by summing the outputs from the two detectors (emissions from the atom *or* cavity). $P_{AC}(n)$ is the photoelectron counting distribution for the summed output in the long-counting-time limit (the dashed curve is a Poisson distribution). The results in Fig. 4 are essentially those for resonance fluorescence [compare Figs. 1(b) and 4(a) in [3]]. The only departure is the less than perfect collapse to the ground state for photon emissions from the cavity, evidenced by the fact that $W_C(0)$ and $W_{AC}(0)$ are not zero. This is a consequence of the small probability for occupying states $|1,u\rangle$ and $|2,l\rangle$. Figure 5 shows a much larger departure from a two-state cavity collapse, and in addition, the modulation at frequency $2g$ is now in evidence. But ringing due to the "two-state" dynamic Stark splitting is also present [compare Fig. 1(c) in [3]].

We now focus on the Stark splitting by simulating a

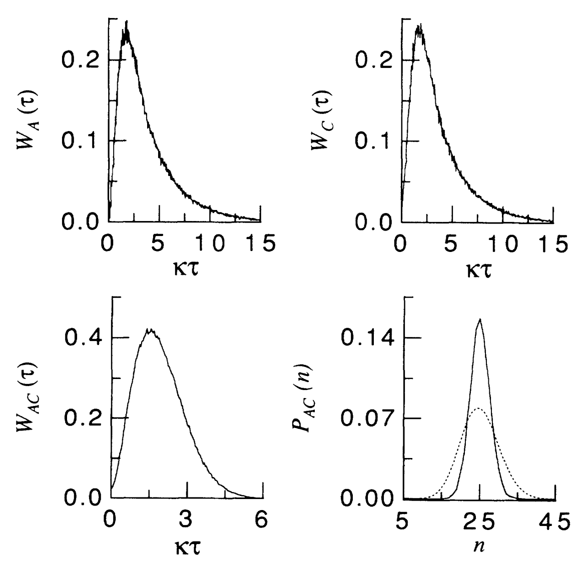

FIG. 4. Results of simulated photoelectron counting for $\mathcal{E}/\kappa=1$.

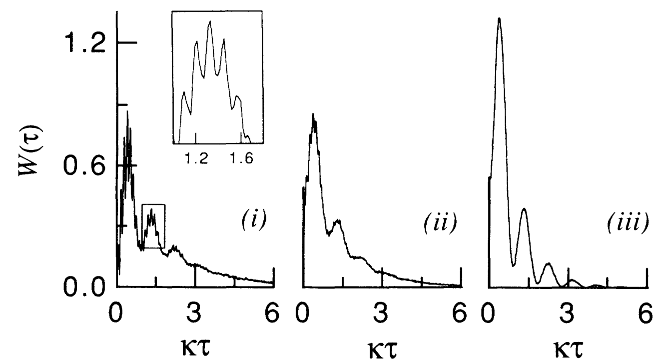

FIG. 5. Results of simulated photoelectron counting for $\mathcal{E}/\kappa=5$: (*i*) $W_A(\tau)$, (*ii*) $W_C(\tau)$, (*iii*) $W_{AC}(\tau)$.

measurement of the optical spectrum. We calculate the spectrum of the light transmitted by the cavity by coupling this light into a second, filter cavity, and simulating quantum trajectories for the coupled cavity and atom plus filter cavity. We write the unnormalized conditioned state as $|\bar\psi_c(t)\rangle|0\rangle_F+|\bar\psi_c^1(t)\rangle|1\rangle_F$, where $|0\rangle_F$ and $|1\rangle_F$ are the vacuum and one-photon states of the filter cavity. For a filter with half-width β and resonance frequency ω, between emissions, Eq. (1) is now solved together with the equation

$$|\dot{\bar\psi}_c^1\rangle=[(1/i\hbar)\hat H-(\beta+i\omega)]|\bar\psi_c^1\rangle+\xi\hat a|\bar\psi_c\rangle, \qquad (8)$$

where $\xi\sim\sqrt{\beta}$ is an arbitrary coupling strength. The spectrum is obtained from the time-averaged conditioned mean photon number for the filter cavity $\langle\bar\psi_c^1(t)|\bar\psi_c^1(t)\rangle/\langle\bar\psi_c(t)|\bar\psi_c(t)\rangle$, for different settings of the filter frequency ω. Figure 6 shows the results of this simulation. Figure 7 shows the spectrum calculated by traditional methods using the quantum regression theorem. (Figure 6 includes a contribution from filtered coherent light which is not shown in Fig. 7.) Although there are departures from two-state behavior, the spectrum shows a clear Mollow triplet with sidebands at frequencies $\omega-\omega_L\approx\pm\sqrt{2}\mathcal{E}$. The additional doublets centered at $\omega-\omega_L=\pm 2g$ come from the high-frequency modulation seen in Figs. 1 and 5. Asymmetries are present due to two effects: an unequal detuning from the state $|2,l\rangle$ for tran-

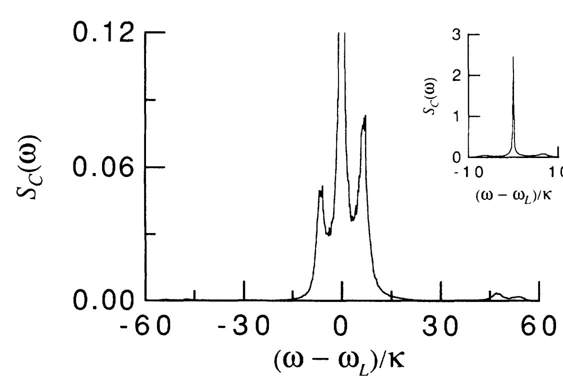

FIG. 6. Simulated measurement of the spectrum of the light transmitted by the cavity for $\mathcal{E}/\kappa=5$.

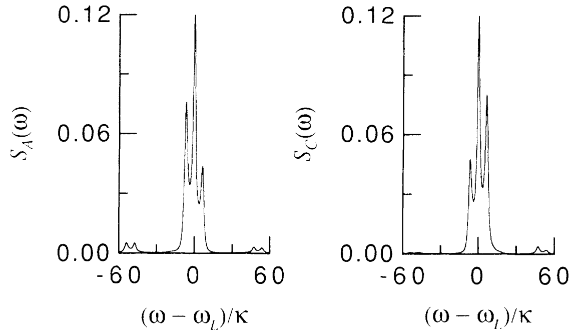

FIG. 7. Incoherent spectrum of the light radiated by the atom, $S_A(\omega)$, and by the cavity, $S_C(\omega)$, for $\mathcal{E}/\kappa=5$.

sitions out of the "dressed" states, and unequal matrix elements between the states $|1,l\rangle$ and $|2,l\rangle$ for the operators $\hat{\sigma}_-$ and $\hat{a}$. We leave these details for discussion elsewhere.

We have used the quantum trajectory approach to show that an optical cavity containing one atom can behave as a two-state system. The quantum trajectory approach allows us to visualize the dynamics of a photoemissive source free from the straightjacket of classical diffusion models. It is particularly useful in cavity quantum electrodynamics where the standard diffusion approximation breaks down.

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