PHYSICAL REVIEW A VOLUME 46, NUMBER 10 15 NOVEMBER 1992

Self-organization and a dynamical transition in traffic-flow models

Ofer Biham and A. Alan Middleton
Department of Physics, Syracuse University, Syracuse, New York 13244

Dov Levine
Department of Physics, Technion, Israel Institute of Technology, Haifa 32000, Israel

(Received 2 June 1992)

A simple model that describes traffic flow in two dimensions is studied. A sharp *jamming transition* is found that separates between the low-density dynamical phase in which all cars move at maximal speed and the high-density jammed phase in which they are all stopped. Self-organization effects in both phases are studied and discussed.

PACS number(s): 02.70.+d, 05.70.Ln, 64.60.Cn, 89.40.+k

Traffic problems have been studied extensively in recent years in order to help in the design of transportation infrastructure and to optimize the allocation of resources. Traffic simulations, based on various hydrodynamic models, have provided much insight and are in good agreement with experiments for simple systems such as a freeway, a tunnel, or a single junction [1]. However, the simulation of traffic flow in a whole city is a formidable task as it involves many degrees of freedom such as local densities and speeds. The availability of powerful supercomputers is likely to make these simulations feasible in the near future, but models that are simpler and more flexible than hydrodynamic models will be needed in order to achieve this task.

Cellular automaton (CA) models [2] are increasingly used in simulations of complex physical systems such as fluid dynamics [3], driven diffusive systems [4], sandpiles [5], and chemical reactions [6]. In some of these systems the cellular automaton models provide only some general qualitative features of the system while in other cases useful quantitative information can be obtained. For some problems involving complex geometries, such as simulations of fluid dynamics in porous media, cellular automata are found to be superior to other methods.

In this paper we present three variants of a simple cellular automaton model that describes traffic flow in two dimensions. The first two variants are three-state CA models on a square lattice. Each site contains either an arrow pointing upwards, an arrow pointing to the right, or is empty. In the first variant (model I) the dynamics is controlled by a traffic light, such that the right arrows move only in even time steps and the up arrows move in odd time steps. On even time steps, each right arrow moves one step to the right unless the site on its right-hand side is occupied by another arrow (which can be either an up or a right arrow). If it is blocked by another arrow it does not move, even if during the same time step the blocking arrow moves out of that site. Similar rules apply to the up arrows, which move upwards. Note that this is a fully deterministic model; randomness enters only through the initial conditions. In this model the traffic problem is reduced to its simplest form while the essential features are maintained. These features include the simultaneous flow in two perpendicular directions of objects that cannot overlap. No attempt is made here to draw a more direct analogy between our model and real traffic problems.

The model is defined on a square lattice of $N \times N$ sites with periodic boundary conditions. Due to the periodic boundary conditions the total number of arrows of each type is conserved. Moreover, the total number of up arrows in each column and the total number of right arrows in each row are conserved, giving rise to $2N$ conservation rules.

The density of right (up) arrows is given by $p_{\rightarrow} = n_{\rightarrow}/N^2$ ($p_{\uparrow} = n_{\uparrow}/N^2$), where $n_{\rightarrow}$ ($n_{\uparrow}$) is the number of right (up) arrows in the system. Here we examine the case where $p_{\rightarrow} = p_{\uparrow} = p/2$. The (average) velocity v of an arrow in a time interval τ is defined to be the number of successful moves it makes in τ divided by the number of attempted moves in τ. It has maximal value $v = 1$, indicating that the arrow was never blocked, while $v = 0$ means that the arrow was stopped for the entire duration τ, and never moved at all. The average velocity $\overline{v}$ for the system is then obtained by averaging v over all the arrows in the system.

We have performed extensive simulations of the model starting with an ensemble of random initial conditions. After a transient period that depends on the system size, on p, and on the random initial condition, the system reaches its asymptotic state. We found two qualitatively different asymptotic states, which are separated by a sharp dynamical transition. Below the transition all the arrows move freely in their turn and the average velocity is $\overline{v} = 1$, while above it they are all stuck and $\overline{v} = 0$ with very high probability. A typical configuration below the transition is shown in Fig. 1, where the system is self organized into separate rows of right and up arrows along the diagonals from the upper-left to the lower-right corners. This arrangement enables the arrows to achieve the maximal speed. When a row of horizontal arrows moves, it makes space for a row of vertical arrows to move in the

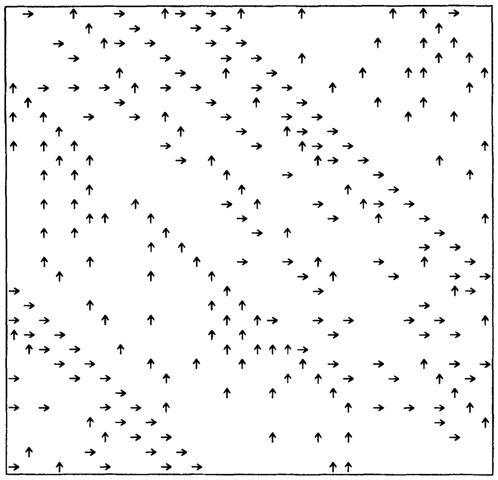

FIG. 1. A typical dynamic configuration in the low-density phase below the transition. The system is self-organized into a pattern of lines of arrows from the upper-left to the lower-right corners and $\overline{v} = 1$. The system size is 32×32 and $p = 0.25$.

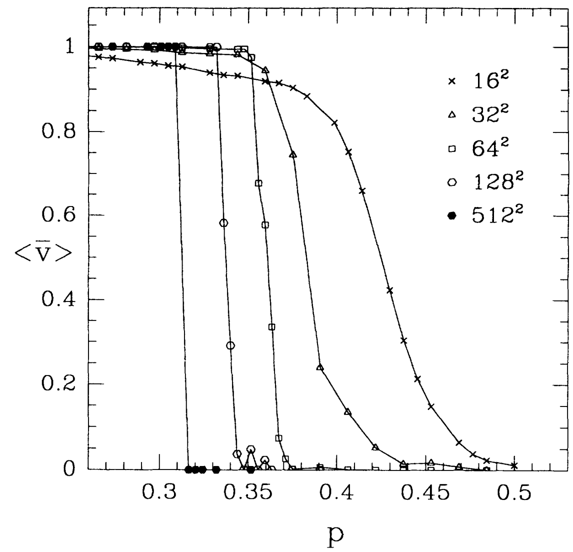

FIG. 3. The ensemble average velocity $\langle\overline{v}\rangle$ as a function of the concentration p for five different system sizes (model I). As the system size increases the transition becomes sharper, and the ensemble-average velocity changes rapidly from $\langle\overline{v}\rangle = 1$ below $p_c(N)$ to $\langle\overline{v}\rangle = 0$ above it.

next step, such that they never collide. Above the transition all the arrows are stopped in a global cluster, shown in Fig. 2 (by global cluster we mean a cluster that connects one side of the system to the other). This global cluster is oriented along the diagonal from the upper-right to the lower-left corners. This way it blocks the paths of all arrows which finally get stopped [7].

These two states are separated by a sharp *jamming transition* in which the ensemble-average velocity changes rapidly from $\langle\overline{v}\rangle = 1$ to $\langle\overline{v}\rangle = 0$ as p is varied (see Fig. 3). Results for five system sizes from 16×16 to 512×512 are presented. Small-size systems (up to 128×128) have been simulated on sequential machines, while the larger systems were simulated on a DECmpp parallel computer with 8k processors. For small-size systems the transition is not sharp but there is a range of densities in which both asymptotically dynamic and asymptotically static states are found with a non-negligible probability (depending on the initial condition). We define $p_c(N)$ to be at the center of this region, which is characterized by very long transients. As the system size increases, $p_c(N)$ tends to decrease while the coexistence region shrinks, giving rise to a sharper transition. From our simulations we have not been able to obtain conclusive results for p_c in the infinite system limit. We find that the transition is very sharp for large systems. However, $p_c(N)$ keeps decreasing as N increases, and we have not been able to determine whether it converges to a finite p_c or to zero in the infinite system limit. The difficulty results from the long equilibration times near the transition (see Fig. 4) and from the slow convergence of $p_c(N)$ as the system size increases.

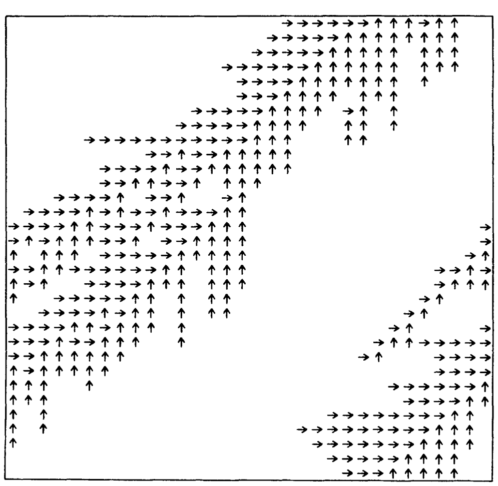

FIG. 2. A typical static configuration in the high-density phase above the transition. Here the global cluster is oriented between the upper-right and the lower-left corners, and blocks the paths of all the arrows until they get stopped. The system size is 32×32 and $p = 418/1024 \approx 0.4082$.

Being a transition as a function of the concentration p, between a state with no global cluster below p_c to a state with a global cluster above p_c, it resembles the percolation transition [8]. However, the percolation transition is a second-order transition and has no dynamics. The jamming transition can also be considered in the context of pinning transitions that occur in extended systems with quenched random impurities such as charge-density waves [9]. In our model there is no quenched component and the two sets of arrows pin each other when the density increases above threshold.

In order to examine the robustness of the jamming transition we have also studied a nondeterministic variant of our model (model II). In model II the traffic light is removed and all arrows move in all time steps (unless

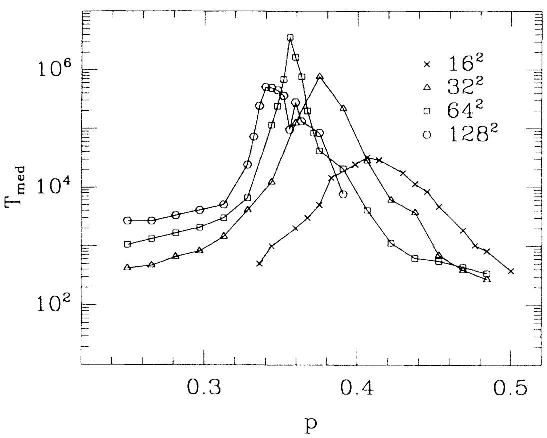

FIG. 4. The median equilibration time $T_{\rm med}$ for model I as a function of p for four different system sizes. The equilibration time is the number of time steps it takes to reach a periodic cycle or to get stuck. The peak around p_c becomes higher and sharper as the system size increases up to 64×64, and then becomes more flat for 128×128. It is not clear how to interpret this behavior for 128×128, although it may be that there is a very narrow peak that we have not been able to resolve.

they are stopped). If both an up and a right arrow try to move to the same site, one of them will be chosen randomly, with equal probabilities [10]. For this model we also find a sharp transition. The values of $p_c(N)$ are smaller than for model I (approximately 0.10 for systems of size 512×512). The value of p_c in the infinite system limit cannot be determined from our data.

By choosing a two-dimensional model that has only right and up arrows, and does not have left and down arrows, we simplify the problem without losing most of its essential features. The essential problem that causes traffic jams is the need of the right and up arrows to cross each other's paths, while each site can be occupied by only one arrow. There is no such problem between the up and down, or between the right and left arrows, as they can move in parallel paths that do not intersect. In models that have both right, up, left, and down arrows one can have a stable finite traffic jam. A simple example is a set of four arrows in which an up arrow blocks a left arrow, which blocks a down arrow, which blocks a right arrow, which blocks the first up arrow. This is the *gridlock* mechanism, which may occur at any density p. In our model gridlocks are not possible, and the jamming transition occurs only when a global cluster forms.

To obtain a better understanding of the model we now describe the one-dimensional analog which can be solved analytically. In one dimension there is only one type of arrows (say right arrows) that move along a closed ring. Every time step each arrow moves to the right unless it is blocked by another arrow [11]. The asymptotic velocity $\overline{v}$ is independent of initial conditions. It is $\overline{v} = 1$ for $p < 1/2$, while for $p > 1/2$ it decreases continuously to zero according to $\overline{v}(p) = (1-p)/p$ [12]. We thus conclude that the sharp first-order transition is indeed a result of the interaction between the horizontal and vertical arrows due to the excluded volume. To clarify this point further we have performed preliminary simulations on a variant (model III) in which a right and an up arrow are allowed to occupy the same site. In this four-state model all arrows try to move at every time step. If both an up arrow and a right arrow try to move to an empty site *at the same time step* they both move in and overlap. On the other hand no arrow can move into a site which is already occupied. This model is designed to have weaker excluded volume effects between arrows of different types. Indeed our simulations show that model III exhibits a continuous transition which is qualitatively similar to the one-dimensional case.

In summary, we have presented a cellular automaton model that describes traffic flow in two dimensions. Our simulations of finite systems up to 512×512 show a sharp transition that separates a low-density dynamical phase in which all cars move at a maximum speed and a high-density static phase in which they are all stuck in a global traffic jam. Such behavior is found both in a deterministic and a nondeterministic variants of the model and we thus believe that it is robust and represents a general feature of traffic flow in two dimensions. We believe that cellular automata provide a useful framework for traffic simulations that should be developed further. These models are especially suitable for simulations on parallel computers, and their flexibility is especially important in the complex geometries of traffic networks.

This work was performed using the computational resources of the Northeast Parallel Architectures Center (NPAC) at Syracuse University.

[1] See, e.g., *Transportation and Traffic Theory*, edited by N.H. Gartner and N.H.M. Wilson (Elsevier, New York, 1987); W. Leutzbach, *Introduction to the Theory of Traffic Flow* (Springer-Verlag, Berlin, 1988); D.L. Gerlough and M.J. Huber, *Traffic Flow Theory* (National Research Council, Washington, D.C., 1975).

[2] S. Wolfram, *Theory and Applications of Cellular Automata* (World Scientific, Singapore, 1986).

[3] See, e.g., *Lattice Gas Methods for PDE's*, Proceedings of the NATO Advanced Research Workshop, edited by G.D. Doolen, [Physica D **47**, 1 (1991)].

[4] B. Schnittman, Int. J. Mod. Phys. B **4**, 2269 (1990).

[5] P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987).

[6] M. Gerhardt and H. Schuster, Physica D **36**, 209 (1989).

[7] Note that the fact that we find static behavior above p_c and dynamic behavior below does not mean that there are no static configurations with $p < p_c$ or dynamic configurations with $p > p_c$. What it means is that these cases are atypical and have very small basins of attraction in

the ensemble of random initial conditions. In fact, the static configuration with the smallest possible $p > 0$ has $p = 2/N \to 0$ as $N \to \infty$, and the dynamic configuration with $\overline{v} = 1$ and the largest possible p has $p = 2/3$.

[8] D. Stauffer, *Introduction to Percolation Theory* (Taylor & Francis, London, 1985).

[9] See, e.g., *Charge Density Waves in Solids*, edited by G. Hutiray and J. Sólyom (Springer-Verlag, Berlin, 1985); G. Grüner, Rev. Mod. Phys. **60**, 1129 (1988); *Charge Density Waves in Solids*, edited by L. P. Gorkov and G. Grüner (Elsevier, New York, 1989).

[10] Note that a similar situation occurs in lattice-gas cellular automata on the triangular lattice, where some states can have several different outcomes. In this case one can use either a deterministic approach such as using different outcomes in even and odd time steps or the nondeterministic approach. The deterministic approach is more efficient in numerical simulations, while there seems to be no significant difference in the results between the two approaches.

[11] This one-dimensional model is identical to Wolfram's CA rule no. 184, which is asymmetric and thus does not belong to the set of 32 legal rules with a three-site neighborhood. This model was previously studied in the context of surface growth through ballistic deposition [J. Krug and H. Spohn, Phys. Rev. A **38**, 4271 (1988)]. Related stochastic models in two dimensions with one type of arrow have also been studied [S.A. Janowsky and J.L. Lebowitz, Phys. Rev. A **45**, 618 (1992)].

[12] This result is obtained analytically by considering the vacant sites as left-moving arrows, with an exchange dynamics such that the numbers of right arrows and left arrows moving in each time step are the same.