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Remote multimode feedback stabilization of plasma instabilities

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An experimental demonstration of a multimode feedback stabilization with a single sensor-suppressor pair is presented. Two modes were simultaneously stabilized with a simple state-feedback-type method where more "state" information was generated from a single-sensor Langmuir probe by appropriate signal processing. A single feedback-modulated ion beam served as the remote suppressor. A simple theory shows good agreement with experiment. This experiment may be considered as a paradigm for controlling certain classes of chaotic systems.

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Feedback methods have been applied to plasmas to control instabilities with rather limited success [1]. Most involved metallic probes or electrodes as sensors and suppressors to stabilize a single mode of plasma instabilities [1–5]. Remote, nonintrusive feedback suppressors are desirable where metallic probes may not survive or may prove too perturbing to the plasma. This motivated a recent experiment of controlling a single plasma instability with a feedback-modulated ion beam [6]. In general, many unstable modes can exist simultaneously and so reducing the consequent fluctuations requires multimode stabilization. Hence, achieving multimode stabilization is crucial if feedback control is to be a viable technique for control of plasma instabilities. However, the general problem of simultaneously stabilizing multimode plasma instabilities using a single sensor-suppressor pair has been unsuccessful because stabilizing one mode tended to destabilize other modes. A single sensor and suppressor are desirable to minimize the perturbation to the plasma.

The control of lumped parameter systems characterized by ordinary differential equations is a well-established science with a strong theoretical foundation and wide technological applications. In contrast, the control of distributed parameter systems (for example, plasmas, fluids, very large elastic structures, biological systems, etc.) characterized by partial differential equations has remained a mostly open problem, especially in the experimental arena. The formal reason for this disparity is that a lumped parameter system has a finite number of dynamic "states" (degrees of freedom). However, a distributed parameter system (continuum), strictly speaking, has an infinite number of dynamic "states," which renders it mathematically and physically intractable. This problem has been made tractable by the use of physically motivated definitions of "normal-mode states" [7]. It can be shown that the dynamical equations of the modal amplitudes of these states form a set of ordinary differential equations [7]. One can then use the formal machinery of the usual control science for stabilization of instabilities in any continuum.

In this Rapid Communication, we follow this procedure for two plasma instabilities and perform an experiment validating this unique formalism and with a single sensor-suppressor pair to stabilize multimode plasma instabilities. This experiment has substantial significance for many plasma devices and in particular for the plasma fusion effort which has been plagued by the observation of anomalous transport in tokamaks. The anomalous transport is generally believed to be caused by instabilities and their stabilization may lead to better plasma confinement in fusion machines and more uniform plasmas in many other devices.

It has been shown theoretically that extra plasma state information may be generated from a single sensor [7]. A "state"-feedback scheme of stabilizing many plasma instabilities simultaneously with a single sensor-suppressor pair with appropriate choices of feedback gain and phase parameters has also been considered [8]. In our experiment, a feedback-modulated ion beam was used as the remote suppressor and a Langmuir probe as the sensor. A state-feedback-type method was implemented where a differentiator was employed to obtain extra information about the dynamic states of the plasma. By analogy with a system in classical mechanics, differentiation with respect to time of the position coordinate generates the momentum, which is another independent coordinate. This additional information of the system can then be used as another control signal. The independent signal generated by differentiation was then combined with the original signal and fed back into the plasma via the ion beam.

The feedback control of multimode plasma instabilities requires the measurement of the complete system dynamics because the control action is based upon them. In an Nth-order lumped parameter system characterized by ordinary differential equations, N independent dynamic variables called "states" must be "observed" (measured) for complete control. For distributed parameter systems such as a plasma, characterized by partial differential equations, the concept of states has been adapted in terms of "normal-mode states" [7]. If a plasma consists of m unstable modes, representing m normal-mode states, a single-sensor signal (for example, from a Langmuir probe) contains linear combinations of all the m states. In order to control the multimode plasma with a single sensor, one must derive an equal number of independent control signals from the sensor signal. We have experimentally accomplished this for a plasma with two modes.

The model for the plasma and the generation of two control signals is shown in Fig. 1. A superposition of the two independent control signals is fed back into the plasma. The transfer function $H(s)$ of a plasma with two unstable modes with real frequencies ω_1 and ω_2 and growth rates γ_1 and γ_2 can be written as

$$H(s)=\frac{N(s)}{[(s-\gamma_1)^2+\omega_1^2][(s-\gamma_2)^2+\omega_2^2]}\,, \tag{1}$$

where $s=\gamma+i\omega$ with ω and γ the real frequency and growth rate, respectively.

From Fig. 1, with feedback, the system equation is given by

$$1-H(s)\left(k_1+\frac{k_2 s}{\omega_s}\right)=0\,, \tag{2}$$

where $k_1=k_{10}\exp[i\phi_1(\omega)]$, $k_2=k_{20}\exp[i\phi_2(\omega)]$ are the complex feedback gains, and ω_s is the bandwidth of the differentiator.

Determining the exact form of $N(s)$ is a difficult problem which requires an elaborate theory including the dynamics of the coupling of the suppressor to the plasma. From Eq. (2), we note that the choice of $N(s)$ has the same effect on both branches of the controller and so its effect can be nullified if relative values of the feedback gain and phases are considered. Hence, for simplicity, we can take $N(s)=1$ and define the relative gain G_R and phases ϕ_{1R},ϕ_{2R} as $G_R\equiv|k_{20}/k_{10}|$, $\phi_{1R}\equiv\phi_2(\omega_1)-\phi_1(\omega_1)$, and $\phi_{2R}\equiv\phi_2(\omega_2)-\phi_1(\omega_2)$.

Now, we assume linear phase shifters of the form $\phi_i(\omega)=\phi_{i0}\omega$. For marginal stability, setting $s=i\omega_1$ and $s=i\omega_2$ in Eq. (2) results in the following four equations for the four variables k_{10}, ϕ_{10}, k_{20}, and ϕ_{20}:

$$k_{10}\cos[\phi_1(\omega_1)]=\gamma_1^2(\omega_2^2-\omega_1^2+\gamma_2^2)-4\gamma_1\gamma_2\omega_1^2+\frac{k_{20}\omega_1}{\omega_s}\sin[\phi_2(\omega_1)]\,, \tag{3}$$

$$k_{10}\sin[\phi_1(\omega_1)]=-2\gamma_1\omega_1(\omega_2^2-\omega_1^2+\gamma_2^2)-2\gamma_1^2\gamma_2\omega_1-\frac{k_{20}\omega_1}{\omega_s}\cos[\phi_2(\omega_1)]\,, \tag{4}$$

$$k_{10}\cos[\phi_1(\omega_2)]=\gamma_2^2(\omega_1^2-\omega_2^2+\gamma_1^2)-4\gamma_1\gamma_2\omega_2^2+\frac{k_{20}\omega_2}{\omega_s}\sin[\phi_2(\omega_2)]\,, \tag{5}$$

$$k_{10}\sin[\phi_1(\omega_2)]=-2\gamma_2\omega_2(\omega_1^2-\omega_2^2+\gamma_1^2)-2\gamma_2^2\gamma_1\omega_2-\frac{k_{20}\omega_2}{\omega_s}\cos[\phi_2(\omega_2)]\,. \tag{6}$$

We note that there may be many possible solutions to the above set of nonlinear equations. Assuming weak instabilities with γ_1/ω_1, $\gamma_2/\omega_2\ll 1$, $\gamma_1\simeq\gamma_2$, $\gamma_1/\omega_1\simeq 0.1$, and with the experimental parameters for $\omega_s\simeq 3\omega_1$ and $\omega_2/\omega_1\simeq 1.5$, the equations were linearized about trial values for $\omega_1\phi_{10}$ and $\omega_1\phi_{20}$, guided by the experimental results and solved for the four unknowns. This solution was then used as an initial guess for a Newton-Raphson solution to the original nonlinear equations (3)-(6). One solution is given by $\omega_1\phi_{20}\sim 227°$, $\omega_1\phi_{10}\sim 9°$, $k_{10}\sim -0.26$, and $k_{20}\sim 0.92$. Negative gain values are allowed because they correspond simply to a phase shift of π. The relative feedback parameters are thus $G_R\sim 3.5$, $\phi_{1R}\sim 218°$, and $\phi_{2R}\sim 327°$.

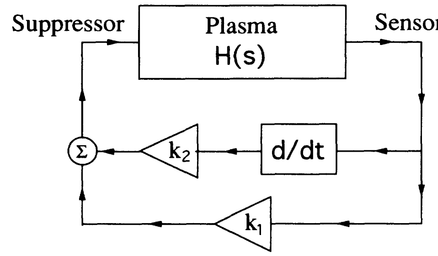

FIG. 1. Model of multimode feedback stabilization with a single sensor-suppressor pair. Another independent signal, representing another "state" of the plasma, is generated by differentiating the original signal.

This experiment was conducted in the Columbia Linear Machine (CLM) [9]. It is a steady-state linear machine with plasma density $\sim 5\times 10^8$ cm^{-3}. The plasma flows from the source region to the experimental cell where the background magnetic field is 1 kG. An independent magnetic mirror coil is also situated in the experimental cell. The plasma is terminated on a conducting endplate attached to the ion-beam source.

This ion-beam source (IBS) has been described elsewhere [10]. It consists of a discharge chamber with an $\mathbf{E}\times\mathbf{B}$ magnetron-type plasma source and dual gridded meshes to extract and modulate the ion beam. The ion-beam energy is governed by the anode bias (discharge chamber wall) which can be varied. The modulation voltage is applied to the screen grid (the inner mesh) biased at around the ion-beam energy. This ion beam is injected axially, along the background magnetic field.

To demonstrate the feasibility of this control scheme of stabilizing multimode plasma instabilities with a single feedback-modulated ion-beam suppressor, two modes were chosen. The ion temperature gradient (ITG) instability was selected because, in CLM, it is usually also accompanied by a rotationally driven $\mathbf{E}\times\mathbf{B}$ mode. The rf transit-time heating method was used to produce the ITG instability [11]. The purpose of the heating was to produce an ion temperature gradient in the radial direction so that the threshold $\eta_{i\parallel}$ for the onset of the instability would be exceeded, where $\eta_{i\parallel}\equiv d\ln T_{i\parallel}/d\ln n$, with n the density and T_i the ion temperature. The magnetic mirror also provided a curvature drive in addition to the $\eta_{i\parallel}$ drive. The $\mathbf{E}\times\mathbf{B}$ mode [12] is usually unstable because of the presence of a radial electric field in CLM resulting in a rotation of the plasma column. This imparts a centrifugal force which destabilizes the mode.

The experimental feedback setup is shown in Fig. 2. The remote ion-beam suppressor, located behind the terminating endplate, was injected around the peak of the mode amplitudes. The plasma density fluctuations due to the instabilities were sensed by a Langmuir probe located around the same radial location. This signal was then processed through the feedback controller. The main section of the feedback controller consisted of two branches.

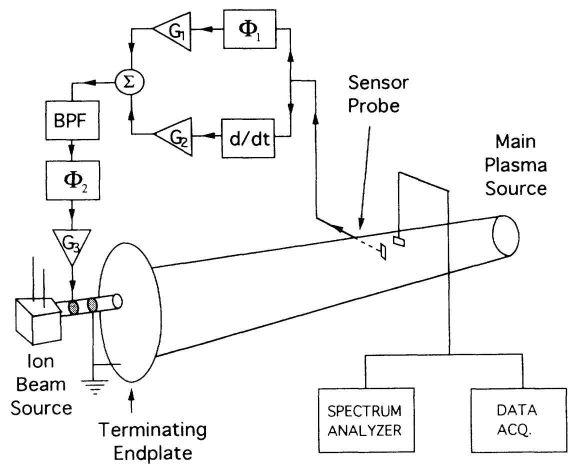

FIG. 2. Schematic of the feedback experimental setup for multimode control. A single feedback-modulated ion beam is the remote suppressor and a single Langmuir probe is the sensor. A second "scanner" Langmuir probe is used to monitor the fluctuations.

One branch contained a phase shifter and an amplifier. The other branch differentiated the same incoming signal before passing it through another amplifier. This phase shifter determined the relative feedback phase between the two branches for given mode frequencies, while the ratio of the two amplifiers gave the relative feedback gain. These two signals were then summed and bandpass filtered. The bandpass filter had a pass band from 30 to 160 kHz, wide enough to pass through both modes. Finally, this reconstructed signal from the superposition of the two signals was amplified and phase shifted again and used to modulate the ion beam via the screen grid. Thus, the sensor signal, appropriately modified by feedback, was carried back into the plasma by the ion beam, completing the feedback loop.

The experimental results are shown below. The ion beam was injected at a radius of 2.0 cm and the Langmuir sensor probe was also positioned at 2.0 cm. A second "scanner" Langmuir probe was then used to monitor the amplitudes of the density fluctuations of the instabilities. The ion-beam energy was 120 eV. This ion-beam

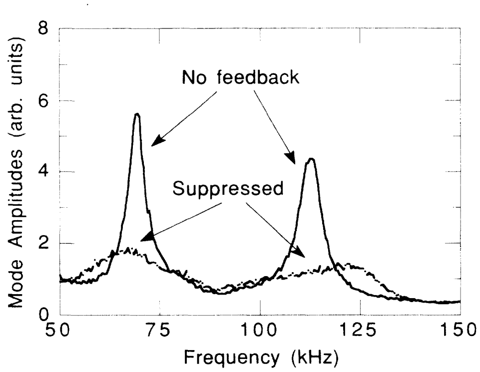

FIG. 3. Spectra of the mode amplitudes for the case of optimal suppression. The mode spectrum without feedback is also indicated.

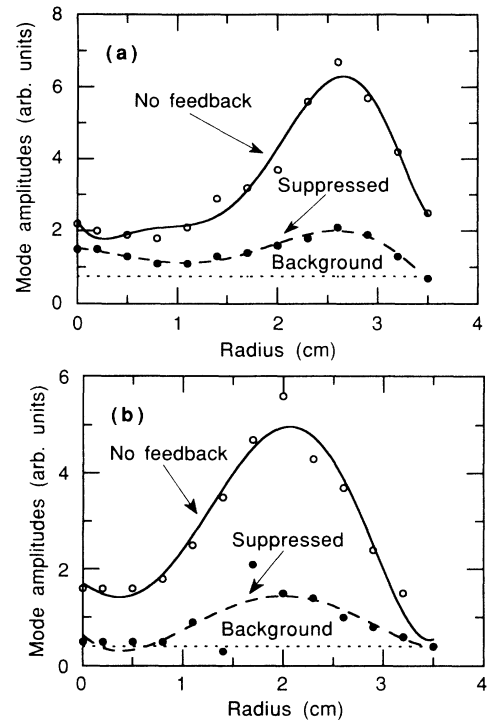

FIG. 4. Radial profiles of the mode amplitudes for the case of optimal suppression. (a) $\mathbf{E}\times\mathbf{B}$ mode corresponding to the mode located at a frequency of 70 kHz. (b) ITG mode corresponding to the mode located at a frequency of 110 kHz.

suppression technique is nonperturbing to the plasma [6] and it was verified that the injection of an unmodulated ion beam did not affect the modes. Plotted in Fig. 3 are the mode spectra with and without feedback obtained from a computer controlled analog spectrum analyzer. The case without feedback corresponds to turning off the modulation of the ion beam. The solid line shows the amplitudes of the modes without feedback. When the feedback is turned on, both modes may be suppressed down to the background noise level, as indicated by the dashed line. Not only has the fluctuational amplitude been reduced, but the overall fluctuational energy has been reduced equally well.

This is also observed in the corresponding radial profiles of the two modes shown in Figs. 4(a) and 4(b). The cases with and without feedback are similarly indicated by the dashed and solid lines, respectively. Figure 4(a) is a plot of the radial mode amplitude for the "first mode," the $m=1$ $\mathbf{E}\times\mathbf{B}$ mode located at a frequency of ~ 70 kHz. Figure 4(b) is a similar plot for the "second mode," the $m=2$ ITG mode located at a frequency of ~ 110 kHz. We note that even though the ion beam is radially localized, the suppression is global.

For optimal multimode suppression, the experimental values for the relative feedback parameters were $G_R \sim 3.6\pm 0.1$, $\phi_{1R}\sim 210^\circ \pm 15^\circ$, and $\phi_{2R}\sim 310^\circ \pm 15^\circ$. These values are in good agreement with the theoretical solution given earlier.

The effect of different suboptimal feedback gain and phase parameters on the mode spectra is shown in Fig. 5.

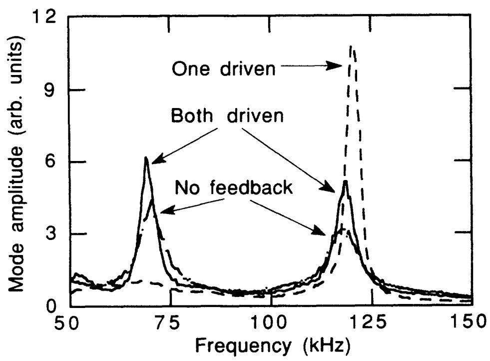

FIG. 5. Spectra for different combinations of feedback parameters: no feedback; both modes driven with $G_R \sim 3.6$, $\phi_{1R} \sim 90°$, and $\phi_{2R} \sim 190°$; and η_i mode driven while $\mathbf{E} \times \mathbf{B}$ is suppressed.

Various combinations of the feedback gains and phases can be set to obtain similar responses. Both modes could be driven simultaneously as shown by the solid curve in Fig. 5 by varying the common phase shifter setting appropriately. The relative feedback parameters in this case were $G_R \sim 3.6$, $\phi_{1R} \sim 90°$, and $\phi_{2R} \sim 190°$. The spectrum without feedback (long-dashed curve) is also shown for comparison. In addition, for other settings of the feedback gains and phases, either one of the modes could be driven while the other mode was suppressed. The spectrum for the case of a driven n_i mode with a suppressed $\mathbf{E} \times \mathbf{B}$ mode is also included in the figure (dashed curve). These results clearly indicate the feedback nature of the interaction of the plasma modes with the feedback system.

In summary, the stabilization scheme using a feedback-modulated ion beam has been extended to multimode control of plasma instabilities. This experiment demonstrated multimode feedback stabilization with a single sensor-suppressor pair. Two modes, the ITG and the $\mathbf{E} \times \mathbf{B}$ instabilities, were stabilized simultaneously. This was effected using a simple state-feedback-type method where more state information was generated from a single-sensor Langmuir probe by differentiation. A single feedback-modulated ion beam served as the remote suppressor. The radial profiles of both modes when they were suppressed indicated that the total fluctuations could be reduced to the background noise level. A simple theory shows good agreement with the experiment.

Our particular setup is not suitable for extension to more than a few modes because differentiation accentuates noise and cannot be performed too often if signal fidelity is to be retained. This problem may be resolved by resorting to more sophisticated designs for the feedback controller [8]. Hence, this experiment gives promising indications that feedback control may be a viable technique to stabilize plasma instabilities which are believed to be responsible for anomalous transport and nonuniformities in plasmas. In a wider sense, this experiment can be considered as a paradigm demonstrating the viability of controlling any continuum medium via a single discrete sensor and suppressor pair. It is also noted that the conceptual and experimental methodologies described here are applicable to certain chaotic systems which are driven by linear instabilities.

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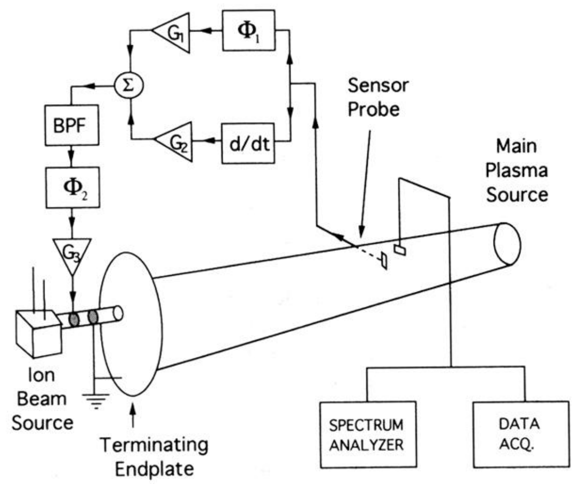

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